Pseudo-Goldstone masses in QCD: Confronting recent full QCD lattice data with Chiral Perturbation Theory

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Abstract

I present a selection of recent lattice data (by the major collaborations) for the pseudo-Goldstone boson masses in dynamical ($N_f = 2$) QCD, where the valence quarks are chosen exactly degenerate with the sea quarks. At least the more chiral points should be consistent with Chiral Perturbation Theory for the latter to be useful in extrapolating to physical masses. I discuss the chiral convergence behaviour and possible reasons why the expected chiral logs are barely visible in the data.

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Unquenching Lattice QCD = going chiral

Extract hadron properties from correlators

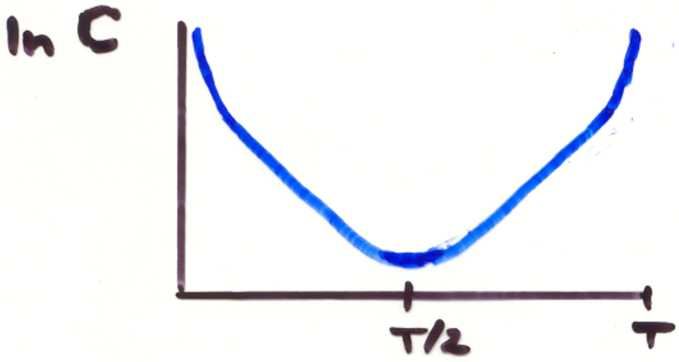
$$C(x) = \langle Q(x)^{+} Q(0) \rangle = \frac{1}{Z} \int DU \, D\bar{q} \, Dq \;\; Q(x)^{+} Q(0) \; e^{-S_G - S_F}$$

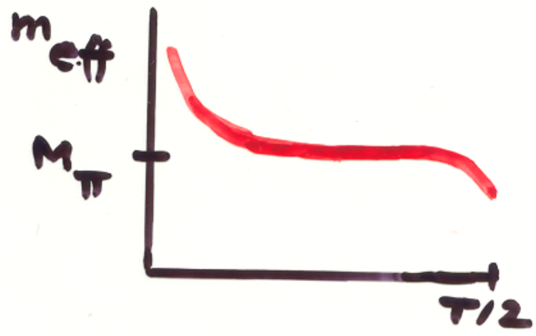
$Q(x) = \bar{q}(x) \, \Gamma \, q(x)$ or smeared

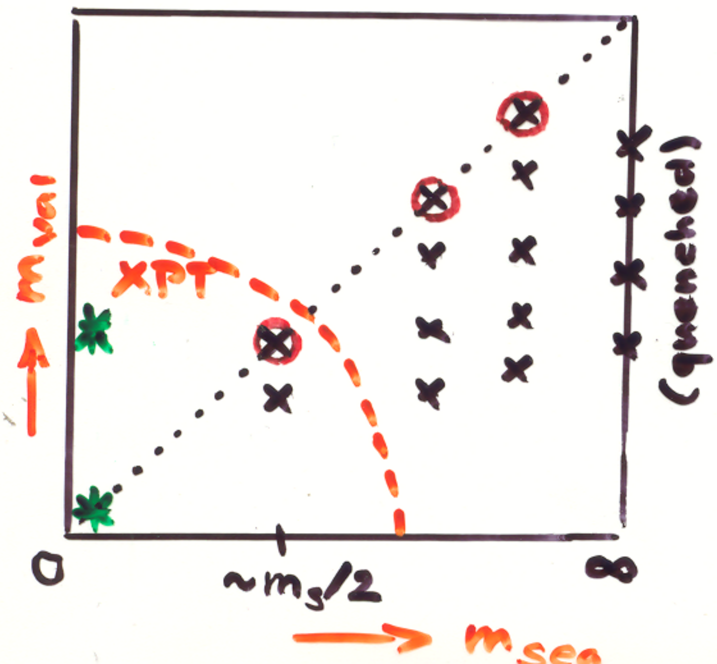
$Q(x) = \bar{d}(x) \, \gamma_5 \, u(x)$ for $\pi^{\pm}$

$$\langle \bar{q}(x) \Gamma_1 q(x) \; \bar{q}(0) \Gamma_2 q(0) \rangle = \frac{1}{Z} \int DU \;\; \det(D + m_{sea})^{N_f} \; e^{-S_G}$$

$$\times \, \mathrm{Tr} \; \Gamma_1 (D + m_{val})^{-1}_{x0} \; \Gamma_2 \, (D + m_{val})^{-1}_{0x}$$

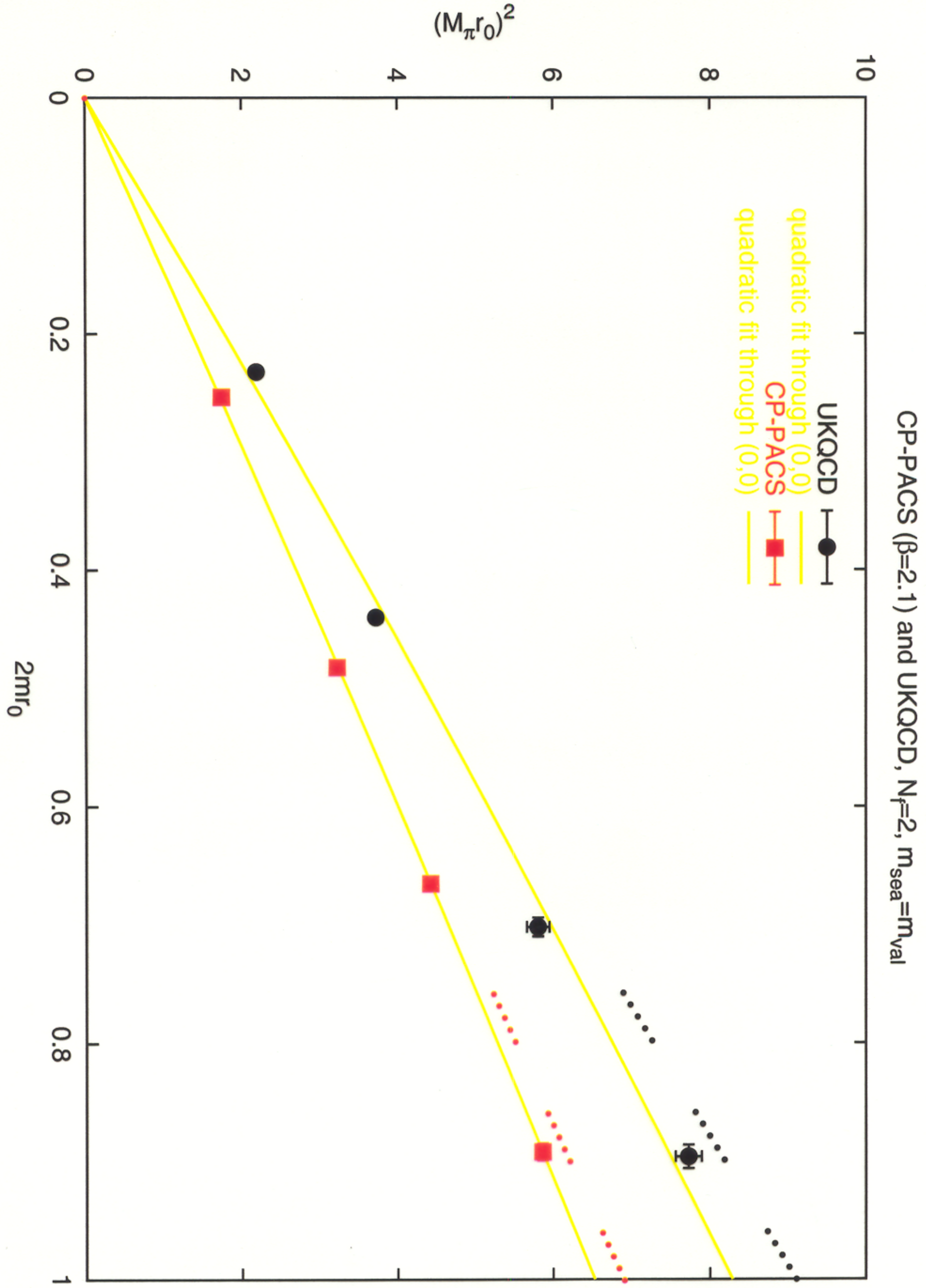






- extrapolate in m_{sea} and m_{val} to physical $m_{ud} \equiv \frac{m_u + m_d}{2}$
- can one use (PQ) XPT to do that ?

→ test using the diagonal (unitary) subset



UKQCD and CP-PACS data converted to the form M_π^2 vs. $2m$ (masses in units of r_0^{-1}, only latter set renormalized). Quadratic fits without a constant part – segments of the asymptotic slope in the chiral limit included to highlight the curvature.

Convergence pattern of χPT

Gasser Leutwyler:

χPT = expansion in p^2, m $\longrightarrow$ $\mathcal{L} = \mathcal{L}^{(2)} + \hbar \mathcal{L}^{(4)} + \dots$

$$\Sigma_\pi^{NLO} = $$

$\left(\begin{array}{l} \bullet : \text{from } \mathcal{L}^{(2)} \\ \square : \text{from } \mathcal{L}^{(4)} \end{array} \right)$

Bernard Golterman Sharpe:

PQ-χPT = generalization to $m_{val} \neq m_{sea}$

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Idea: Test for overlap in the restrictive case $m_{val} = m_{sea}$

(χPT: need $m \ll \Lambda_{QCD}$, Lat: cheaper for $\Lambda_{QCD} < m$)

LO: $M_\pi^2 = 2mB \equiv M^2$ $\quad \left(m \equiv \frac{m_u + m_d}{2} \right)$

NLO: $M_\pi^2 = M^2 \left(1 - \frac{M^2}{32\pi^2 F^2} \log\left(\frac{\Lambda_3^2}{M^2} \right) \right)$

NNLO: $M_\pi^2 = M^2 \left(1 - \dots + \frac{M^4}{256\pi^4 F^4} \left(\frac{17}{8} \log^2\left(\frac{\Lambda_M^2}{M^2} \right) + k_M \right) \right)$

↑ Bürgi Colangelo

$\Lambda_3 = 0.6\text{ GeV}\ {}^{+1.4}_{-0.4}\text{ GeV}$

$\Lambda_M = 0.6\text{ GeV} \pm 0{,}03\text{ GeV}$

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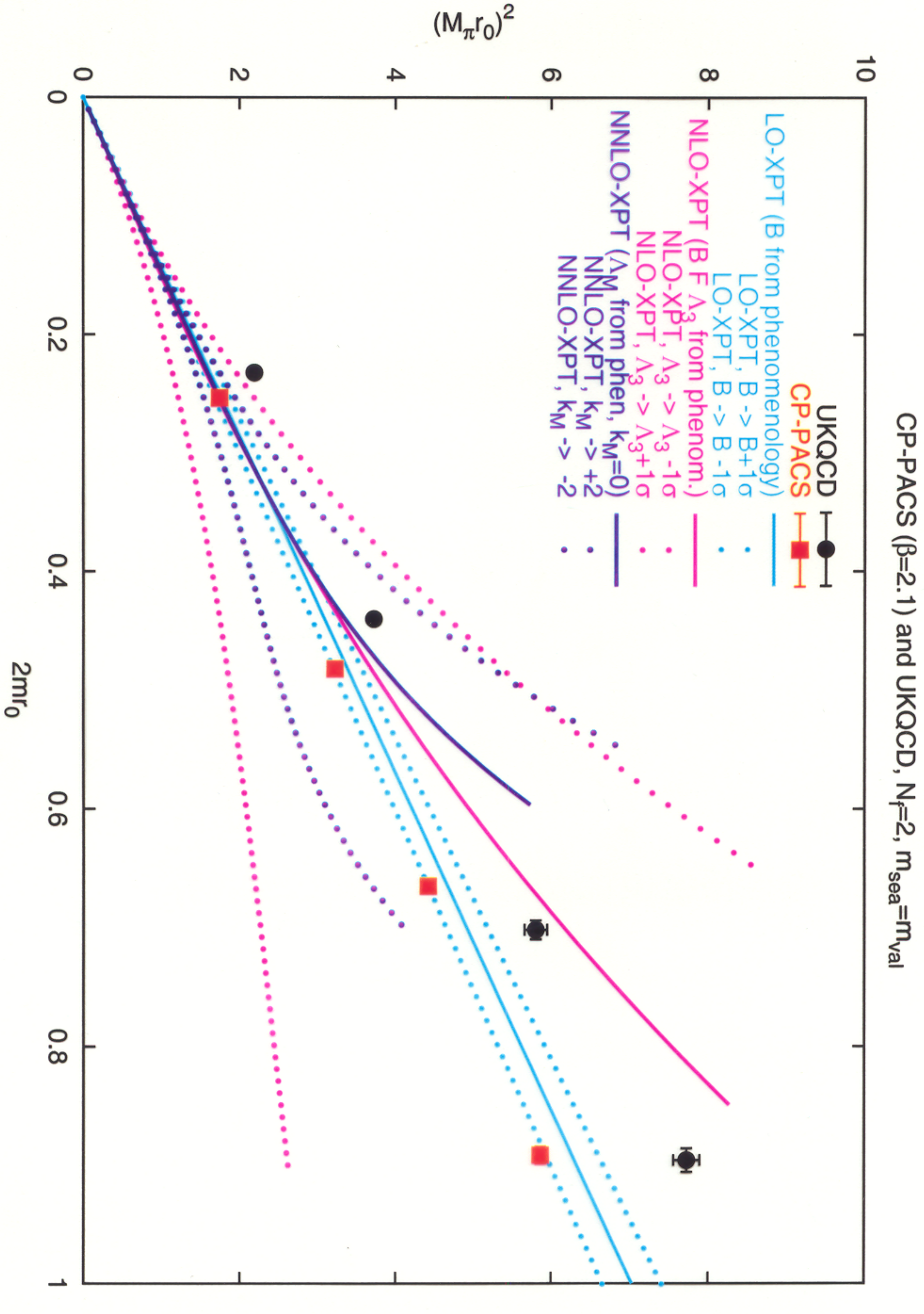
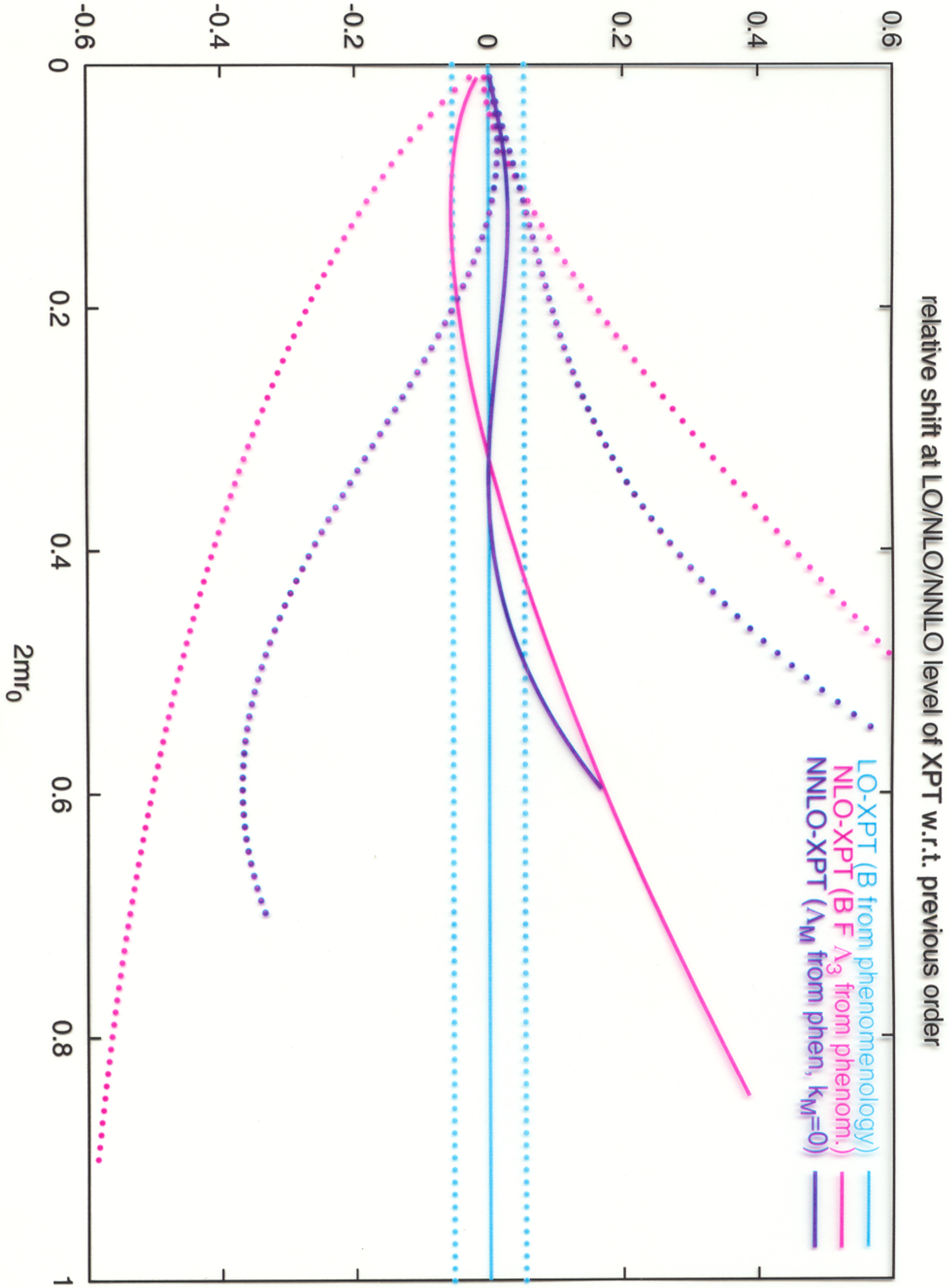
Asymtotic series: behaviour suggests NLO useful if

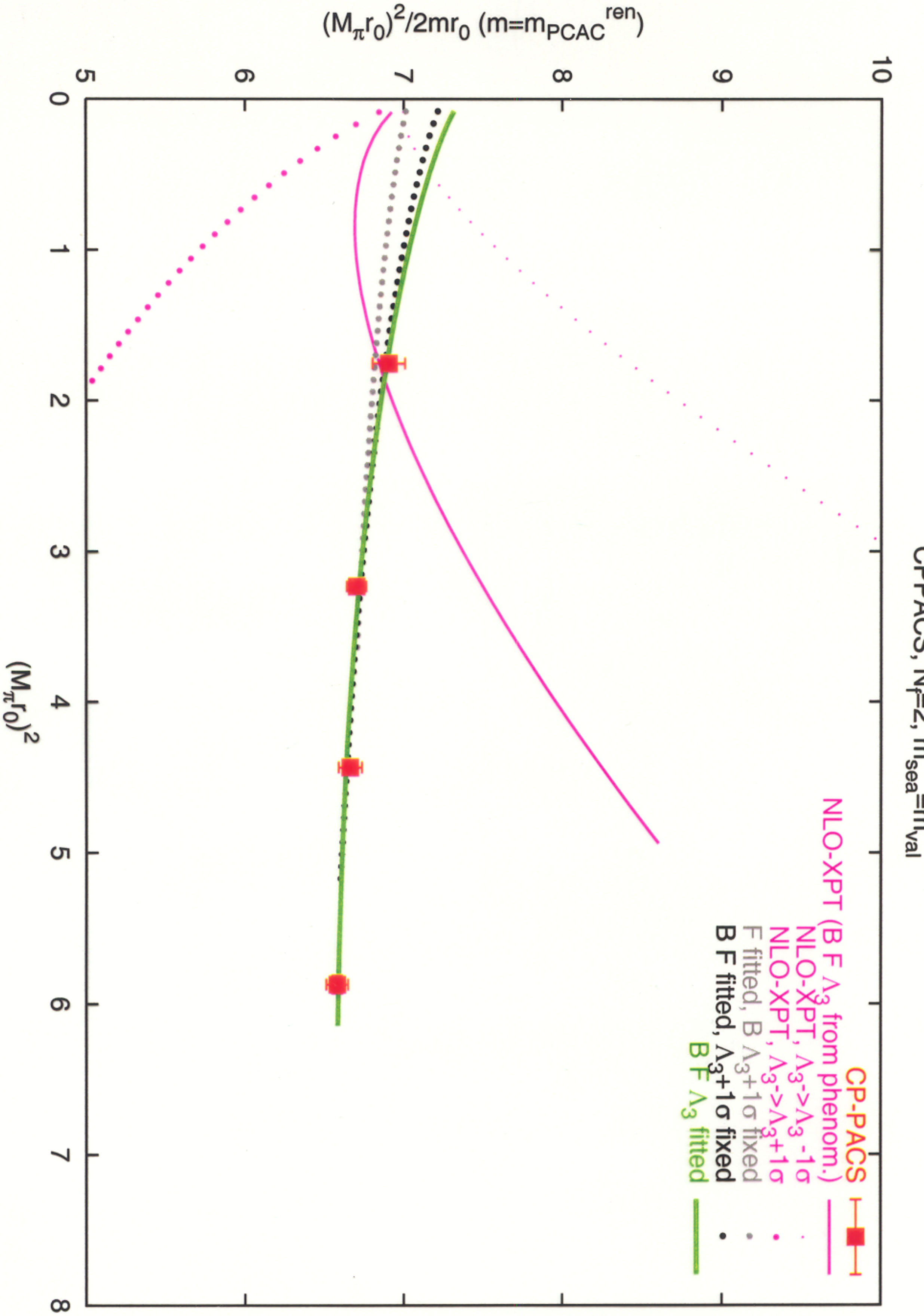
$2mr_0 < 0.25 \Leftrightarrow (M_\pi r_0)^2 < 2 \Leftrightarrow M_\pi < 560\text{ MeV}$



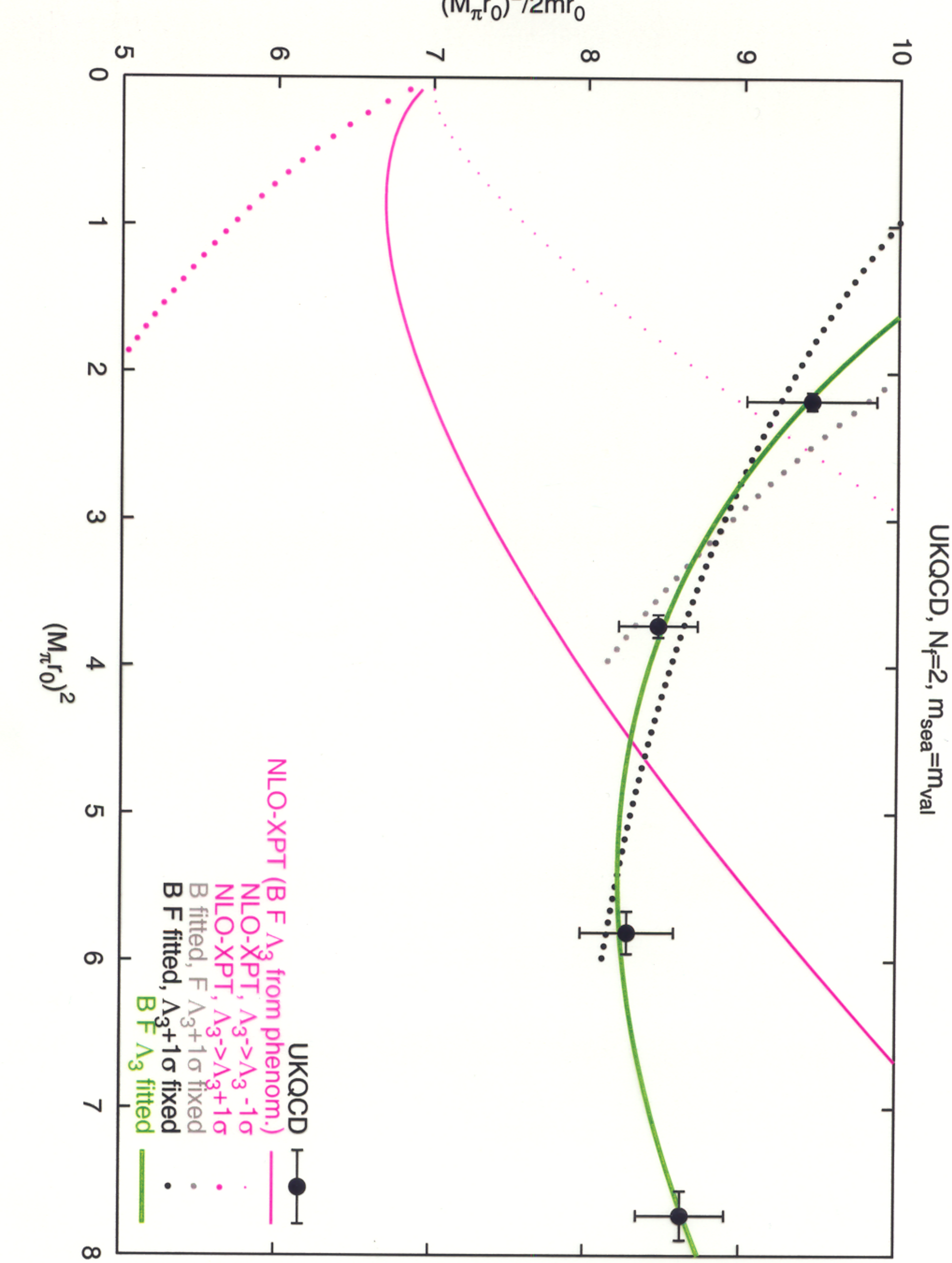
UKQCD and CP-PACS data converted to the form M_π^2 vs. $2m$. LO/NLO/NNLO chiral predictions with phenomenological values for $B, F, \Lambda_3, \Lambda_M, k_M$ (parameter-free predictions – no fits !).



Relative shifts in M_π^2 versus $2mr_0$ in XPT; due to uncertainty in B at LO (light), due to NLO contribution added to LO formula with B fixed at central value, Λ_3 varied within 1σ bound (pink), due to NNLO contribution added to NLO formula with B and Λ_3 fixed at central values, $k_M \in \{0, \pm 2\}$ (dark).



Attempts to fit the CP-PACS data with the NLO functional form with various constraints on B, F, Λ_3 *(relaxing them from the phenomenological* $B, F, \Lambda_3{+}\Delta\Lambda_3$ *values). Note that there is a priori no reason why non-continuum extrapolated data with such heavy quark masses should follow the chiral prediction.*



Attempts to fit the UKQCD data with the NLO functional form with various constraints on B, F, Λ_3 *(relaxing them from the phenomenological* $B, F, \Lambda_3 + \Delta\Lambda_3$ *values). Note that there is a priori no reason why non-continuum extrapolated data with such heavy quark masses should follow the chiral prediction.*

		Br_0	$F_\pi r_0$	$\Lambda_3 r_0$	χ^2/d.o.f.
CP-PACS	(i)	(7.04)	0.692	(5.03)	0.86/1
	(ii)	7.25	0.539	(5.03)	0.27/1
	(iii)	7.37	0.453	4.34	0.21/1
UKQCD	(i)	14.3	(0.233)	(5.03)	1.07/1
	(ii)	11.1	0.319	(5.03)	0.84/1
	(iii)	13.4	0.210	3.83	0.05/1
Pheno (GL)		7.04	0.233	1.51	—

NLO:
$$\frac{(M_\pi r_0)^2}{2 m r_0} = Br_0 - \frac{(M_\pi r_0)^2 \, Br_0}{32\pi^2 (F_\pi r_0)^2} \log\left(\frac{(\Lambda_3 r_0)^2}{(M_\pi r_0)^2}\right)$$

- Agenda: include NP renormalization for UKQCD data as soon as available

$$m^{\overline{MS}}(\mu = \tfrac{1}{a}) = Z_A / Z_P \cdot (1 + \ldots)\; m^{lat}$$

- No conclusion at this time!

  Encouraging: None of the fitted parameters off by orders of magnitude

- Long term: Clean determination of Λ_3

$$l_3^r = 8(2L_6^r - L_4^r) + 4(\underbrace{2L_8^r - L_5^r}) - \frac{1}{576\pi^2}\left(\log\frac{M_\eta^2|_{m_{ud}=0}}{\mu^2} + 1\right)$$

constrains KM ambiguity (strong CP by $m_u = 0$)

Summary

- to date „one-point" overlap of unquenched ($N_f = 2$, clover-improved) lattice data with region where χPT holds

- warranted use of χPT to extrapolate to physical m_{ud} only after more chiral points have been added

- unwarranted NLO fits to existing data work surprisingly well; situation may change as error-bars shrink

- besides going more chiral, computing the renormalization factors non-perturbatively and doing the continuum-limit seems crucial

Acknowledgements:

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