SUSY GRAND UNIFIED THEORIES

K.S. BABU
OKLAHOMA STATE UNIVERSITY

ICHEP 2002
AMSTERDAM

OUTLINE

- MOTIVATIONS
- SYMMETRY BREAKING
- QUARK AND LEPTON MASSES
- PROTON DECAY
- REALISTIC EXAMPLES

Structure of Matter Multiplets

$$\boxed{Q} = \begin{pmatrix} u_1 & u_2 & u_3 \\ d_1 & d_2 & d_3 \end{pmatrix} \sim (3, 2, \frac{1}{6})$$

$$\boxed{u^c} = (\, u_1^c \quad u_2^c \quad u_3^c \,) \sim (\overline{3}, 1, \frac{-2}{3})$$

$$\boxed{d^c} = (\, d_1^c \quad d_2^c \quad d_3^c \,) \sim (\overline{3}, 1, \frac{1}{3})$$

$$\boxed{L} = \begin{pmatrix} \nu \\ e^- \end{pmatrix} \sim (1, 2, \frac{-1}{2})$$

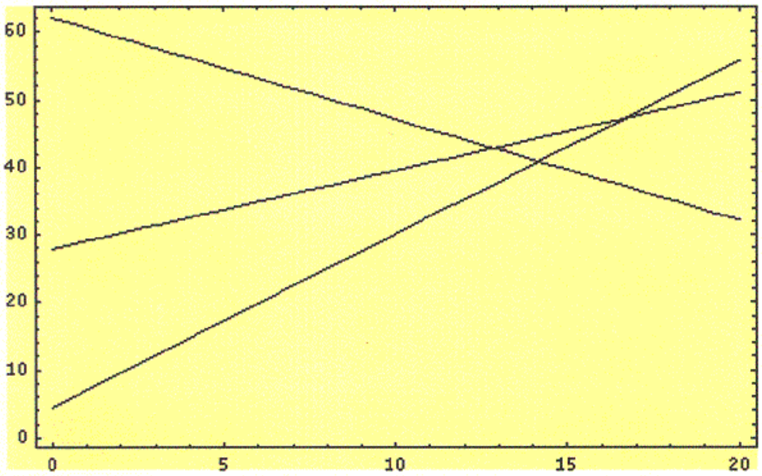
$$\boxed{e^c} \qquad \sim (1, 1, +1)$$

$$\boxed{\nu^c} \qquad \sim (1, 1, 0)$$

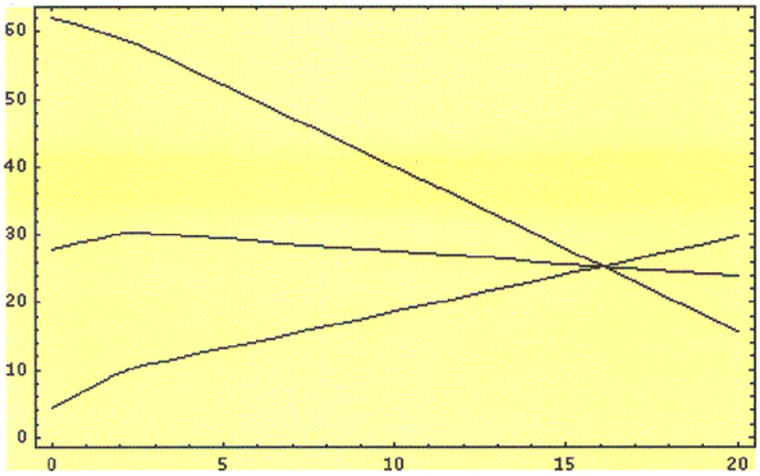
$$
\begin{array}{rcl}
u_1 & : & |\uparrow\downarrow\uparrow\uparrow\downarrow> \\
u_2 & : & |\uparrow\downarrow\uparrow\downarrow\uparrow> \\
u_3 & : & |\uparrow\downarrow\downarrow\uparrow\uparrow> \\
d_1 & : & |\downarrow\uparrow\uparrow\uparrow\downarrow> \\
d_2 & : & |\downarrow\uparrow\uparrow\downarrow\uparrow> \\
d_3 & : & |\downarrow\uparrow\downarrow\uparrow\uparrow> \\
u_1^c & : & |\downarrow\downarrow\uparrow\downarrow\downarrow> \\
u_2^c & : & |\downarrow\downarrow\downarrow\uparrow\downarrow> \\
u_3^c & : & |\downarrow\downarrow\downarrow\downarrow\uparrow> \\
d_1^c & : & |\uparrow\uparrow\uparrow\downarrow\downarrow> \\
d_2^c & : & |\uparrow\uparrow\downarrow\uparrow\downarrow> \\
d_3^c & : & |\uparrow\uparrow\downarrow\downarrow\uparrow> \\
\nu & : & |\uparrow\downarrow\downarrow\downarrow\downarrow> \\
e & : & |\downarrow\uparrow\downarrow\downarrow\downarrow> \\
e^c & : & |\downarrow\downarrow\uparrow\uparrow\uparrow> \\
\nu^c & : & |\uparrow\uparrow\uparrow\uparrow\uparrow>
\end{array}
$$

Experimental evidence for supersymmetry: the running of coupling constants for the Standard Model and its minimal supersymmetric extension

The Standard Model



MSSM



These figures are taken from U. Amaldi, W. de Boer, and H. Furstenau, *Phys. Lett.* **B260** (1991) 447-455. For the MSSM, the slope of the running of the coupling constant has a discontinuity at M_{SUSY}. Here, M_{SUSY} was adjusted such that all three coupling constants will actually meet at some M_{GUT}. Because we had one free paramtameter to adjust, the fact that coupling constant converged at a single point alone is not especially remarkable. What makes MSSM special is the fact that it predicts the proton decay life time of the order $10^{33.2+/-1.2}$ yr which does not contradict the experimental limits of $t > 10^{32}$ yr.

MINIMAL SU(5)

QUARKS AND LEPTONS $\sim \{10_i + \bar{5}_i\}$

$$10_i = \begin{bmatrix} 0 & u_3^c & -u_2^c & u_1 & d_1 \\ & 0 & u_1^c & u_2 & d_2 \\ & & 0 & u_3 & d_3 \\ & & & 0 & e^c \\ & & & & 0 \end{bmatrix} \qquad \bar{5}_i = \begin{bmatrix} d_1^c \\ d_2^c \\ d_3^c \\ e^- \\ \nu \end{bmatrix}$$

HIGGS $\sim \{24_H + 5_H + \bar{5}_H\}$

$$\mathcal{L}_{YUKAWA} = f_{ij}\, 10_i\, 10_j\, 5_H + h_{ij}\, 10_i\, \bar{5}_j\, \bar{5}_H$$

$$M_{up} = f v_u \qquad M_d = h v_d$$

$$M_\ell = M_d^T$$

$$\Rightarrow \quad \boxed{m_b = m_\tau} \qquad m_s = \frac{m_\mu}{3} \qquad m_d = 3 m_e$$

SO(10)

- QUARKS AND LEPTONS $\sim \{16_i\}$
- CONTAINS ν_R AND SEESAW MECHANISM
- HIGGS $\sim \{45_H + 16_H + \overline{16}_H + 10_H\}$

$$\mathcal{L}_{YUKAWA} = f_{ij}\, 16_i\, 16_j\, 10_H$$

$$\Rightarrow \quad M_{up} = M_\nu^D \qquad M_d = M_\ell$$

ALSO $M_{up} \propto M_{down}$

$$\mathcal{L}_{Majorana} = h_{ij}\, 16_i\, 16_j\, \overline{16}_H\, \overline{16}_H / M_{Pl}$$

$$\Rightarrow \quad m^D_{\nu_\tau} \simeq m_t$$

$$m^M_{\nu_{\tau R}} \simeq h_{33} \frac{M^2_{GUT}}{M_{Pl}}$$

$$\Rightarrow \quad m_{\nu_\tau} = \frac{m_t^2}{m^M_{\nu_{\tau R}}} \simeq 0.05\ \mathrm{eV} \quad (h_{33} \sim 1)$$

FITS ATMOSPHERIC ν- DATA WELL.

MSSM HIGGS DOUBLETS HAVE

COLOR TRIPLET PARTNERS IN GUTs

$H \; (1,2,\tfrac{1}{2}) \oplus H_c \, (3,1,-\tfrac{1}{3}) = 5$ OF $SU(5)$

$\bar{H} \; (1,2,-\tfrac{1}{2}) \oplus \bar{H}_c \, (\bar{3},1,\tfrac{1}{3}) = \bar{5}$

- $H, \bar{H}$ MUST REMAIN LIGHT $\sim M_W$
- $H_c, \bar{H}_c$ MUST HAVE GUT SCALE MASS

TO PREVENT RAPID PROTON DECAY

$\Downarrow$

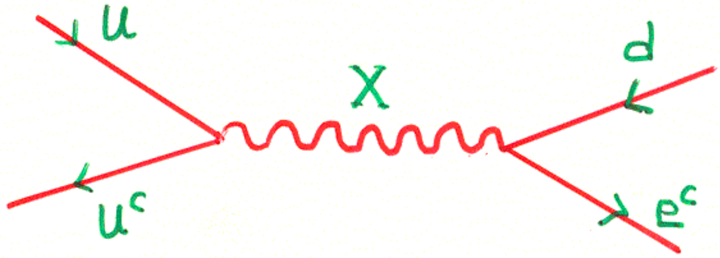
DOUBLET TRIPLET SPLITTING

EVEN IF COLOR TRIPLETS HAVE GUT SCALE

MASS, $d = 5$ PROTON DECAY PROBLEMATI

NUCLEON DECAY IN SUSY GUTs

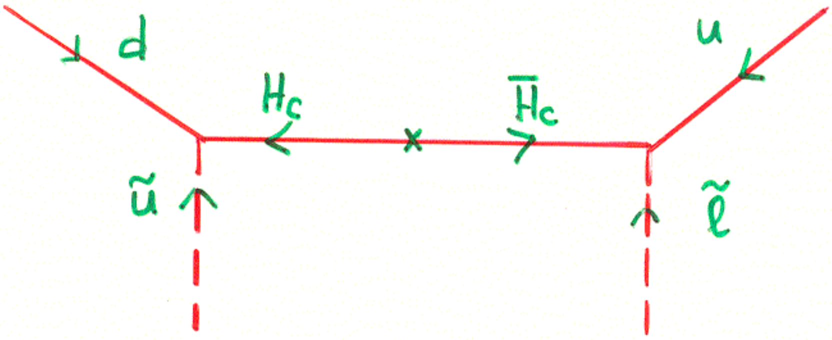
1.

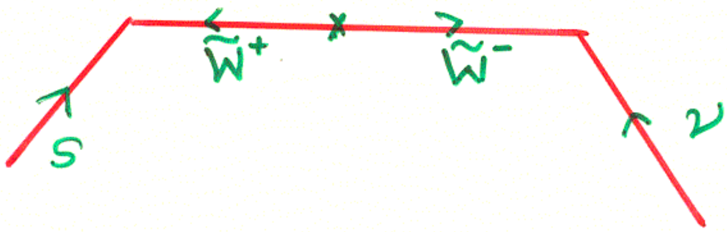


$p \to e^+ \pi^0$ $$\tau_p^{-1} \approx \left[\frac{g^2}{M_X^2}\right]^2 m_p^5 = \left[10^{36\pm1}\ yr\right]^{-1}$$

2. HIGGSINO EXCHANGE

Sakai, Yanagida
Weinberg





$p \to \bar{\nu} K^+$

$$\tau_p^{-1} \approx \left[\frac{f^2}{M_{H_c} M_{SUSY}}\right]^2 \left(\frac{\alpha}{4\pi}\right)^2 m_p^5$$

$$\approx \left[\left(10^{28} - 10^{34}\right) \text{yr}\right]^{-1}$$

SYMMETRY BREAKING

SUSY SU(5)

$$W_{D\text{-}T} = \bar{5}_H \left(\lambda\, 24_H + M \right) 5_H$$

$$\langle 24_H \rangle = \begin{bmatrix} 1 & & & & \\ & 1 & & & \\ & & 1 & & \\ & & & -\frac{3}{2} & \\ & & & & -\frac{3}{2} \end{bmatrix} V$$

$$M_{H_C} = \lambda V + M \longrightarrow \mathcal{O}(M_{GUT})$$

$$M_H = -\frac{3}{2}\lambda V + M$$

FINE-TUNED TO $\mathcal{O}(M_W)$

THE GOOD

(1) PREDICTS UNIFICATION OF COUPLING

(2) USES ECONOMIC HIGGS SECTOR

THE BAD

(1) UNNATURAL FINE TUNING

(2) LARGE P-DECAY RATE

ARE THERE SUSY GUTs SATISFYING:

(1) NO FINE TUNING

(2) COMPLETE SUPPRESSION OF $d=5$ PROTON DECAY

(3) UNIFICATION OF GAUGE COUPLINGS AS A PREDICTION ($\Rightarrow$ NO LIGHT EXOTICS)

(4) ECONOMY OF LARGE HIGGS REP. (NO MORE THAN A SINGLE ADJOINT)

ANSWER IS YES IN SO(10)

(NO IN SU(5))

SUSY SO(10)

$$W_{D-T} = \lambda(10_H\, 45_H\, 10'_H) + \cdots$$

$$\langle 45_H \rangle = \begin{bmatrix} a &&&& \\ & a &&& \\ && a && \\ &&& 0 & \\ &&&& 0 \end{bmatrix} \otimes i\tau_2 \quad \propto (B-L)$$

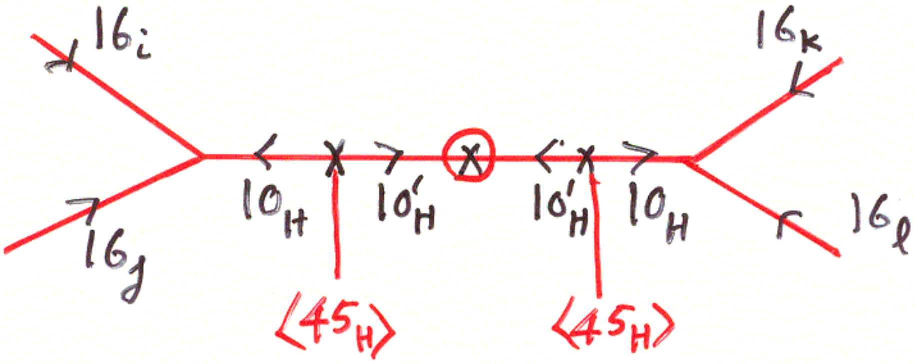
- → B-L VEV GIVES MASS TO TRIPLETS ONLY (DIMOPOULOS-WILCZEK)
- → IF 10_H ONLY COUPLES TO FERMIONS, NO $d=5$ PROTON DECAY
- → PASSES 3 OF 4 CRITERIA
- → DOUBLETS FROM 10_H AND $10'_H$ LIGHT (⇒ 4 DOUBLETS, UNIFICATION UPSET)

⇓

ADD MASS TERM FOR $10'_H$

$$W_{D-T} = \lambda(10_H\, 45_H\, 10'_H) + M\, 10'_H\, 10'_H$$

$d=5$ PROTON DECAY REAPPEARS



IN SUSY SO(10) WITH A SINGLE

ADJOINT HIGGS $\langle 45_H \rangle \propto (B-L)$

$d=5$ P-DECAY CANNOT BE ELIMINATED.

REASON: ONLY 2 TYPES OF MASS TERMS

EXIST: 5-MASS OR 3-MASS

SU(5) INVARIANT

ONLY FOR COLOR TRIPLETS

PREDICTIONS

$$m_b^0 \simeq m_\tau^0$$

$$m_s(1\,\mathrm{GeV}) \simeq 116\ \mathrm{MeV}$$

$$V_{cb} \simeq 0.043$$

$$\begin{cases} \sin^2 2\theta_{\mu\tau} = (0.96,\ 0.91,\ 0.86,\ 0.83,\ 0.81) \\ \dfrac{m_{\nu_\mu}}{m_{\nu_\tau}} = \left(\dfrac{1}{10},\ \dfrac{1}{15},\ \dfrac{1}{20},\ \dfrac{1}{25},\ \dfrac{1}{30}\right) \end{cases}$$

$$m_d(1\,\mathrm{GeV}) \simeq 8\ \mathrm{MeV}$$

$$\theta_c \simeq \left| \sqrt{m_d/m_s} - e^{i\phi}\sqrt{\frac{m_u}{m_c}} \right|$$

$$\left|\frac{V_{us}}{V_{cs}}\right| \simeq \sqrt{\frac{m_u}{m_c}} \simeq 0.07$$

$$\tau(p \to \bar{\nu} K^+) \lesssim 10^{34}\ \mathrm{yr}$$

$$\mathrm{Br}\,(p \to \mu^+ K^0) \sim 10\,\%$$

REALISTIC SO(10) MODEL

(Pati, Wilczek, K.B)

$$U = \begin{bmatrix} 0 & \epsilon' & 0 \\ -\epsilon' & 0 & \epsilon+\sigma \\ 0 & -\epsilon+\sigma & 1 \end{bmatrix} m_U \, ; \quad D = \begin{bmatrix} 0 & \epsilon'+\eta' & 0 \\ -\epsilon'+\eta' & 0 & \epsilon+\eta \\ 0 & -\epsilon+\eta & 1 \end{bmatrix} m_D$$

$$N = \begin{bmatrix} 0 & -3\epsilon' & 0 \\ 3\epsilon' & 0 & -3\epsilon+\sigma \\ 0 & 3\epsilon+\sigma & 1 \end{bmatrix} m_U \, ; \quad L = \begin{bmatrix} 0 & -3\epsilon'+\eta' & 0 \\ 3\epsilon'+\eta' & 0 & -3\epsilon+\eta \\ 0 & 3\epsilon+\eta & 1 \end{bmatrix} m_D$$

$$M_\nu^R = \begin{bmatrix} x & 0 & z \\ 0 & 0 & y \\ z & y & 1 \end{bmatrix} M_R$$

"1" : $16_3\, 16_3\, 10_H$

"ϵ" : $16_2\, 16_3\, (10_H \times 45_H)/M$

"σ" : $16_2\, 16_3\, (10_H \times 1_H)/M$

"η" : $16_2\, 16_3\, 16_H\, 16_H\, /M$

$$\langle 45_H \rangle \propto (B-L)$$

SO(10) MODELS WITH $\langle 45_H \rangle \propto I_{3R}$

(Barr, Babu, 2002)

$$\langle 45_H \rangle = \begin{bmatrix} 0 & & & & \\ & 0 & & & \\ & & 0 & & \\ & & & a & \\ & & & & a \end{bmatrix} \otimes i\tau_2$$

$10_H\, 45_H\, 10'_H \Rightarrow$ { NO MASS TO TRIPLETS / GUT SCALE MASS TO DOUBLETS

$\overline{16}_H\, 45_H\, 16_H \Rightarrow$ { NO MASS TO DOUBLETS / GUT MASS TO TRIPLETS

$16_H \to 10_H + \bar{5}_H + 1_H$ UNDER SU(5)

$\overline{16}_H \to \overline{10}_H + 5_H + \bar{1}_H$

CAN BE MSSM HIGGS

THESE THEORIES HAVE

5- MASS ← SAME FOR 3 AND 2

3- MASS ← COLOR TRIPLETS ONLY

2- MASS ← DOUBLETS ONLY

$\Rightarrow$ $d = 5$ PROTON DECAY CAN BE REMOVED

REALISTIC SO(10) WITH NO d=5 P-DECAY

BARR-RABY

$$W_{D-T} = \overline{16}_H(45_H + 1_H)16'_H + \overline{16}'_H(45_H + 1'_H)16_H + \overline{16}'_H 45_H 16'_H + m\,\overline{16}''_H 16''_H + 16_H 16_H 10_H + \overline{16}_H \overline{16}_H 10_H + [16''_H 16_H]_{10}[45_H 10_H]_{10}$$

$\Rightarrow$

$$\left(\bar{5}_{16_H}\ \ \bar{5}_{16'_H}\ \ \bar{5}_{16''_H}\ \ \bar{5}_{10_H}\right)$$

$$\begin{bmatrix} 0 & \langle 45_H + 1_H\rangle & 0 & \langle 16_H\rangle \\ \langle 45_H + 1'_H\rangle & \langle 45_H\rangle_3 & 0 & 0 \\ 0 & 0 & m & \langle 45_H 16_H\rangle_2 \\ \langle \overline{16}_H\rangle & 0 & 0 & 0 \end{bmatrix} \begin{pmatrix} 5_{\overline{16}_H} \\ 5_{\overline{16}'_H} \\ 5_{\overline{16}''_H} \\ 5_{10_H} \end{pmatrix}$$

$$\mathrm{Det} = m\,\langle 16_H\rangle\langle \overline{16}_H\rangle\langle 45_H\rangle_3$$

$\Rightarrow$ ALL COLOR TRIPLETS MASSIVE

ONE PAIR OF DOUBLETS LIGHT

$$H_u = \overline{16}'_H + \overline{16}''_H + 10_H$$

$$H_d = 16'_H + 10_H$$

$16'_H$ AND $\overline{16}''_H$ COUPLE TO FERMION $\Rightarrow$ NO P-DK.

2^{ND} EXAMPLE

$$W_{D-T} = \overline{16}_H (45_H + 1_H) 16'_H + \overline{16}'_H (45_H + 1'_H) 16_H + \overline{16}''_H 45_H 16'_H + m\, \overline{16}''_H 16''_H + 16''_H 16_H 10_H + \overline{16}_H \overline{16}_H 10_H + 16_H 16_H 45_H 10_H$$

$\Rightarrow$

$$\left(\bar{5}_{16_H} \;\; \bar{5}_{16'_H} \;\; \bar{5}_{16''_H} \;\; \bar{5}_{10_H} \right)
\begin{bmatrix} 0 & \langle 45_H + 1'_H \rangle & 0 & \langle 45_H 16_H \rangle_2 \\ \langle 45_H + 1'_H \rangle & 0 & \langle 45_H \rangle_3 & 0 \\ 0 & 0 & m & \langle 16_H \rangle \\ \langle \overline{16}_H \rangle & 0 & 0 & 0 \end{bmatrix}
\begin{pmatrix} 5_{\overline{16}_H} \\ 5_{\overline{16}'_H} \\ 5_{\overline{16}''_H} \\ 5_{10_H} \end{pmatrix}$$

$$\mathrm{Det} = \langle \overline{16}_H \rangle \langle 16_H \rangle \langle 45_H + 1_H \rangle \langle 45_H \rangle_3$$

$\Rightarrow$ ALL TRIPLETS HEAVY

LIGHT DOUBLETS: $H_d = 16'_H + 10_H$

$H_u = \overline{16}'_H + \overline{16}''_H + 10_H$

$16'_H \oplus \overline{16}'_H$ COUPLE TO FERMIONS $\Rightarrow$

NO $d=5$ PROTON DECAY

PREDICTIVE QUARK AND LEPTON MASSES

K.S.B, S. BARR, Phys. Lett. B525, 289, (200

$$L = \begin{bmatrix} 0 & 0 & \delta' \\ 0 & \delta & -\epsilon' \\ \rho' & \rho-\epsilon & 1 \end{bmatrix} m_D \qquad D = \begin{bmatrix} 0 & 0 & \rho' \\ 0 & \delta & \rho-\epsilon' \\ \delta' & -\epsilon & 1 \end{bmatrix} m_D$$

LOPSIDED

$$N = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & \epsilon' \\ 0 & \epsilon & 1 \end{bmatrix} m_U \qquad U = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & \epsilon' \\ 0 & \epsilon & 1 \end{bmatrix} m_U$$

$$W_{Yuk} = 16_3\, 16_3\, 10_H + 16_2\, 16_3\, 10_H\, 45_H + 16_2\, 16_3\, 16_H\, 16'_H + 16_1\, 16_3\, 16_H\, 16'_H + 16_2\, 16_2\, 16_H\, 16'_H$$

10 PARAMETERS VS 20 OBSERVABLES

$\Rightarrow$ PREDICTIONS

FIT

$$1 \sim \rho \sim \rho' >> \epsilon, \epsilon' >> \delta, \delta'$$

$$m_b \simeq m_\tau = \sqrt{1+\sigma^2}\, m_D$$ ----- BURAS et. al

$$\frac{m_s}{m_b} \simeq \frac{|\sigma\epsilon + \delta^* \rho/\sigma|}{1+\sigma^2}$$

GEORGI-JARLSKOG

$$\frac{m_\mu}{m_\tau} \simeq \frac{|\sigma\epsilon' + \delta^* \rho/\sigma|}{1+\sigma^2}$$

$$m_d m_s m_b \simeq m_e m_\mu m_\tau$$

$$\frac{m_c}{m_t} \simeq \epsilon\epsilon' \qquad \frac{m_u}{m_t} \simeq 0$$

$$|V_{us}| \simeq \frac{\delta'}{\epsilon + \delta\rho/\sigma^2} \qquad |V_{ub}| \simeq \frac{\delta'}{1+\sigma^2}$$

$$|V_{ub}| \simeq \frac{|\epsilon(2+\sigma^2) - \delta^*\rho|}{1+\sigma^2} \qquad \eta_{CP} \simeq \frac{2\epsilon\rho \ \mathrm{Im}(\delta)}{\sigma^2(1+\sigma^2)|V_{cb}|^2}$$

NOTE $\qquad m_\mu \neq m_s$

$$\tan\theta_{atm} \simeq \sigma \equiv \sqrt{\rho^2 + \rho'^2} \simeq \frac{m_s}{m_b}\left|\frac{V_{us}}{V_{ub}}\right|$$

$$\tan\theta_\odot \simeq (0.4 - 0.6)$$

$$|U_{e3}| \simeq 0.06$$

REASONABLY GOOD FIT.

SUMMARY

- REALISTIC SO(10) SUSY GUTs CAN BE BUILT WITHOUT $d=5$ PROTON DECAY
- SINGLE ADJOINT HIGGS SUFFICIENT
  $\langle 45_H \rangle \propto I_{3R}$
- PREDICTIVE FERMION MASSES EASY TO INCORPORATE
- 4-d SUSY GUTs ALIVE AND WELL
- HIGHLY DESIRABLE TO PROBE $p \to e^+ \pi^0$ TO THE LEVEL $\tau \sim 10^{36}$ yr.