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Measurement of Inclusive $\mathbf{f_1(1285)}$ and $\mathbf{f_1(1420)}$ Production in Z Decays with the DELPHI Detector

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Abstract

DELPHI results are presented on the inclusive production of two $(K\bar{K}\pi)^0$ states in the mass region 1.22–1.56 GeV/c^2 in hadronic Z decays at LEP I. The measured masses (widths) are 1274 ± 6 MeV/c^2 (29 ± 12 MeV/c^2) and 1426 ± 6 MeV/c^2 (51 ± 14 MeV/c^2) respectively. A partial-wave analysis of the $(K\bar{K}\pi)^0$ system shows that the first peak is consistent with the quantum numbers $I^G(J^{PC}) = 0^+(0^{-+}/1^{++})$ and the second with $I^G(J^{PC}) = 0^+(1^{++})$. The total hadronic production rates per hadronic Z decay are (0.159 ± 0.051) and (0.051 ± 0.011) respectively. These measurements are consistent with the two states being the $f_1(1285)$ and $f_1(1420)$ mesons.

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1 Introduction

The inclusive production of mesons has been a subject of long-standing study at LEP, as it provides insight into the nature of fragmentation of quarks and gluons into hadrons. So far studies have been done on the S-wave mesons (both $^1S_0$ and $^3S_1$) such as π and ρ, as well as certain P-wave mesons $f_2(1270)$ and $K_2^*(1430)$ (i.e. $^3P_2$) and $f_0(980)$ and $a_0(980)$ ($^3P_0$) [1, 11]. Very little is known about the production of mesons belonging to other P-wave multiplets (i.e. $^3P_1$ and $^1P_1$). For the first time, we present in this paper a study of the inclusive production of two $J^{PC} = 1^{++}$ mesons, the $f_1(1285)$ and the $f_1(1420)$ (i.e. $^3P_1$).

There are at least four known nonstrange isoscalar mesons [2], $I^G(J^{PC}) = 0^+(1^{++})$ and $I^G(J^{PC}) = 0^+(0^{-+})$, in the mass region between 1.2 and 1.5 GeV/c^2, which couple strongly to the decay channel $(K\bar{K}\pi)^0$. These are $f_1(1285)$, $\eta(1295)$, $f_1(1420)$ and $\eta(1440)$. All are seen prominently in the peripheral production from π^-p interactions [2], indicating that, despite their decay into $(K\bar{K}\pi)^0$, they are mostly $n\bar{n}$ states, where $n = \{u, d\}$. There exist possibly two additional states, $I^G(J^{PC}) = 0^-?(1^{+-})$ $h_1(1380)$ and $I^G(J^{PC}) = 0^+(1^{++})$ $f_1(1510)$, which may harbor a large $s\bar{s}$ content, as they are produced with considerable cross-sections in the peripheral reactions involving K^-p interactions [2]. Given this complexity in the $(K\bar{K}\pi)^0$ systems, it is important to find which resonances among these are readily excited in inclusive hadron Z decays.

The DELPHI data for this study are based on the neutral $K\bar{K}\pi$ channel in the reaction

$$Z \to (K_s K^{\pm}\pi^{\mp}) + X^0 \tag{1}$$

Section 2 is devoted to the selection process for the event sample collected for this study. The $K\bar{K}\pi$ mass spectra is studied in Section 3. It is shown that selection of the events with low $M(K_s\,K^{\pm})$ mass is the crucial criterion to reveal the presence of two signals in the $f_1(1285)$ and $f_1(1420)$ mass regions. A partial-wave analysis, carried out to explore the spin-parity content of the two signals, is described in Section 4. The measurement of the production rates and differential cross sections is presented in Section 5. Conclusions are given in Section 6.

2 Experimental Procedure

The analysis presented here is based on a data sample of 3.4 million hadronic Z decays collected from 1992 to 1995 with the DELPHI detector. A detailed description of the DELPHI detector and its performance can be found elsewhere [3, 4].

The charged particle tracks have been measured in the 1.2 T magnetic field by a set of tracking detectors. The average momentum resolution for charged particles in hadronic final states, $\Delta p/p$, is usually between 0.001 and 0.01, depending on their momentum as well as on which detectors are included in the track fit.

A charged particle has been accepted in this analysis if its momentum p is greater than 100 MeV/c, its momentum error $\Delta p/p$ is less than 1 and its impact parameter with respect to the nominal crossing point is within 4 cm in the transverse (xy) plane and 4 cm/$\sin\theta$ along the beam direction (z-axis), θ being the polar angle of the track.

Hadronic events are then selected by requiring at least 5 charged particles, 3 GeV as minimum energy of the charged particles in each hemisphere of the event (defined with respect to the beam direction) and total energy of the charged particles of at least 12% of the center-of-mass energy. The contamination from events due to beam-gas scattering and to γ-γ interactions is estimated to be less than 0.1% and the background from $\tau^+\tau^-$ events is less than 0.2% of the accepted events.

After the event selection, in order to ensure a better signal-to-background ratio for the resonances in the $K_S K^{\pm}\pi^{\mp}$ invariant mass system, tighter requirements have been imposed on the track impact parameters with respect to the nominal crossing point, i.e. they have to be within 0.2 cm in the transverse plane and 0.4 cm/$\sin\theta$ along the beam direction.

$K^{\pm}$ identification has been provided by the Barrel Ring Imaging Cherenkov (BRICH) detector for particles with momenta above 700 MeV/c, while the ionization loss measured in the Time Projection Chamber (TPC) has been used for momenta above 100 MeV/c. The corresponding identification tags are based on the combined probabilities, derived from the average Cherenkov angle and the number of observed photons in the BRICH detector, as well as the measured $\mathrm{d}E/\mathrm{d}x$ in the TPC. Cuts on the tags have been applied to achieve the best signal-to-background ratio, while rejecting $e^{\pm}$, $\mu^{\pm}$, p and $\bar{p}$ tracks. A more detailed description of the identification tags can be found in Ref. [1]. In the present case, the $K^{\pm}$ identification efficiency (typically 50% over the kaon momentum range of this analysis [5]) has been estimated by comparing the ϕ (1020) to K^+K^- signal in the experimental data with a sample of simulated events generated with JETSET [6] tuned with the DELPHI parameters [7] and passed through the detector simulation program DELSIM [4]. Agreement within 4% is observed between the data and the simulation.

The K_S candidates are detected by their decay in flight into $\pi^+\pi^-$. The details of the reconstruction method and the various cuts applied are described in Ref. [8]. Our selection process consists of taking the V^0's passing the standard criteria for quality of the reconstruction plus a mass cut given by $0.45 < M(\pi^+\pi^-) < 0.55$ GeV/c^2.

After all the above cuts, only events with at least one $K_S K^+\pi^-$ or $K_S K^-\pi^+$ combination have been kept in the present analysis, corresponding to a sample of 547k events.

3 $K_S K^{\pm}\pi^{\mp}$ Mass Spectra

The $K_S\, K^{\pm}\, \pi^{\mp}$ invariant mass distribution is shown in Fig. 1. Also shown in the figure is the same mass spectrum with a $K^*(892)$ selection ($0.822 < M(K\pi) < 0.962$ GeV/c^2), which would be appropriate if the decay of a resonance had proceeded through a $K^*(892)$ intermediate state. Neither histogram shows a visible enhancement in the mass region between 1.25 to 1.45 GeV/c^2. This is due to the enormous background in this mass region coming from the high number of $K_S K^{\pm}\pi^{\mp}$ combinations per event (11 on average) in inclusive Z decays. The key to a successful study of the $f_1(1285)$ and $f_1(1420)$ under the circumstances is to select events with low $M(K_S\, K^{\pm})$ mass. This has the effect of selecting both the decay mode $a_0(980)^{\pm}\pi^{\mp}$ while reducing substantially the general background for the $K\bar{K}\pi$ system, and the interference region of the two $K^*(892)$ bands on the Dalitz plot. Varying this cut on the Monte Carlo generated events suggests a mass cut $M(K_S\, K^{\pm}) \leq 1.04$ GeV/c^2 to maximize both $f_1(1285)$ and $f_1(1420)$ signals over background. The application of this cut on the experimental data is shown in Fig. 2, where two clear peaks are now seen in this mass region. Based on the Monte Carlo generated

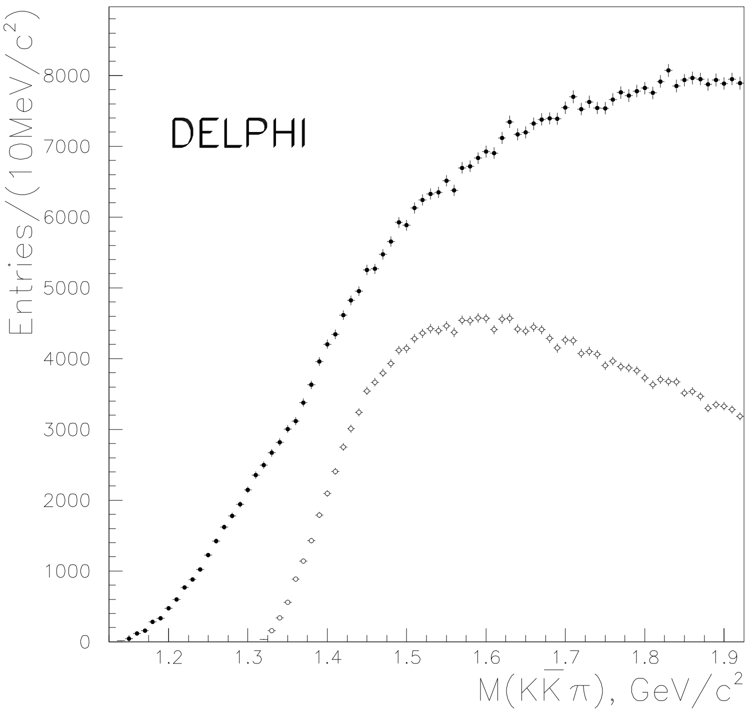

Figure 1: $K_s\, K^{\pm}\, \pi^{\mp}$ invariant mass distributions from the Z decays with the DELPHI detector at LEP I. The histogram with solid circles is for the full data sample, that one with open circles is for data with a $0.822 < M(K\pi) < 0.962$ GeV/c^2 K^* selection.

event sample, we have verified that none of the two signals was a reflection of resonances whose mass is in the 1.0 to 1.5 GeV/c^2 range, such as the ϕ (1020) and the $K_1(1270)$ mesons or was faked by a possible misidentification of kaons or pions coming from the decay of these resonances.

To estimate the background under the signals, we have used the Monte Carlo generated events from which we have removed all mesons with a major decay mode into $(K\bar{K}\pi)^0$ in the mass region 1.25 to 1.45 GeV/c^2, i.e. $f_1(1285)$, $h_1(1380)$ and $f_1(1420)$. The smooth curve shown in Fig. 2 has been obtained by fitting the mass spectrum of the aforementioned Monte Carlo sample between 1.15 to 1.7 GeV/c^2 with a background function

$$f_b(M) = (M - M_0)^{\alpha_1} \exp(\alpha_2\, M + \alpha_3\, M^2) \tag{2}$$

where M and M_0 are the effective masses of the $(K\bar{K}\pi)^0$ system and its threshold, respectively, and α_i are the fitted parameters.

Then we have fitted the $(K\bar{K}\pi)^0$ spectrum adding two S-wave Breit-Wigner forms to the background $f_b(M)$, given by

$$f_r(M) = \frac{\Gamma_r^2}{(M - M_r)^2 + (\Gamma_r/2)^2} \tag{3}$$

where M_r and Γ_r are respectively the mass and the width to be determined experimentally. The results are shown in Fig. 2 and also in Table I.

Table I. Fitted parameters and numbers of events

Mass (MeV/c^2)	Width (MeV/c^2)	Events
1274 ± 6	29 ± 12	358 ± 93 (stat) $\pm$ 59 (sys)
1426 ± 6	51 ± 14	870 ± 128 (stat) $\pm$ 136 (sys)

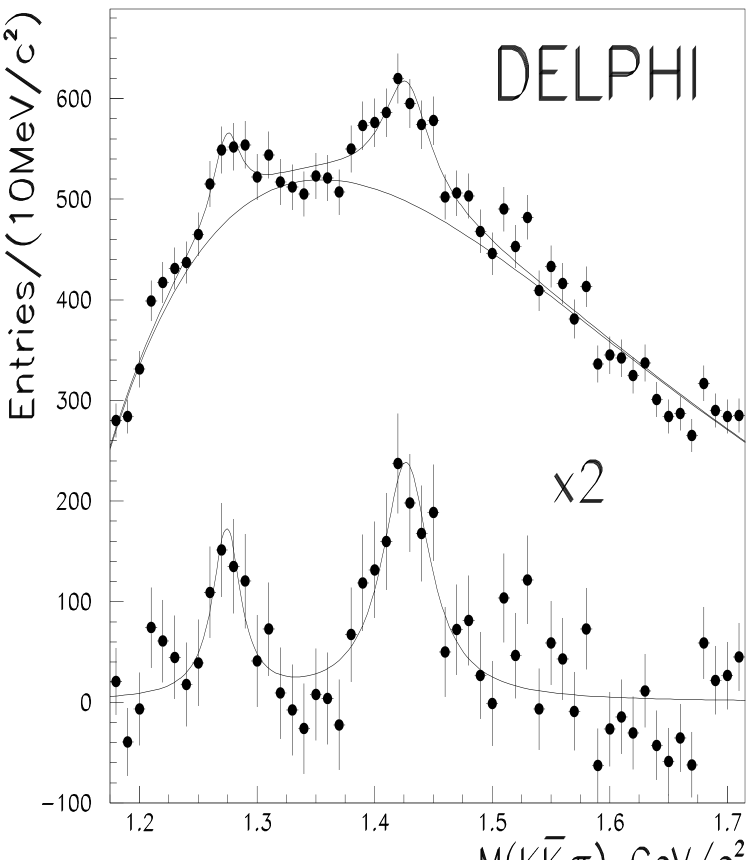

Figure 2: Invariant mass distributions for the system $K_S\, K^{\pm}\, \pi^{\mp}$ with a mass cut $M(K_S\, K^{\pm}) < 1.04$ GeV/c^2. The two solid curves in the upper part of the histogram describe Breit-Wigner fits over a smooth background (see text). The lower histogram and the solid curve give the same fits with the background subtracted and amplified by a factor of two.

Taking into account the mass resolution (8 and 9 MeV/c^2 in the 1280 and 1420 MeV/c^2 mass regions respectively), one sees that the fitted masses and widths are well compatible with the PDG [2] values for the $f_1(1285)$ and $f_1(1420)$ resonances and, for the first peak, not with those of the $\eta(1295)$. The number of events in Table I correspond to a fit where the width of the two peaks has been fixed to the fitted value and the background parameters left free.

The main sources of systematic uncertainty come from the various cuts and selection criteria applied for the V^0 reconstruction and the charged K identification on the one hand and the conditions of the fit procedure on the other. To estimate the first type of error, we have compared the $K_s K^{\pm}$ mass distributions of the simulated sample with the real data. Normalized to the same number of events, both distributions agree within 7%, in the low $K_s K^{\pm}$ mass region.

The $f_1(1285)$ and $f_1(1420)$ signals show up over a large background ($\sim 80\%$). Variations of the background shape and amplitude induce sizable fluctuations of the fitted numbers of signal events. To quantify this effect, we have performed various series of fits, one varying the mass range of the fit, another leaving free the background parameters while fixing the width of the signals, another with a polynomial shape for the background, thereby allowing the background level and shape to fluctuate. In this way we estimate the uncertainty of the number of fitted events to be 15% for the $f_1(1285)$ and 14% for the $f_1(1420)$. The systematic uncertainties have been added quadratically and are shown in Table 1.

The overall efficiencies for the two states have been estimated from the Monte Carlo simulated events to be:

$$\begin{array}{rcll} (0.063 & \pm & 0.003)\% & \text{for} \quad f_1(1285) \\ (0.45 & \pm & 0.02)\% & \text{for} \quad f_1(1420) \end{array} \tag{4}$$

The quoted numbers include the following corrections for the $f_1(1285)$ and $f_1(1420)$ respectively: branching ratios to $K\bar{K}\pi$ $(0.09, 1.)$, fractions of final states with charged pion $(1/2, 2/3)$, branching ratio of $K^0 \to \pi^+\pi^- = 1/2 \times 0.686 = 0.343$, reconstruction and identification efficiency for the selected events $(0.058, 0.061)$ and $M(K_s\, K^{\pm}) \leq 1.04$ GeV/c^2 mass cut correction factor $(0.70, 0.32)$.

4 Partial-wave Analysis

In order to get more information on the spin-parity content of the two signals we have performed a partial-wave analysis of the $K_s K^{\pm}\pi^{\mp}$ system. There have been many 3-body partial-wave analyses; the reader may consult PDG [2] for earlier references, for example, on $a_1(1260)$, $a_2(1320)$, $K_1(1270/1400)$ or $K_2(1770)$. For the first time, we apply the same technique to a study of the $(K\bar{K}\pi)^0$ system from the inclusive decay of the Z at LEP.

A spin-parity analysis of the system composed of three pseudoscalars requires five variables, which may be chosen to be the three Euler angles describing the orientation of the 3-body system in its suitably-chosen rest frame and two effective masses. We have chosen to employ the so-called Dalitz plot analysis, integrating over the three Euler angles. This entails an essential simplification in the number of parameters required in the analysis, as the decay amplitudes involving the D-functions defined over the three Euler angles and their appropriate decay-coupling constants, are orthogonal for different spins and parities [9]. The actual fitting of the data is done by using the maximum-likelihood method, in which the normalization integrals are evaluated with the Monte Carlo events [10], thus taking into account the finite acceptance of the detector and the event selection.

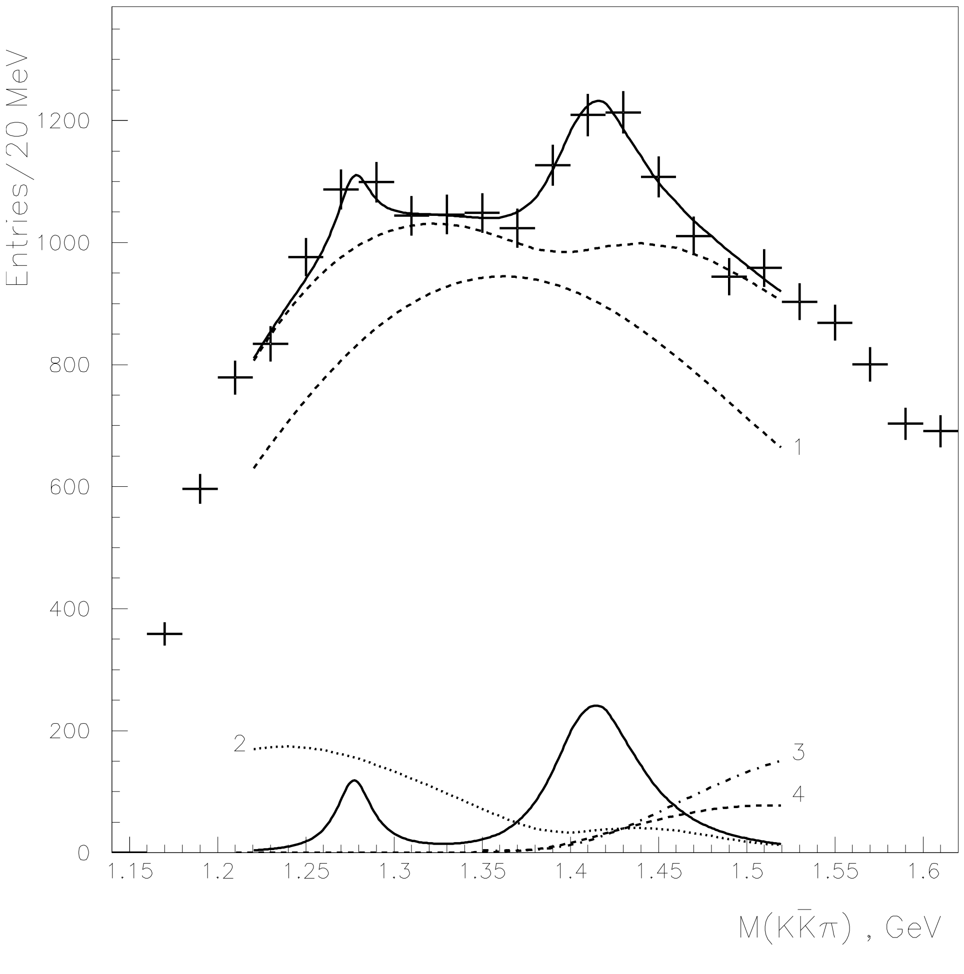

Figure 3: $M(K_s\,K^{\pm}\,\pi^{\mp})$ distributions per 20 MeV/c^2 with a breakdown into the partial-waves for the signals and the background. The signals consist of $1^{++}\,a_0(980)\pi$ for the first peak and $1^{++}\,K^{*}(892)\bar{K}$ for the second peak. The background consists of a non-interfering superposition of an isotropic distribution (1) and of non resonant $1^{++}\,a_0(980)\pi$ (2), $1^{++}\,K^{*}(892)\bar{K}$ (3) and $1^{+-}\,K^{*}(892)\bar{K}$ (4) waves.

The background under the two f_1's is very large. It is assumed that this represents essentially different processes with, for example, different overall multiplicities so that the background does not interfere with the signals. We assume further that the background itself is a non-interfering superposition of a constant three-body phase space term and the partial waves $I^G(J^{PC}) = 0^+(1^{++})\, a_0(980)\pi$, $0^+(1^{++})\,(K^*(892)\bar{K} + c.c.)$ and $0^-(1^{+-})\,(K^*(892)\bar{K} + c.c.)$. We have verified that these amplitudes give a good description of the three background regions for $M(K\bar{K}\pi)$ in $1.22 \to 1.26$, $1.30 \to 1.38$ and $1.48 \to 1.56$ GeV/c^2, respectively.

The signal regions, for $M(K\bar{K}\pi)$ in $1.26 \to 1.30$ and $1.38 \to 1.48$ GeV/c^2, have been fitted, one by one, with a non-interfering superposition of the partial waves $I^G(J^{PC}) = 0^+(1^{++})$, $0^+(1^{+-})$ and $0^+(0^{-+})$, where the decay channels $a_0(980)\pi$ and $K^*(892)\bar{K} + c.c.$ are allowed to interfere within a given J^{PC}. All other possible partial waves have been found to be negligible in the signal regions. The fit results can be summarized as follows: (1) the maximum likelihood is found to be the same for $I^G(J^{PC}) = 0^+(1^{++})\, a_0(980)\pi$ and for $0^+(0^{-+})\, a_0(980)\pi$, i.e. the 1.28 GeV/$c^2$ region is equally likely to be the $f_1(1285)$ or the $\eta(1295)$; (2) in the 1.4 GeV/c^2 region, the maximum likelihood is significantly better (by about 14 for $\Delta \ln \mathcal{L}$) for $I^G(J^{PC}) = 0^+(1^{++})\; f_1(1420)$ than $I^G(J^{PC}) = 0^+(0^{-+})$ which excludes the $\eta(1440)$. The $I^G(J^{PC}) = 0^-(1^{+-})\; h_1(1380)$ is also ruled out in this analysis (by about 23 for $\Delta \ln \mathcal{L}$).

The results of the partial-wave analysis are summarized in Fig. 3. It should be emphasized that both the fit of the $K_S\, K^{\pm}\pi^{\mp}$ mass spectrum described in Section 2 and the PWA mass-dependent fit give compatible results. The masses, the widths and the numbers of events found in the PWA fit, shown as solid curves in Fig. 3, are statistically consistent with those given in Table I, even though the overall background shape—see the dotted curve just below the histogram with error bars—exhibits a slight dip around 1.4 GeV/c^2 which is *not present* in Fig. 2.

5 Production Rates and Differential Cross Sections

We have measured the production rate $\langle n \rangle$ per hadronic Z decay for $f_1(1285)$ and $f_1(1420)$. The results are

$$\begin{aligned} \langle n \rangle &= 0.165 \pm 0.051 \quad \text{for} \quad f_1(1285) \\ \langle n \rangle &= 0.056 \pm 0.012 \quad \text{for} \quad f_1(1420) \end{aligned} \tag{5}$$

taking a $K\bar{K}\pi$ branching ratio of $(9.0 \pm 0.4)\%$ for the $f_1(1285)$ and 100% for the $f_1(1420)$ [2]. The total production rate per spin state [i.e. divided by $(2J + 1)$] has been studied in Ref. [11]. In Fig. 4 is given all the available data for those mesons with a 'triplet' $q\bar{q}$ structure, i.e. $S = 1$. To this figure we have added our two mesons for comparison. It is seen that both $f_1(1285)$ and $f_1(1420)$ come close to the line corresponding to mesons whose constituents are thought to be of the type $n\bar{n}$. This suggests that both $f_1(1285)$ and $f_1(1420)$ have little $s\bar{s}$ content. Indeed, the two states which are thought to be pure $s\bar{s}$ mesons, the ϕ and the $f_2'(1525)$, are down by a factor $\gamma^k \approx 1/4$ ($\gamma = 0.50 \pm 0.02$ and $k = 2$, k being the number of s and $\bar{s}$ quarks in the meson), as shown in Fig. 4. A high strange quark content is highly unlikely given the production rate (5).

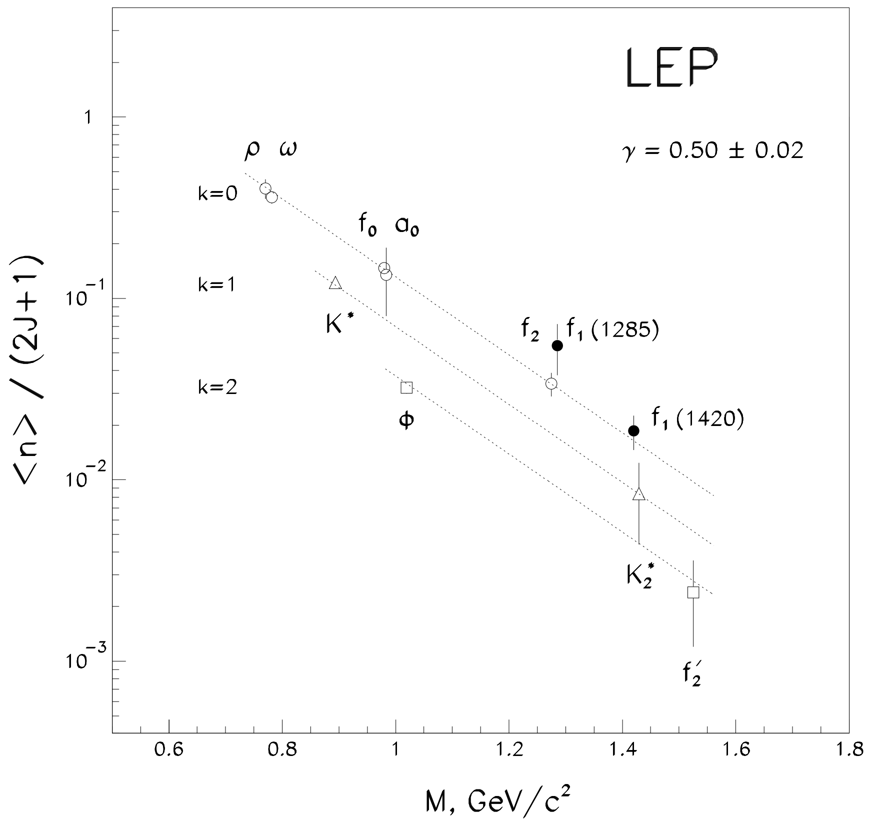

Figure 4: Total production rate per spin state and isospin for scalar, vector and tensor mesons as a function of the mass (open symbols). The two solid circles correspond to the $f_1(1285)$ and the $f_1(1420)$.

For completeness, we give in Fig. 5 and in Table II the $f_1(1285)$ and $f_1(1420)$ differential rates and cross-sections as a function of the scaled momentum x_p ($x_p = p_{K\bar{K}\pi^0}/p_{beam}$), for $x_p > .05$ as the signal to background ratio is too small for lower momenta. Comparison with JETSET predictions is not possible in a meaningful way as there was no tuning for $f_1(1285)$ and $f_1(1420)$ and the implementation of the $(K\bar{K}\pi)^0$ decay of both resonances in JETSET is according to phase space i.e not according to the correct spin-parity matrix element.

Table II. Measured production rates per hadronic event and differential cross-sections for the $f_1(1285)$ and $f_1(1420)$. The errors are the quadratic sum of statistical and systematic uncertainties

x_p range	$f_1(1285)$ rate	$(1/\sigma)(d\sigma/dx_p)$
.05-.10	0.038±0.021	0.76±0.43
.10-.20	0.056±0.024	0.56±0.24
.20-1.0	0.056±0.023	0.07±0.03
x_p range	$f_1(1420)$ rate	$(1/\sigma)(d\sigma/dx_p)$
.05-.10	0.017±0.005	0.33±0.10
.10-.20	0.017±0.004	0.17±0.04
.20-1.0	0.018±0.006	0.02±0.01

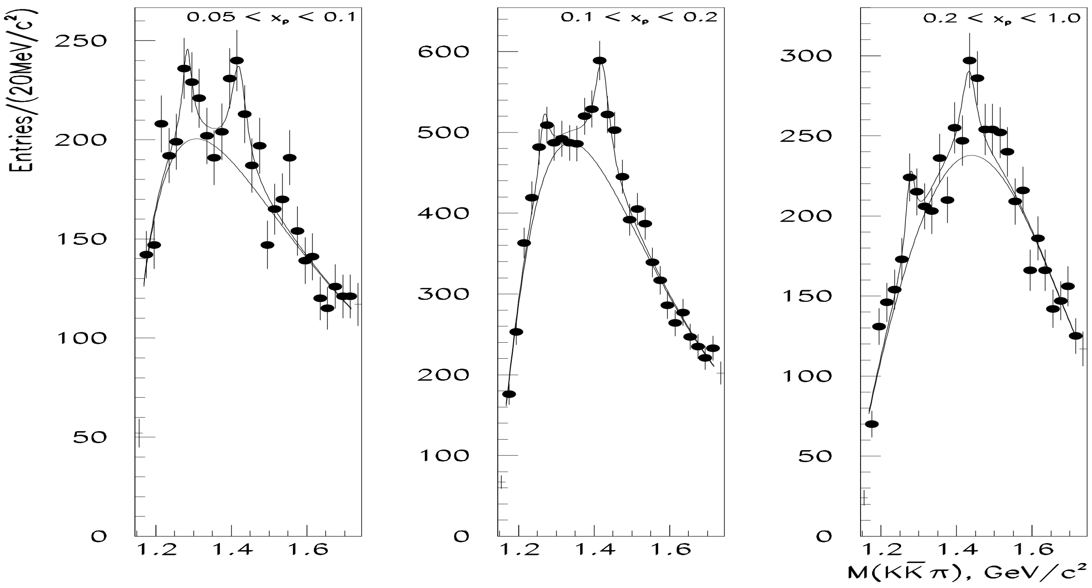

Figure 5: The $K_S\,K^{\pm}\,\pi^{\mp}$ invariant mass spectra for various x_p ranges as indicated. Dots are the data, the solid lines show the result from the fit and the background contribution.

6 Conclusions

We have studied the inclusive production of two $(K\bar{K}\pi)^0$ states in Z decays at LEP I. The measured masses and widths are 1274 ± 6 and 29 ± 12 MeV/c^2 for the first peak and 1426 ± 6 and 51 ± 14 MeV/c^2 for the second one, compatible with those of the $f_1(1285)$ and $f_1(1420)$ mesons [2]. For the first time, a partial-wave analysis has been carried out on the $(K\bar{K}\pi)^0$ system from the inclusive Z decay. While the results cannot disentagle between $I^G(J^{PC}) = 0^+(1^{++})$ and $0^+(0^{-+})$ in the 1.28 GeV/c^2 region, the second peak is consistent with $I^G(J^{PC}) = 0^+(1^{++})$. On the other hand, the comparison of the hadronic production rate of these two states with a previous study of the production rate [11] for the $S = 1$ mesons (which included $^3S_1$, $^3P_0$ and $^3P_2$) suggests that their quantum numbers are very probably $I^G(J^{PC}) = 0^+(1^{++})$ and that their quark constituents are mainly of the type $n\bar{n}$, where $n = \{u, d\}$ and thus confirms that these states are very likely the $f_1(1285)$ and $f_1(1420)$ mesons. Finally, we conclude that the mesons $\eta(1295)$, $\eta(1440)$ and $h_1(1380)$ are less likely to be produced in the inclusive Z decays compared to $f_1(1285)$ and $f_1(1420)$.

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