Lattice Study of the Coleman–Weinberg mass in the SU(2)-Higgs model

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1. Introduction
2. Lattice formulation
3. Simulations
4. Results and conclusion

1. Introduction

Symmetry breaking is a fundamental concept

Popular mechanisms: Higgs mechanism, radiative symmetry breaking (Coleman-Weinberg mechanism)

Connected to the problem of vacuum stability, i.e. the value of the minimal Higgs mass

We study symmetry breaking on the lattice in the bosonic sector of the SU(2)-Higgs model

Difficult two scale problem

2. Lattice formulation

In perturbation theory SB occurs through radiative corrections if the mass parameter (m_0) is zero.
The resulting Higgs mass is of the order of the smallest possible mass value in the broken case.
In the same sense m_0=0 is close to the border of the SSB and no symmetry breaking case.

The lattice action in standard notation is:

$$S[U,\varphi] = \beta \sum_{pl} \left(1 - \frac{1}{2} \operatorname{Tr} U_{pl} \right) +$$

$$+ \sum_x \left\{ \frac{1}{2} \operatorname{Tr} \left(\varphi_x^\dagger \varphi_x \right) + \lambda \left[\frac{1}{2} \operatorname{Tr} \left(\varphi_x^\dagger \varphi_x \right) - 1 \right]^2 - \kappa \sum_{\mu=1}^{4} \operatorname{Tr} \left(\varphi_{x+\hat{\mu}}^\dagger U_{x\mu} \varphi_x \right) \right\}$$

The continuum bare parameters are given:

$g^2 = 4/\beta$, $\lambda_c = \lambda/4/\kappa^2$, $\varphi_c = \sqrt{2\kappa/a}\varphi$ and $a^2 m_0^2 = (1 - 2\,\lambda)/\kappa - 8$.

The action is bounded from below for positive λ, while it is not for $\lambda < 0$.

Thus λ=0 is the interesting point relevant to the Coleman-Weinberg mass. Since the Higgs mass is a monotonous function of λ/κ^2, the limit gives the smallest Higgs mass possible. Our aim is to determine the Higgs mass in this region and try to interpret the results in terms of the CW mechanism.

3. Simulations

It is essential to simulate along a line of constant physics (LCP).
For CW we define the LCP with
λ=0, g_R^2=fixed and κ tuned to the finite temperature phase transition point.
We simulate on $L_t \cdot L_s^3$ hot lattices ($L_t << L_s$) and perform an $L_s \to \infty$ extrapolation.
Next we extrapolate to the continuum limit ($L_t \to \infty$).

In practice we take λ very small ($5 \cdot 10^{-6}$, $2.875 \cdot 10^{-5}$, $5.25 \cdot 10^{-5}$), fix $\beta = 8$ and find κ_{cr} by constrained simulation.

The physical scale is found for each λ, β, κ_{cr} by performing simulations at T=0 and fixing M_W=80 GeV. The quantity we extrapolate is $R_{HW} = M_H/M_W$.

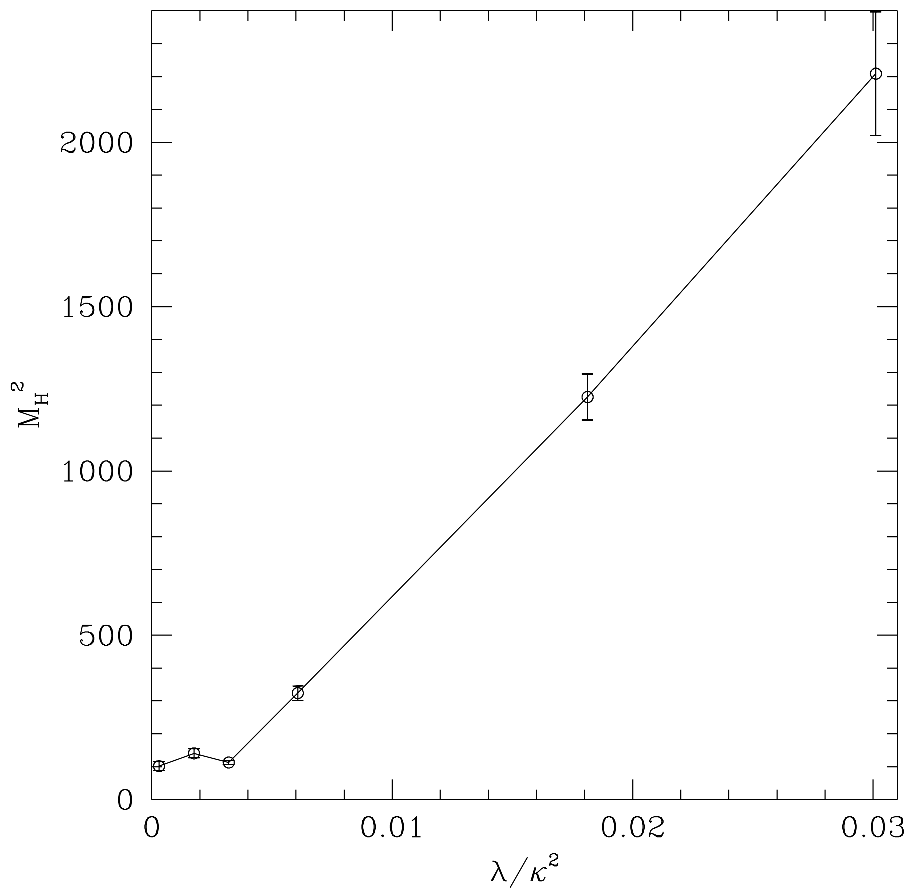
4. Results and conclusion

$L_t = 2$	κ_∞	R_{HW}
$\lambda = 5.25 \cdot 10^{-5}$	0.12822578(95)	0.1849(31)
$\lambda = 2.875 \cdot 10^{-5}$	0.12816615(128)	0.1726(41)
$\lambda = 5 \cdot 10^{-6}$	0.12811980(160)	0.1330(107)

$L_t = 3$	κ_∞	R_{HW}
$\lambda = 5.25 \cdot 10^{-5}$	0.12801466(21)	0.1615(50)
$\lambda = 2.875 \cdot 10^{-5}$	0.12792890(25)	0.1609(64)
$\lambda = 5 \cdot 10^{-6}$	0.12782043(24)	0.1246(245)

$L_t = 4$	κ_∞	R_{HW}
$\lambda = 5.25 \cdot 10^{-5}$	0.12797273(19)	0.1449(27)
$\lambda = 2.875 \cdot 10^{-5}$	0.12787487(18)	0.1531(225)
$\lambda = 5 \cdot 10^{-6}$	0.12774775(21)	0.1278(55)

$L_t = 5$	κ_∞	R_{HW}
$\lambda = 5.25 \cdot 10^{-5}$	0.12794617(14)	0.1505(61)
$\lambda = 2.875 \cdot 10^{-5}$	0.12783292(12)	0.1488(24)
$\lambda = 5 \cdot 10^{-6}$	0.12768277(20)	0.1240(141)



Higgs mass squared as a function of λ/κ_{cr}^2.

In one loop perturbation theory one gets

$$M_H^2 = v^2 \cdot (8\lambda_c + 12B), \quad B = \frac{9g^4}{1024\pi^2} \tag{1}$$

The CW case corresponds to $\lambda_c = -B/2$. The minimal Higgs mass is for $\lambda_c = -B$.

We get $M_{H,min}^2 = 115.78 \pm 4.95$ GeV^2 to be compared to the perturbative value: 22.80 GeV^2 (91.20 GeV^2).