Subleading Shape Functions and the Determination of V_{ub}

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Work in Collaboration with

C. W. Bauer and M. Luke (hep-ph/0102089, hep-ph/0205150)

see also A. Leibovitch, Z. Ligeti and M. Wise (hep-ph/0205148)

- $B \to X_u \ell \bar{\nu}_\ell$ versus $B \to X_s \gamma$
- Subleading shape functions
- V_{ub} from $B \to X_u \ell \bar{\nu}_\ell$ and $B \to X_s \gamma$

$B \to X_u \ell \bar{\nu}_\ell$ versus $B \to X_s \gamma$

- End-point region of $B \to X_s \gamma$ and $B \to X_u \ell \bar{\nu}_\ell$:
  - The invariant mass of the final state hadrons is "small"

$$(p_B - q)^2 \sim \mathcal{O}\sqrt{\Lambda_{QCD} m_b}$$

  - The hadronic energy is "large"

$$M_B - v \cdot q \sim \mathcal{O}(m_b)$$

- Expand in $m_b - n \cdot q$: Twist Expansion
  (analogous to Deep Inelsatic Scattering)

- In this region and to leading twist and leading order $\alpha_s(m_b)$:
  Both the Photon spectrum in $B \to X_s\gamma$ and the Lepton-Energy Spectrum in $B \to X_u \ell \bar{\nu}_\ell$ depend on the same non-perturbative input:

$$\frac{d\Gamma^{B\to X_s\gamma}}{dx} = \frac{G_F^2 \alpha m_b^5}{32\pi^4} |V_{ts}V_{tb^*}|^2 |C_7|^2 f(m_b[1-x]) \qquad x = \frac{2E_\gamma}{m_b}$$

$$\frac{d\Gamma^{B\to X_u\ell\bar{\nu}_\ell}}{dy} = \frac{G_F^2 m_b^5}{96\pi^3} |V_{ub}|^2 \int_{-m_b(1-y)}^{M_B-m_b} d\omega\, f(\omega) \qquad y = \frac{2E_\ell}{m_b}$$

where

$$2M_B f(\omega) = \langle B(v)|\bar{Q}_v \delta(\omega + iD_+) Q_v|B(v)\rangle \ : \ \text{Shape Function}$$

D_+: light-cone component of the heavy quark residual momentum.

Observables for the V_{ub} determination

- Model independent determination of $V_{ub}/(V_{ts}^* V_{tb})$

$\rightarrow$ Define Observables (E_c: Energy cut)

$$\Gamma_s(E_c) = \frac{2}{m_b} \int_{E_c}^{m_B/2} dE_\gamma (E_\gamma - E_c) \frac{d\Gamma^{B \to X_s \gamma}}{dE_\gamma}$$

$$\Gamma_u(E_c) = \int_{E_c}^{m_B/2} dE_\ell \frac{d\Gamma^{B \to X_u \ell \bar{\nu}_\ell}}{dE_\ell}$$

- To leading order we have

$$\left| \frac{V_{ub}}{V_{tb} V_{ts}^*} \right|^2 = \rho^2 + \eta^2 = \frac{3\alpha}{\pi} |C_7|^2 \frac{\Gamma_u(E_c)}{\Gamma_s(E_c)} + O(\alpha_s) + O\left(\frac{\Lambda_{QCD}}{m_B} \right)$$

Subleading Shape Functions

- Leading order shape function:
  Fourier transform of a Wilson line:

$$f(\omega) = \int \frac{dt}{2\pi} \langle B(v)|\bar{Q}_v(0)\mathcal{P}\exp\left(-i\int_0^t ds\, n\cdot A(n\cdot s)\right) Q_v(n\cdot t)|B(v)\rangle$$

- Subleading shape functions: Fourier Transforms of
  Wilson lines with insertions of derivatives: e.g.

$$\int \frac{dt}{2\pi} \langle B(v)|\bar{Q}_v(0)\mathcal{P}\exp\left(-i\int_0^t ds\, n\cdot A(n\cdot s)\right) (iD_\mu) Q_v(n\cdot t)|B(v)\rangle$$

- Subleading shape functions:
  Matrix elements of these non-local operators
- $G_2(\omega)$: Generalized kinetic energy operator
- $t(\omega)$: Insertions of the $1/m_b$ terms of the Lagrangian
- $H_2(\omega)$ and $h_1(\omega)$: Spin dependent functions
- $f(\omega)$ is linked to subleading functions via reparametrization invariance: Campanario, M., ...

$$F(\omega) = f(\omega) + \frac{1}{2m_b}\left[t(\omega) - 2\, G_2(\omega)\right]$$

In total three non-perturbative functions up to $1/m_b$

Moment Expansion

- From the ususal $1/m_b$ Expansion we know the moments:

$$
\begin{aligned}
f(\omega) &= \delta(\omega) - \frac{\lambda_1}{6}\delta''(\omega) - \frac{\rho_1}{18}\delta'''(\omega) + \dots \\
G_2(\omega) &= -\frac{2\lambda_1}{3}\delta'(\omega) + \dots \\
t(\omega) &= -(\lambda_1 + 3\lambda_2)\,\delta'(\omega) + \frac{\tau}{2}\delta''(\omega) + \dots \\
h_1(\omega) &= \lambda_2\,\delta'(\omega) + \frac{\rho_2}{2}\delta''(\omega) + \dots \\
H_2(\omega) &= -\lambda_2\,\delta'(\omega) + \dots\,,
\end{aligned}
$$

V_{ub} from $B \to X_u \ell \bar{\nu}_\ell$ and $B \to X_s \gamma$

- Including the subleading functions:

$$\frac{d\Gamma^{B\to X_s\gamma}}{dE_\gamma} = \frac{\Gamma_0^{B\to X_s\gamma}}{m_b}\Bigg[(4E_\gamma - m_b)F(m_b - 2E_\gamma) + \frac{1}{m_b}\left[h_1(m_b - 2E_\gamma) + H_2(m_b - 2E_\gamma)\right]\Bigg] + O\left(\frac{\Lambda_{QCD}^2}{m_b^2}\right)$$

$$\frac{d\Gamma^{B\to X_u\ell\bar{\nu}_\ell}}{dE_\ell} = \frac{4\,\Gamma_0^{B\to X_u\ell\bar{\nu}_\ell}}{m_b}\int d\omega\,\theta(m_b - 2E_\ell - \omega) \left[F(\omega)\left(1 - \frac{\omega}{m_b}\right) - \frac{1}{m_b}h_1(\omega) + \frac{3}{m_b}H_2(\omega)\right] + O\left(\frac{\Lambda_{QCD}^2}{m_b^2}\right)$$

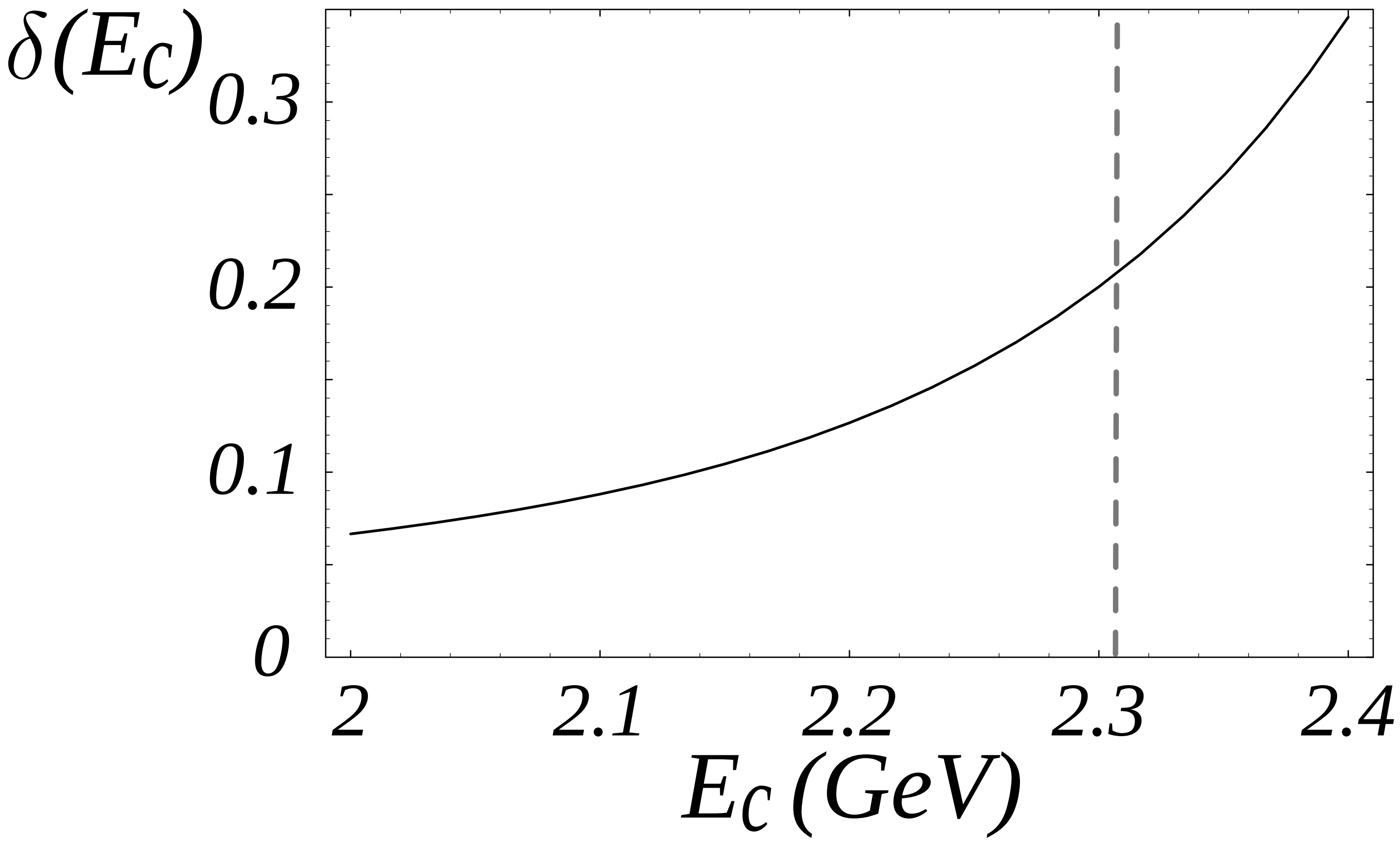
- In terms of the partially integrated rates:

$$\left|\frac{V_{ub}}{V_{tb}V_{ts}^*}\right| = \left(\frac{3\alpha}{\pi}|C_7^{\rm eff}|^2\frac{\Gamma_u(E_c)}{\Gamma_s(E_c)}\right.$$

$$\left.\left[1+\frac{1}{m_b}\frac{\int_{E_c}^{m_B/2}dE\int_{-\infty}^{m_b-2E}d\omega\left[2h_1(\omega)-2H_2(\omega)-\omega f(\omega)\right]}{\int_{E_c}^{m_B/2}dE\int_{-\infty}^{m_b-2E}d\omega F(\omega)}\right]\right)^{\frac{1}{2}}$$

$$\equiv \left(\frac{3\alpha}{\pi}|C_7^{\rm eff}|^2\frac{\Gamma_u(E_c)}{\Gamma_s(E_c)}\right)^{\frac{1}{2}}(1+\delta(E_c))\,,$$

- Model the subleading shape functions:
  (Keeping the correct moments)



Conclusions

- Lowering E_c: Less sensitivity to (the higher moments of) the (subleading) shape functions !

→ Transitons region from the shape function regime to the regime of the $1/m_b$ expansion

- Taking the size of the modelled subleading terms may be too conservative: leads to sizable uncertainties $\Delta V_{ub}/V_{ub} \sim 15$ %
- Moments are known: May lead to lower uncertainties if E_c can be lowered Neubert
- More investigations are needed / underway:
  Finally $\Delta V_{ub}/V_{ub} \leq 10$ % may become possible.