Horizontal Theories and Soft Breaking

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Flavour Physics

- 3 families with identical gauge numbers ?
- Origin of the mass hierarchy ?
- Intergenerational mixing ?

SM

12 masses
10 mixing angles
and phases . . .

MSSM

fermion + sfermion
masses + mixings
= 124 !!!!

Minimal Supergravity

Canonical Kähler potential $K = \sum_i |\phi^i|^2$ and Superpotential

$$W = W^{hid}(\chi_k) + \left(\frac{\theta}{M}\right)^{\alpha(i,j)} H_a Q_{Li} q^c_{Rj} + \ldots$$

with θ/M generate Yukawa structure. Scalar potential (F-terms):

$$V = e^K \left[\sum_i \left| \frac{\partial W}{\partial \phi_i} + \phi^{i*} W \right|^2 - 3|W|^2 \right]$$

with $F_{\phi_i} = -e^{K/2}\,(W_i^* + \phi_i W^*)$.

after SUSY breaking in hidden sector $\langle W \rangle \simeq m_{3/2}$. Generic F-terms determined by $\langle \phi_i W^* \rangle = m_{3/2} \langle \phi_i \rangle \leq m_{3/2} M_{Pl}$

Assuming $F_\theta \simeq m_{3/2}\langle\theta\rangle$, trilinear couplings,

$$\begin{aligned} A_{ij}\; Y_{ij}\; H_a \tilde{Q}_{Li} \tilde{q}^c_{Rj} &= \left(F^{\chi_k} \hat{K}_{\chi_k} Y^{ij} + F^m \partial_m Y^{ij} \right) \; H_a \tilde{Q}_{Li} \tilde{q}^c_{Rj} \\ &= \left(A_0 Y_{ij} + \alpha_{ij} m_{3/2} Y^{ij} \right) \; H_a \tilde{Q}_{Li} \tilde{q}^c_{Rj} \end{aligned}$$

Yukawa matrices as effective operators as powers of scalar vevs.

Irrespective of the source of SUSY breaking

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$$\boxed{\text{Nonuniversal trilinear terms !!}}$$

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$$\boxed{\text{New sources of FCNC}}$$

Yukawa Textures and Soft breaking

- Explain fermion masses and mixings in terms of a few fundamental parameters
- Off-diagonal entries small relative on-diagonal
- Approximate texture zeros: mixings in terms of masses

$$\frac{M}{m_3} = \begin{pmatrix} 0 & b\ \epsilon^3 & c\ \epsilon^3 \\ b'\ \epsilon^3 & d\ \epsilon^2 & a\ \epsilon^2 \\ f\ \epsilon^m & g\ \epsilon^n & 1 \end{pmatrix}$$

with $\epsilon_u \simeq 0.05$ and $\epsilon_d \simeq 0.15$

if $\epsilon = \langle\theta\rangle/M$ we obtain as trilinear

$$(Y^A)_{ij} \equiv Y_{ij}A_{ij} = A_0 Y_{ij} + Y_{33} m_{3/2} \begin{pmatrix} 0 & 3\ b\ \epsilon^3 & 4\ c\ \epsilon^4 \\ 3\ b'\ \epsilon^3 & 2\ d\ \epsilon^2 & 2\ a\ \epsilon^2 \\ m\ f\ \epsilon^m & n\ g\ \epsilon^n & 0 \end{pmatrix}$$

Non diagonal Trilinear couplings in SCKM $\Rightarrow$ Left-Right FCNC

$$(\tilde{Y}^A)_{31} = Y_{33}\ m_{3/2}\ m\ \epsilon^m + \ldots \qquad (\tilde{Y}^A)_{32} = Y_{33}\ m_{3/2}\ g\ n\ \epsilon^n + \ldots$$

$$(\tilde{Y}^A)_{21} = Y_{33}\ m_{3/2}\ (b'\ \epsilon^3 + a\ \frac{b'}{d}\ g\ n\ \epsilon^{3+n} - a\ f\ m\ \epsilon^{2+m} + \ldots)$$

FCNC and CP Phenomenology

- Soft breaking terms at high scale $\sim M_{GUT}$
- RGE evolution to electroweak scale.

1.- Diagonal elements receive large gaugino contribution

$$m^2_{\tilde{q}} \simeq 6\ m^2_{1/2} + m^2_0 \simeq 19\ m^2_{3/2} = C_q\ m^2_{3/2}$$
$$m^2_{\tilde{l}} \simeq 1.5\ m^2_{1/2} + m^2_0 \simeq 5.5\ m^2_{3/2} = C_l\ m^2_{3/2}$$

2.- Diagonal trilinear couplings receive large gaugino contribution and align with gaugino masses. Pure trilinear contributions unchanged in down but reduced in up $A_t \supset 0.25 A_0,\ A_u \supset 0.60 A_0$

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"String CP problem" and MI bounds

Mass Insertion limits

Mass Insertions

$$\left(\delta^f_A\right)_{ij} = \frac{(m^2_{\tilde{f}_A})_{ij}}{m^2_{\tilde{f}}} \Rightarrow \left(\delta^f_{LR}\right)_{i\neq j} = \frac{v_f(\tilde{Y}^A_f)_{ij}}{C_f\ m^2_{3/2}}$$

Using $m_{\tilde{q}} \simeq 500$ GeV $\Rightarrow m_{3/2} \simeq 120$ GeV.

$$(\delta^d_{LR})_{21} \simeq \frac{m_{3/2}\ m_b}{19\ m^2_{3/2}}(b'\ \epsilon^3_d + a\ \frac{b'}{d}\ g\ n\ \epsilon^{3+n}_d - a\ f\ m\ \epsilon^{2+m}_d) \simeq$$

$$(b'\ \epsilon_d + a\ \frac{b'}{d}\ g\ n\ \epsilon^{1+n}_d - a\ f\ m\ \epsilon^m_d)\ 5\times 10^{-5} \simeq \frac{b'}{2}\ 1.5\times 10^{-5} + \ldots$$

Sizeable contributions to $(\delta^d_{LR})_{21}$ naturally expected. With Im $b' \neq 0$, (observable phase) large contribution to ε'/ε

No strong constraints on unknown elements $m \geq 1$ and $n \geq 0$

$$(\delta^d_{LR})_{32} \simeq \frac{m_{3/2}\ m_b\ g\ n\ \epsilon_d^n}{19\ m_{3/2}^2} \simeq 2.2 \times 10^{-3}\ g\ n\ \epsilon_d^n$$

However minimal suppression in down $m_b/(19 m_{3/2}) \lesssim 2 \times 10^{-3}$, with $m_{\tilde{q}} = 500$ GeV (smaller suppression a priori in up sector).

Leptonic sector

$$(\delta^e_{LR})_{12} \simeq \frac{m_{3/2}\ m_\tau}{5.5\ m_{3/2}^2}(b'\ \epsilon_d^3 + a\ \frac{b'}{d}\ g\ n\ \epsilon_d^{3+n} - a\ f\ m\ \epsilon_d^{2+m}) \simeq$$

$$(b'\ + a\ \frac{b'}{d}\ g\ n\ \epsilon_d^n - a\ f\ m\ \epsilon_d^{m-1})\ 8.7 \times 10^{-6}$$

with $m_{\tilde{l}} = 280$ GeV, ($m_{3/2} = 120$ GeV), MI bound with photino,

$$(\delta^e_{LR})_{12} \leq 7 \times 10^{-7}\ \left(\frac{280}{100}\right)^2 = 5.5 \times 10^{-6}$$

Requires larger sfermion masses: OK for $m_{3/2} = 170$ GeV, corresponding to $m_{\tilde{q}} = 600$ GeV

MI limits

x	$\sqrt{\left\|\mathrm{Im}\,(\delta^d_{LR})^2_{12}\right\|}$	$\sqrt{\left\|\mathrm{Re}\,(\delta^d_{LR})^2_{13}\right\|}$	$\left\|(\delta^l_{LR})_{12}\right\|$	$\left\|(\delta^l_{LR})_{23}\right\|$
0.3	1.1×10^{-5}	1.3×10^{-2}	6.9×10^{-7}	8.7×10^{-3}
1.0	2.0×10^{-5}	1.6×10^{-2}	8.4×10^{-7}	1.0×10^{-2}
4.0	6.3×10^{-5}	3.0×10^{-2}	1.9×10^{-6}	2.3×10^{-2}
$\times(m_{\tilde{q}}\ (\mathrm{GeV})/500)^2$			$\times(m_{\tilde{l}}\ (\mathrm{GeV})/100)^2$	

x	$\|\mathrm{Im}(\delta^d_{11})_{LR}\|$	$\|\mathrm{Im}(\delta^u_{11})_{LR}\|$	$\|\mathrm{Im}(\delta^d_{22})_{LR}\|$
0.3	4.3×10^{-8}	4.3×10^{-8}	3.6×10^{-6}
1	8.0×10^{-8}	8.0×10^{-8}	6.7×10^{-6}
3	1.8×10^{-7}	1.8×10^{-7}	1.6×10^{-5}
$\times(m_{\tilde{l}}\ (\mathrm{GeV})/100)$			

“String CP problem”

- Yukawa matrices contain $\mathcal{O}(1)$ CP violating phases (δ_{CKM})

Super CKM basis

$$V_{qL}^{\dagger} Y_q V_{qR} = D_i = \mathrm{Diag}(h_1, h_2, h_3) \qquad \tilde{Y}_q^A = V_{qL}^{\dagger} Y_q^A V_{qR}$$

- If nonuniversal, even real A can overproduce EDMs

However from θ dependence,

$$\left(V_L \cdot \theta \frac{\partial Y}{\partial \theta} \cdot V_R^{\dagger} \right)_{ii} = V_{L\,ij} \frac{\theta \partial V_{L\,ij}^{*}}{\partial \theta} D_{ii} + \frac{\theta \partial D_{ii}}{\partial \theta} + D_{ii} \frac{\theta \partial V_{R\,ij}}{\partial \theta} V_{R\,ij}^{*}$$

Dominant order in θ fixed by D_{ii}, mixing matrices contribute only to higher orders. $\Rightarrow$ Diagonal elements in Y^A are real at leading order

Subdominat contributions observable in SCKM,

$$\mathrm{Im}\left(\delta^d_{LR}\right)_{11} \simeq \left(\epsilon^n_d \; n \; \mathrm{Im}(\mathbf{g}) + \epsilon^{m-1}_d \; (m-1) \; \mathrm{Im}(\mathbf{f})\right) \; 3.9 \times 10^{-6}$$
$$\mathrm{Im}\left(\delta^u_{LR}\right)_{11} \simeq \left(\epsilon^n_u \; n \; \mathrm{Im}(\mathbf{g}) + \epsilon^{m-1}_u \; (m-1) \; \mathrm{Im}(\mathbf{f})\right) \; 1.9 \times 10^{-6}$$
$$\mathrm{Im}\left(\delta^e_{LR}\right)_{11} \simeq \left(\epsilon^n_d \; n \; \mathrm{Im}(\mathbf{g}) + \epsilon^{m-1}_d \; (m-1) \; \mathrm{Im}(\mathbf{f})\right) \; 6.7 \times 10^{-7}$$

$n = 0$, $m = 1$ still allowed, but otherwise $n \geq 2$, $m \geq 3$ from Hg atom EDMs constraints
Similarly, subdominant corrections to Y_{22}, Y_{12} and Y_{21} constrained to be ϵ^4 and ϵ^5 respectively

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“String CP problem” avoided in reasonable flavour models

Conclusions

- Hierarchical Yukawa structure suggests ordering by $(\theta/M)^{\alpha}$
- Trilinear terms nonuniversality determined by this structure
- Large effects naturally expected in ε'/ε
- $\mu \to e\gamma$ can already constrain the allowed sfermion masses, for large F-terms.
- Third generation decays not sensitive enough.
- "String CP problem" solved in these models, although restricts structure of allowed textures.