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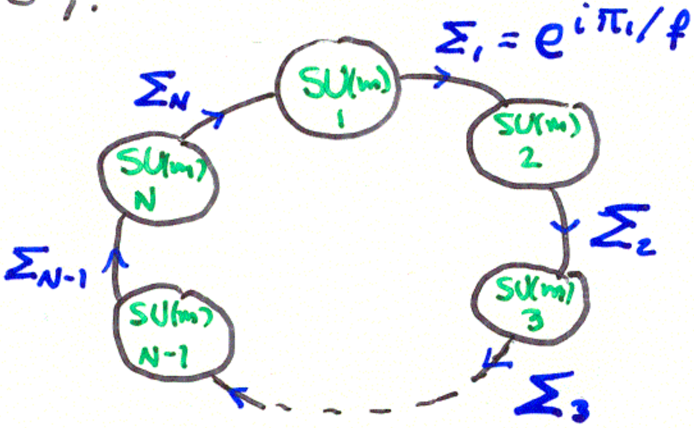
? WHAT IS DIM'L DECO'N

(Arkani-Hamed, et al. ; Hill et al.)

CERTAIN RENORMALIZABLE, ASYMPTOTICALLY FREE 4-DIM'AL FIELD THEORIES LOOK, FOR A RANGE OF ENERGIES, LIKE HIGHER-DIM'AL THEORIES IN WHICH THE EXTRA DIM'NS ARE COMPACTIFIED (AND DISCRETIZED — USUALLY ON A PERIODIC LATTICE).

e.g.



$$m_k^2 = g^2 f^2 \sin^2\left(\frac{\pi k}{N}\right)$$
$(k = 1, \dots, N)$.
$$\sim g^2 f^2 \pi^2 k^2/N^2$$
FOR $k \ll N$.

- $d>4$ COMPONENTS OF GAUGE BOSONS
  $\longleftrightarrow$ 4-d GOLDSTONE BOSONS
  $\Rightarrow$ M_π^2 SMALL, PROTECTED BY SYMMETRIES RESEMBLING $d>4$ GAUGE INVARIANCE

- PGB QUARTIC INTERACTIONS (λ) — FORM & STRENGTH — DEDUCIBLE FROM CORRESPONDING INT'NS OF $d>4$ GAUGE BOSONS.

WHAT IS DD GOOD FOR?

– CONSTRUCTING TRULY DYNAMICAL MODELS OF LIGHT COMPOSITE HIGGS BOSONS, H!

① TRULY DYNAMICAL — NATURAL —
H IS COMPOSITE, BOUND BY STRONG DYNAMICS AT ENERGY SCALE $\Lambda \simeq 4\pi f$ – UV COMPLETION

② LIGHT – $M_H^2 \equiv M_+^2 - M_-^2$ with $M_H^2 \sim M_\pm^2 \ll \Lambda^2$

③ HIGGS – $v^2 \simeq M_H^2/\lambda \ll \Lambda^2$

① REQUIRES TC OR SUSY OR "FUNDAMENTAL SCALE" AT Λ FOR THE UV COMPLETION.

② REQUIRES H = PGB ($d > 4$ GAUGE BOSON)

③ REQUIRES $\lambda \simeq M_H^2/v^2$ NOT $\ll O(1)$
→ MODERATELY STRONG, NON-DERIVATIVELY COUPLED GAUGE BOSON SELF-INT'NS.

The d=6 Toroidal Moose Model

(Arkani-Hamed, Cohen, Georgi - hep-ph/0105239)
(K. Lane - hep-ph/0202093)

To get ~ strong, non-derivative quartic Higgs interactions:

- 6-d SU(m) gauge theory with ~ weak coupling g
- Compactify $d=5,6$ on torus w/ circumferences R
- Discretize torus w/ $N \times N$ sites, $a = \frac{R}{N}$.
- This is a 4-d moose model w/ weak SU(m) at N^2 sites linked by $2N^2$ nonlinear σ-model fields (Fig. 4)

Vertical: $U_{k\ell} = e^{i\pi_{u,k\ell}/f} \rightarrow W_{k\ell}\, U_{k\ell}\, W^{\dagger}_{k,\ell+1}$

Horizontal: $V_{k\ell} = e^{i\pi_{v,k\ell}/f} \rightarrow W_{k\ell}\, V_{k\ell}\, W^{\dagger}_{k,\ell+1}$

N^2-1 GBs eaten to give "KK" spectrum.

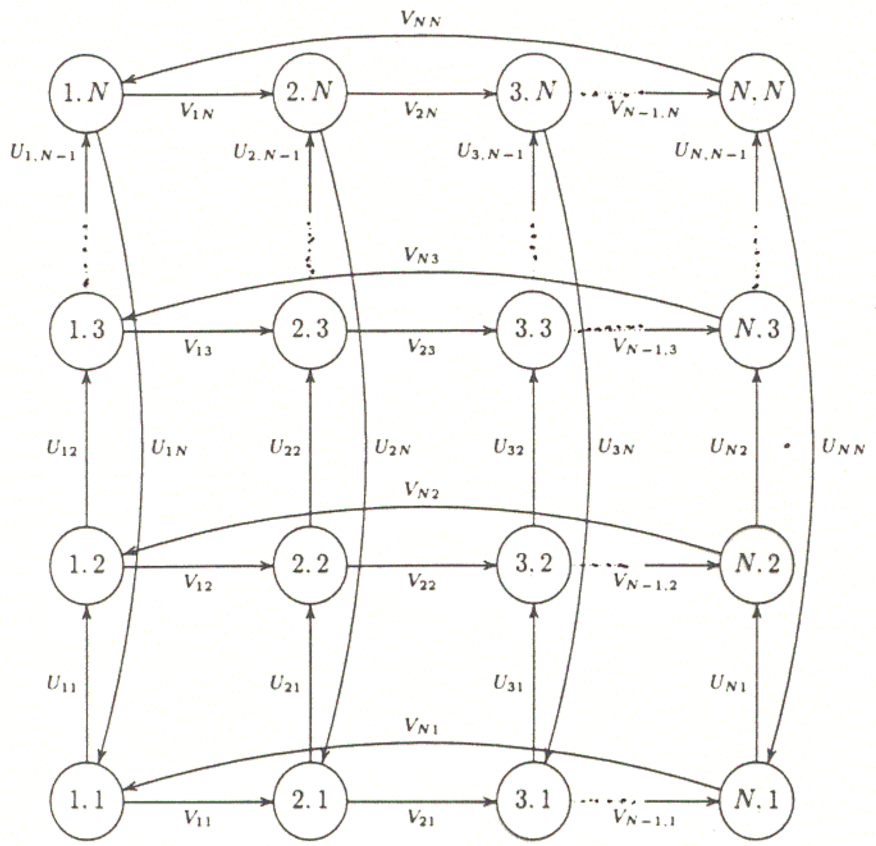


Figure 4: The condensed moose for the toroidal model of Ref. [2]. The weak $SU(m)_{kl}$ group is denoted by a circle at the site (k,l). The site (k,l) is identified with the sites $(k+N,l)$ and $(k,l+N)$. Nonlinear sigma model link–fields U_{kl} and V_{kl} transform according to Eq. (7).

DD $\Longrightarrow$

① 2 (P) GB SU(m) ADJOINTS π_u^a , π_v^a
CORRESPONDING TO A_5^a , A_6^a :

$$A^a_{5,6} = g\,\pi^a_{u,v}/N \; ; \quad g_6 = ga \equiv \frac{1}{f} \simeq \frac{4\pi}{\Lambda} .$$

$$\pi_{u,v} = \frac{1}{N}\sum_{k,l} \pi_{u,v\;kl}$$ ⤳ INCOMPRESSIBLE LOOPS AROUND TORUS IN U, V- DIRECTIONS.

② $\pi_{u,v}$ MASSES FROM WILSON LOOPS :

$$\left|\mathbb{P}\exp\left(i\oint dx_5 A_5\right)\right|^2 = \left|\mathrm{Tr}\left\{\left[\exp(i\pi_u/Nf)\right]^N\right\}\right|^2$$
$$= \left|\mathrm{Tr}\left(U_{k1}U_{k2}\cdots U_{kN}\right)\right|^2$$

$\rightarrow$ $M^2_{\pi_{u,v}} = O\left(\frac{g^4 f^2}{N^4}\right)$ FOR $N \geq 2$

IMPORTANT
CONTROVERSIAL
NOT PREDICTED BY DD

③ QUARTIC INT'N $\lambda\,\mathrm{Tr}\left([\pi_u,\pi_v]^2\right)$ FROM

$$S_{56} = \int d^4x \cdot a^2\,\frac{1}{g_6^2}\,\mathrm{Tr}\,F_{56}^2 = \int d^4x\,\frac{1}{g_6^2 a^2}\sum_{k,l}\underbrace{\mathrm{Tr}\left(U_{kl}V_{k,l+1}U^\dagger_{k+1,l}V^\dagger_{kl}\right)}_{\text{"PLAQUETTE" } \square_{kl}}$$

$\rightarrow$ $\lambda = \frac{1}{g_6^2 a^2}\cdot\frac{N^2}{2N^4 f^4} = \frac{g^2}{2N^2}$ $\rightarrow$ $v^2 \sim \frac{M^2_{\pi_{u,v}}}{\lambda} \sim \frac{N^2 g^2 f^2}{N^4} \ll f^2, \Lambda^2$

Test DD w/ "QCD" UV Completion

Philosophy: If DD is a good prescription for model-building, its predictions ought to be independent of the UV completion.

UV Model: $2N^2$ strong $SU(n)$ at $(k, \ell+\frac{1}{2})$; $(k+\frac{1}{2}, \ell)$.

$\psi_{R(k,\ell+\frac{1}{2})} \varepsilon (n,m,1)$; $\psi_{L(k,\ell+\frac{1}{2})} \varepsilon (n,1,m)$ of $SU(n)_{k,\ell+1/2} \otimes SU(m)_{k\ell} \otimes SU(m)_{k,\ell+1}$

(Fig. 5)

$\psi_{R(k+\frac{1}{2},\ell)} \varepsilon (n,m,1)$; $\psi_{L(k+\frac{1}{2},\ell)} \varepsilon (n,1,m)$ of $SU(n)_{k+\frac{1}{2},\ell} \otimes SU(m)_{k\ell} \otimes SU(m)_{k+1,\ell}$

(see moose - Fig. 5)

$\langle\Omega| \bar\psi_{L(k,\ell+\frac{1}{2})} \psi_{R(k,\ell+\frac{1}{2})} |\Omega\rangle = -\Delta \leftrightarrow -4\pi f^3 U_{k\ell}$

etc.

$\rightarrow$ $2N^2$ GB multiplets $\pi_{u,k\ell}$; $\pi_{v,k\ell}$.
N^2-1 eaten by $SU(m)$ gauge bosons.

$\rightarrow$ $M^2_{\pi_{u,v}} = O\left(\frac{g^4}{16\pi^2} f^2\right)$

$\oplus$ other $M^2_\pi = O\left(\frac{g^4}{16\pi^2} f^2\right)$ too!

$\rightarrow$ $\lambda = O\left(\frac{g^4}{16\pi^2}\right) \Rightarrow v^2 = O(f^2)$

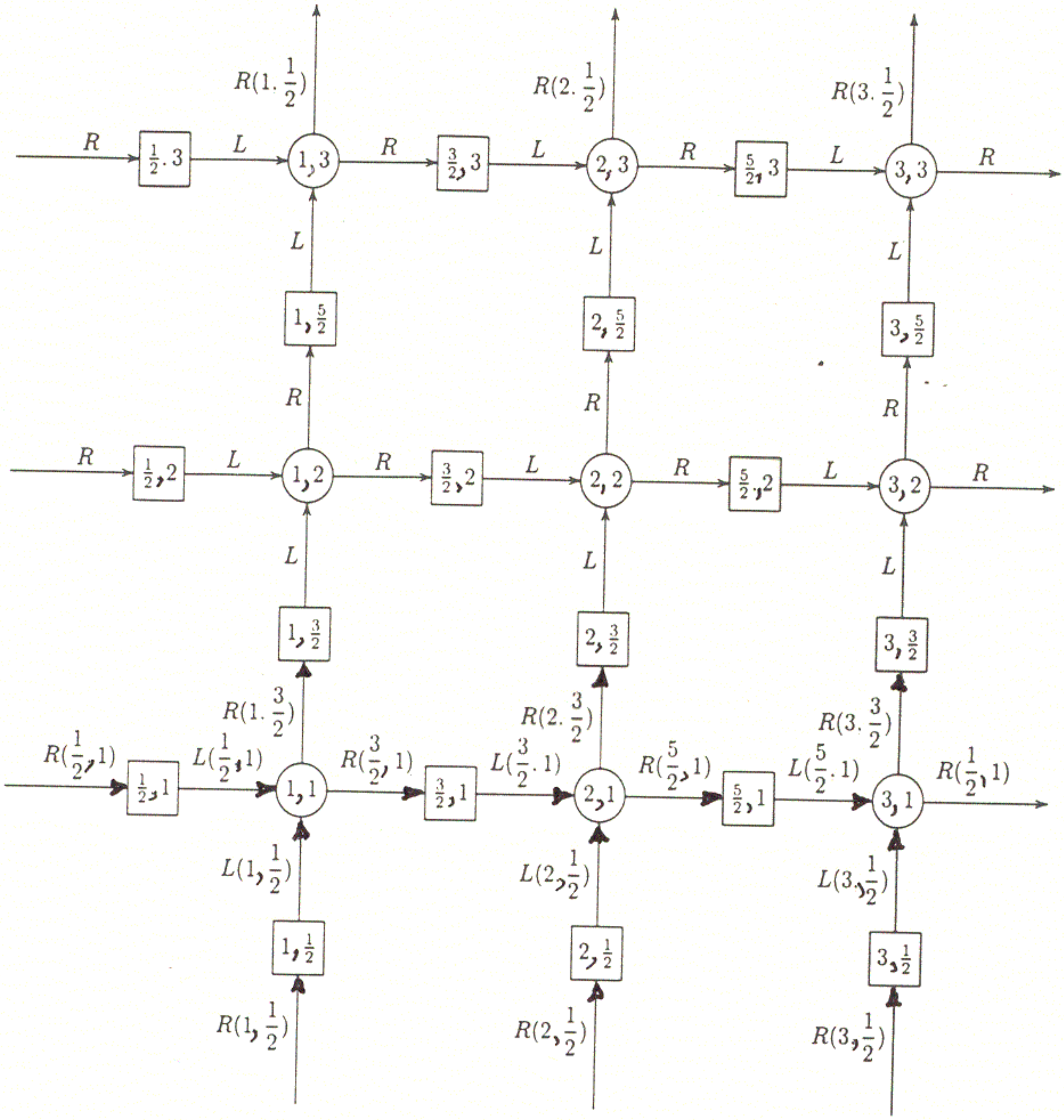


Figure 5: The complete moose for the toroidal model of Ref [2] with a QCD-like UV completion. The weak $SU(m)_{kl}$ gauge groups are as in Fig. 4, and the strong $SU(n)_{k-\frac{1}{2},l}$ and $SU(n)_{k,l-\frac{1}{2}}$ gauge groups are indicated by squares. Fermions transform as in Eq. (17).

CONCLUSIONS

① QCD COMPLETION PREDICTS MANY EXTRA PGBs, w/ $M_{\pi}^2 = O(g^4 f^2/16\pi^2)$ – SAME AS $\Pi_{u,v}$ (AKA A_5, A_6).

② ALSO PREDICTS $\lambda = O(g^4/16\pi^2)$, SO — ASSUMING $M_-^2 \sim M_{\pi}^2$ — $v^2 = O(f^2) \gg M_H^2$.

③ ALTERNATIVE: SIMPLY CHOOSE OPERATORS LIKE $\boxed{\ }_{RE}$ WITH $\lambda_{RE} \simeq \lambda/N^2 = O(g^2/N^4)$, OR EVEN $O(1/N^2)$, SO LONG AS CHOICE IS CONSISTENT W/ GAUGE + CHIRAL SYMMETRIES AND RENORM'S ARE SMALL ("NATURAL" SIZES).

BUT, THIS EFFECTIVE-$\mathcal{L}$ APPROACH DOES **NOT** FOLLOW FROM DD.

i.e., DD FAILS AS A PREDICTIVE SCHEME.

④ NEVERTHELESS, THE EFFECTIVE-$\mathcal{L}$ APPROACH IS VERY PROMISING FOR BUILDING LCH MODELS. THAT'S WHAT NIMA, ANDY, ANN, ... ARE DOING NOW.

STILL ... WHAT'S THE UV COMPLETION ??