QCD Calculations by Numerical Integration: Coulomb Gauge

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July 2002

- Overview of the method.
- Setting up the calculation.
- Why Coulomb gauge?
- The gluon propagator in Coulomb gauge.
- Numerical Coulomb gauge results.
- Wish list.

Overview

• I discuss a method for calculating infrared safe observables at next-to-leading order in field theories, emphasizing recent work on Coulomb gauge.

• The application so far is to QCD jet cross sections *etc.* in e^+e^- annihilation.

• See `M. Kramer and D. E. Soper, hep-ph/0204113` and D. E. Soper,

`Phys. Rev. D` **64**`, 034018 (2001),`
`Phys. Rev. D` **62**`, 014009 (2000),`
`Phys. Rev. Lett.` **81**`, 2638 (1998).`

• A program, *beowulf*, with documentation is available at http://zebu.uoregon.edu/~soper/beowulf/

• It should be possible to extend the method to
$e + p \to e + jets + X$.
$\bar{p} + p \to jets + X$.

• We do not need analytical calculations of loop diagrams.
• Thus the method is very flexible.

• For example, adding masses, $SU(2) \times U(1)$ interactions, SUSY interactions, ...should be straightforward.

Setting up the calculation

• Existing program works for infrared-safe, three-jet-like observables in electron-positron annihilation at order α_s^2.

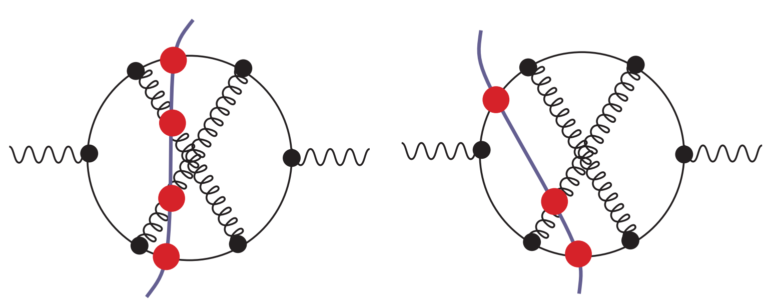
• Ingredients: Feynman diagrams, measurement functions $\mathcal{S}_3(\vec{k}_1, \vec{k}_2, \vec{k}_3)$ and $\mathcal{S}_4(\vec{k}_1, \vec{k}_2, \vec{k}_3, \vec{k}_4)$.

$$
\begin{aligned}
\sigma = & \frac{1}{3!}\int d\vec{k}_1 d\vec{k}_2 d\vec{k}_3 \; \frac{d\sigma_3}{d\vec{k}_1 d\vec{k}_2 d\vec{k}_3} \, \mathcal{S}_3(\vec{k}_1, \vec{k}_2, \vec{k}_3) \\
& + \frac{1}{4!}\int d\vec{k}_1 d\vec{k}_2 d\vec{k}_3 d\vec{k}_4 \; \frac{d\sigma_4}{d\vec{k}_1 d\vec{k}_2 d\vec{k}_3 d\vec{k}_4} \, \mathcal{S}_4(\vec{k}_1, \vec{k}_2, \vec{k}_3, \vec{k}_4).
\end{aligned}
$$

The functions $\mathcal{S}$ are infrared safe:

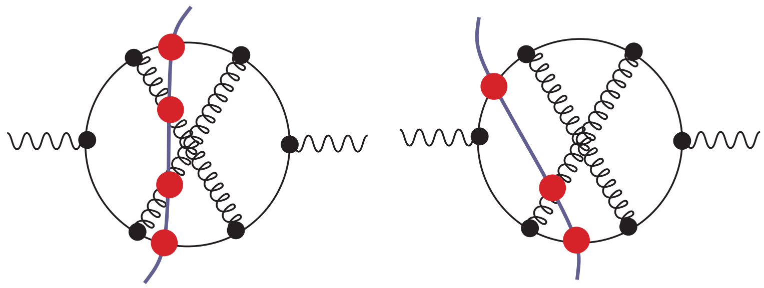
$$
\mathcal{S}_4(\vec{k}_1, \vec{k}_2, \lambda\vec{k}_3, (1-\lambda)\vec{k}_3) = \mathcal{S}_3(\vec{k}_1, \vec{k}_2, \vec{k}_3).
$$

• Think of σ as computed from cut Feynman diagrams.



• The red dots indicate the measurement functions $\mathcal{S}$.

The integrations

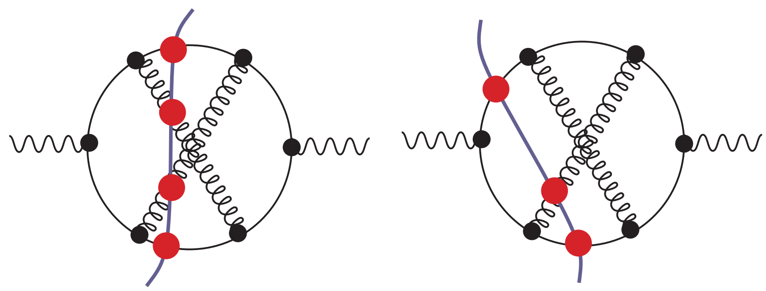


- Arrange the calculation such that the result is expressed as an integral over three loop 3-momenta

$$\int d\vec{\ell}_1 \int d\vec{\ell}_2 \int d\vec{\ell}_3 \cdots$$

- Perform these by Monte Carlo numerical integration.
- This generates "events" with ($+$ or $-$) weights.

Why Coulomb gauge?



• If you include everything at order α_s^2 and no more, then your calculation is gauge invariant.

• Then you may as well use Feynman gauge, since it is computationally simple.

• But you may want to include some higher order effects in an approximate way.

- Running α_s.
- Parton splittings.
- *Cf.* Dokshitser, Dyakonov and Troyan (1980).

• Then you must either make your approximations gauge invariant, or you must choose a gauge.

• In Coulomb gauge (and other "physical" gauges), only physically polarized gluons propagate over long distances.

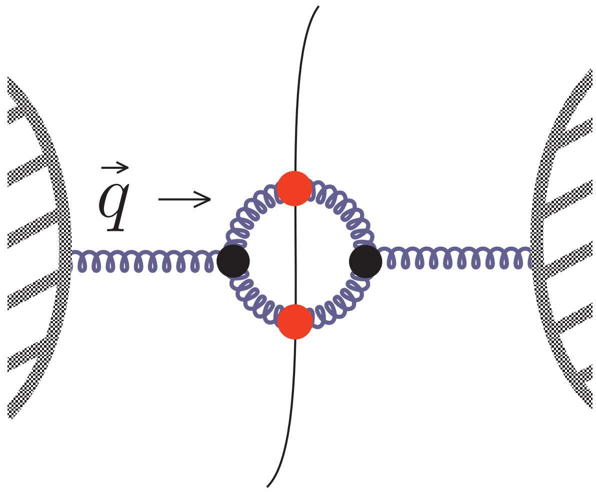
On Coulomb gauge, see

• N.H. Christ and T. D. Lee, Phys. Rev. D **22**, 939 (1980).

• G. Leibbrandt and J. Williams, Nucl. Phys. **B475**, 469 (1996) and subsequent papers.

The gluon propagator in Coulomb gauge

Cut self-energy diagram

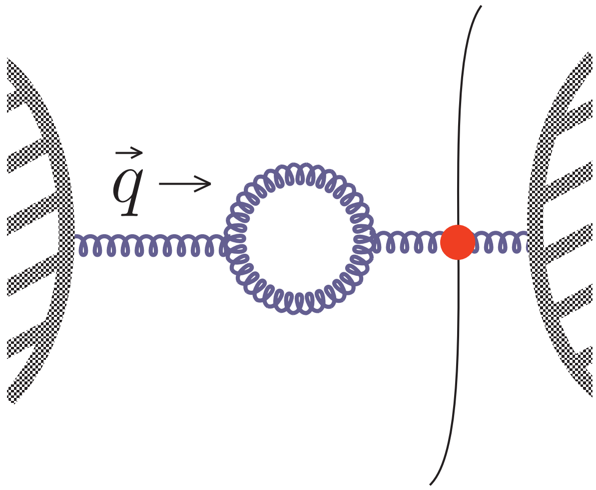


$$\mathcal{I}[\text{real}] = \frac{1}{2|\vec{q}|} \int_0^\infty \frac{d\bar{q}^2}{\bar{q}^2} \int_0^1 dx \int_0^{2\pi} \frac{d\phi}{2\pi}$$
$$\times \mathcal{M}_g^{\mu\nu}(\bar{q}^2, x, \phi)\, R_{\mu\nu}(\bar{q}^2, x, \phi).$$

with a certain choice of integration variables $\vec{\ell} \to \{\bar{q}^2, x, \phi\}$. Here $R_{\mu\nu}$ represents the rest of the graph, including the measurement function, $\bullet$.

The tensor $\mathcal{M}_g^{\mu\nu}$ is complicated, but easy to calculate.

Virtual self-energy diagram



Study

$$F_g^{\mu\nu}(q) = \frac{1}{q^2}\, D(q)^{\mu}_{\mu'}\, \Pi(q)^{\mu'\nu'}\, D(q)^{\nu}_{\nu'} .$$

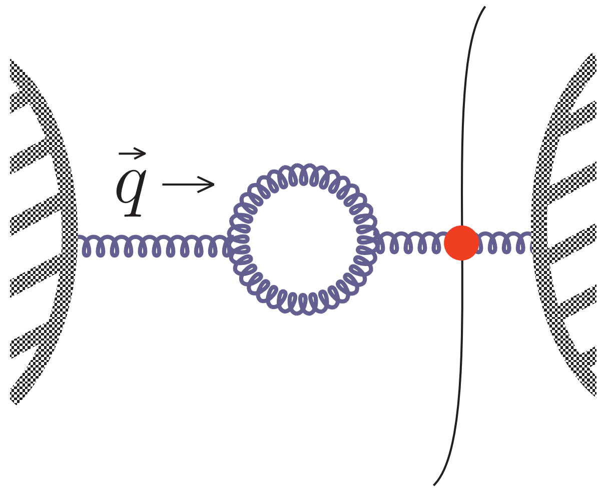
Its form is

$$F_g^{\mu\nu}(q) = D(q)^{\mu\nu} A_T(q^2) + n^\mu n^\nu \frac{q^2}{|\vec{q}|^2} \left[A_E(q^2) - A_T(q^2) \right] .$$

We need $A_T(q^2)$ at $q^2 = 0$.

$A_T(q^2)$ and the definition of Coulomb gauge.

• We perform the integration over loop energy
• This leaves an integral $d\vec{\ell}$ to be done numerically.



• We encounter divergent integrals.
• Use contour integration.
• Average between closing the contour in the upper-half plane and closing the contour in the lower-half plane.

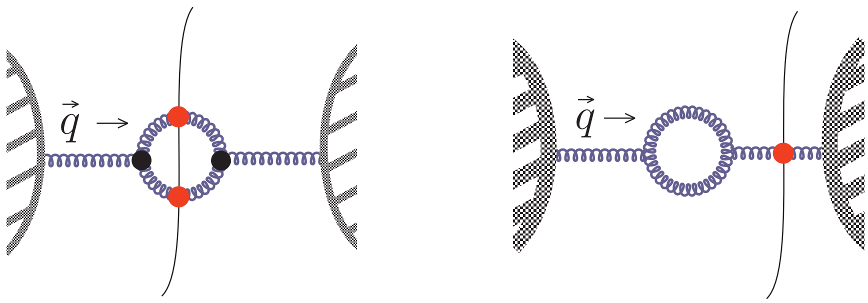
$$\int dE \, \frac{E}{E^2 - \omega^2 + i\epsilon} = 0.$$

$$\int dE \; 1 = 0.$$

(*cf.* Leibbrandt prescription.)

• Some further manipulations are needed to put the integral in a good form.

Result



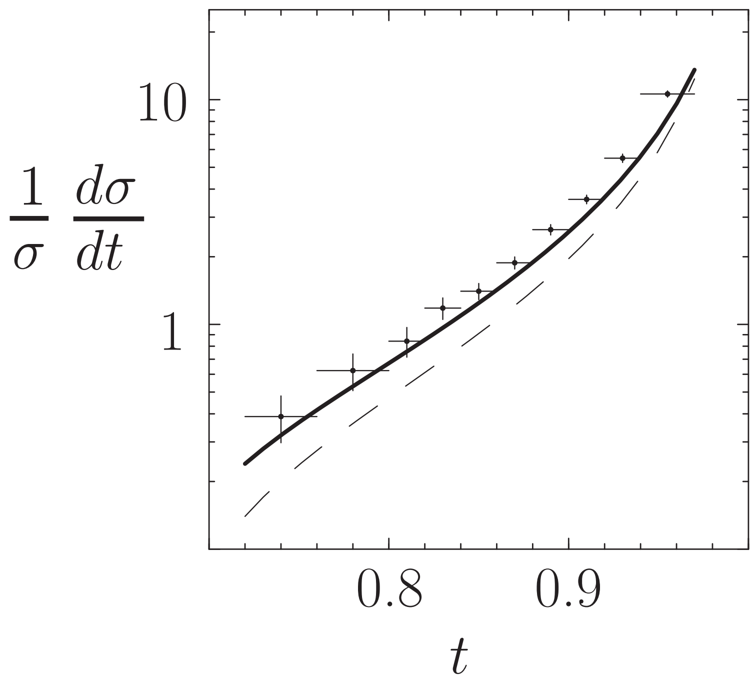
$$\mathcal{I}[\text{real}] + \mathcal{I}[\text{virtual}] = \frac{1}{2|\vec{q}|}\int_0^\infty \frac{d\bar{q}^2}{\bar{q}^2}\int_0^1 dx \int_{-\pi}^{\pi}\frac{d\phi}{2\pi}$$
$$\times \Big\{\mathcal{M}_g^{\mu\nu}(\bar{q}^2,x,\phi)\, R_{\mu\nu}(\bar{q}^2,x,\phi)$$
$$+\,\mathcal{W}_g^{\mu\nu}(\bar{q}^2,x,\phi)\, R_{\mu\nu}^{(0)}\Big\}.$$

- $R_{\mu\nu}^{(0)}$ is $R_{\mu\nu}(\bar{q}^2,x,\phi)$ at $\bar{q}^2 = 0$.
- $\mathcal{M}_g^{\mu\nu}$ and $\mathcal{W}_g^{\mu\nu}$ are known functions.
- There is no problem with being at $q^2 = 0$.
- The integrals are ultraviolet finite.
- For $\bar{q}^2 \to 0$ there is an automatic *real-virtual* cancellation:

$$\mathcal{M}^{\mu\nu} \sim D(q)^{\mu\nu}\,\frac{\alpha_s}{2\pi}\,\frac{1}{2}\,\tilde{P}_{g/g}(x)$$
$$\mathcal{W}^{\mu\nu} \sim -D(q)^{\mu\nu}\,\frac{\alpha_s}{2\pi}\,\frac{1}{2}\,\tilde{P}_{g/g}(x).$$

Results: the thrust distribution

One can use the program to calculate observables like the thrust distribution – using Coulomb gauge.



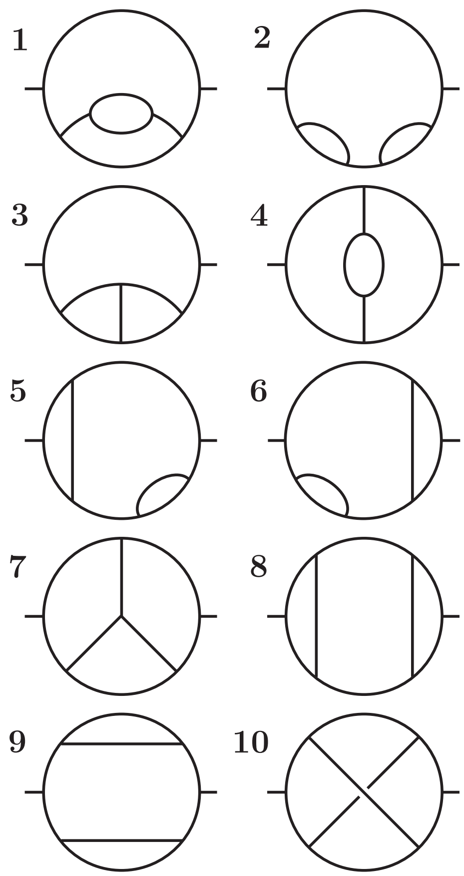
- The data are from the Opal Collaboration.
- The dashed line is the leading order theory.
- The solid curve is the next-to-leading order calculation.
- We choose $\overline{\mathrm{MS}}$ renormalization scale $\mu = \sqrt{s}/6$.

Moments of the thrust distribution

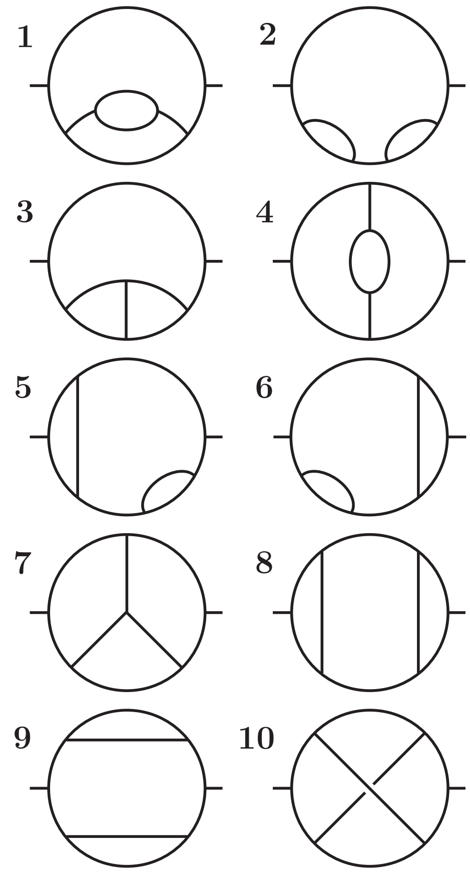
$$\mathcal{I}_n = \frac{1}{\sigma_0}\int_0^1 dt\ (1-t)^n\,\frac{d\sigma}{dt}$$

$$\mathcal{I}_n = \left(\frac{\alpha_s}{\pi}\right) A_n + \left(\frac{\alpha_s}{\pi}\right)^2 B_n + \cdots$$

- First, we calculate B_2 for each graph topology



- Then we will calculate B_n sumed over all graphs



B_2 for each graph topology.

graph	Coulomb gauge	Feynman Gauge
1	−0.327	0.113
2	−0.014	−0.032
3	1.031	0.208
4	−0.196	0.123
5	−0.183	−0.160
6	−0.183	−0.160
7	1.584	1.820
8	−0.141	−0.222
9	0.034	−0.012
10	−0.045	−0.123
sum	1.560	1.555

$$\mathcal{I}_n = \frac{1}{\sigma_0}\int_0^1 dt\ (1-t)^n\,\frac{d\sigma}{dt}$$

$$\mathcal{I}_n = \left(\frac{\alpha_s}{\pi}\right) A_n + \left(\frac{\alpha_s}{\pi}\right)^2 B_n + \cdots$$

B_n in Coulomb gauge and Feynman gauge.

n	gauge	result
1.5	Coulomb	$4.118 \pm 0.010 \pm 0.020$
1.5	Feynman	$4.127 \pm 0.008 \pm 0.025$
2.0	Coulomb	$1.561 \pm 0.003 \pm 0.006$
2.0	Feynman	$1.565 \pm 0.002 \pm 0.007$
2.5	Coulomb	$(6.423 \pm 0.013 \pm 0.021) \times 10^{-1}$
2.5	Feynman	$(6.439 \pm 0.010 \pm 0.022) \times 10^{-1}$
3.0	Coulomb	$(2.816 \pm 0.006 \pm 0.009) \times 10^{-1}$
3.0	Feynman	$(2.822 \pm 0.005 \pm 0.009) \times 10^{-1}$
3.5	Coulomb	$(1.294 \pm 0.003 \pm 0.004) \times 10^{-1}$
3.5	Feynman	$(1.296 \pm 0.002 \pm 0.004) \times 10^{-1}$
4.0	Coulomb	$(6.156 \pm 0.015 \pm 0.018) \times 10^{-2}$
4.0	Feynman	$(6.159 \pm 0.011 \pm 0.016) \times 10^{-2}$
4.5	Coulomb	$(3.010 \pm 0.008 \pm 0.009) \times 10^{-2}$
4.5	Feynman	$(3.009 \pm 0.006 \pm 0.007) \times 10^{-2}$
5.0	Coulomb	$(1.503 \pm 0.004 \pm 0.004) \times 10^{-2}$
5.0	Feynman	$(1.501 \pm 0.003 \pm 0.003) \times 10^{-2}$

Wish list

$e^+ + e^- \to 3$ jets **at NLO**

- Add masses. (M. Krämer and M. Ramtohul, Edinburgh)
- Add interactions: $SU(2) \times U(1)$, SUSY.

Add initial state hadrons

Add more partons

$e^+ + e^- \to 3$ jets **at NNLO**

Go beyond fixed order perturbation theory

- Bubble sums /running coupling for power corrections.
- Add leading log parton showers to NLO calculation.

- Preliminary step (this talk):
  $e^+ + e^- \to 3$ jets at NLO in Coulomb gauge.
- Work underway on adding showers with M. Krämer.