CKM matrix elements and Lattice QCD

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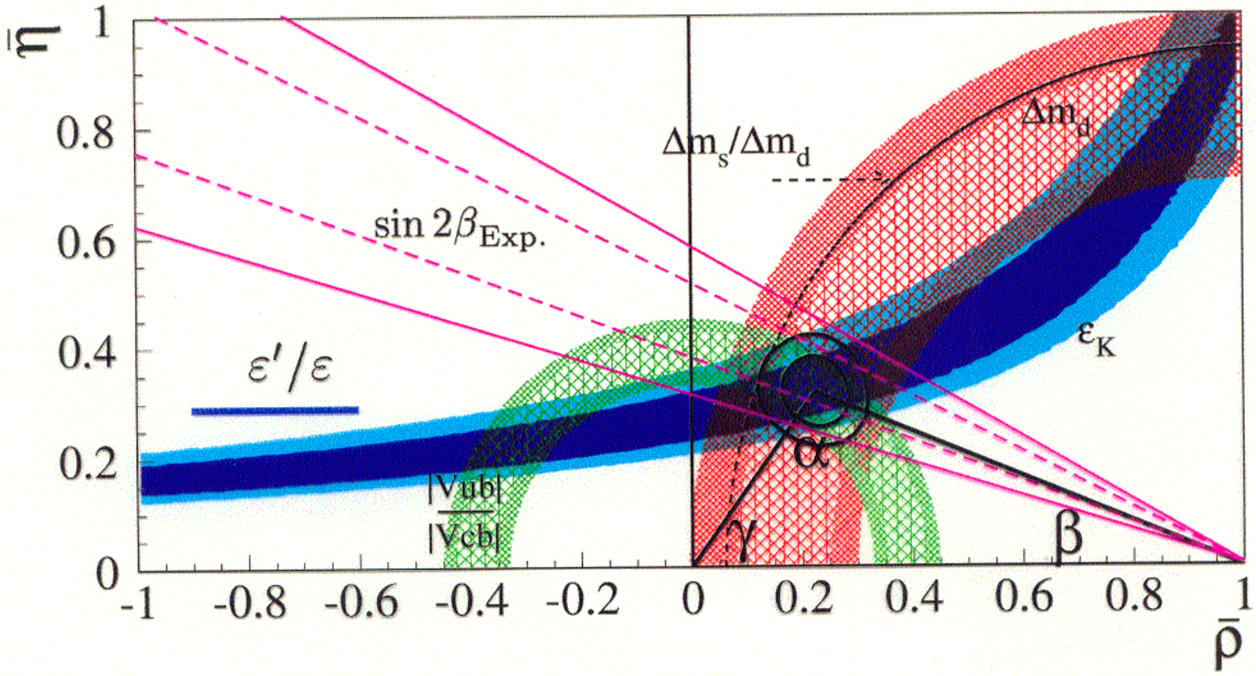
CKM Unitarity Triangle



Main constraints

$$|\varepsilon_K| = C_K \hat{B}_K A^2\lambda^6 \bar{\eta} \left[A^2\lambda^4 (1-\bar{\rho}) F_{tt} + F_{tc}\right]$$

$$\Delta m_d = C_B m_{B_d} f_{B_d}^2 \hat{B}_{B_d} A^2\lambda^6 [(1-\bar{\rho})^2 + \bar{\eta}^2]$$

$$\frac{\Delta m_d}{\Delta m_s} = \frac{m_{B_d}}{m_{B_s}} \frac{f_{B_d}^2 \hat{B}_{B_d}}{f_{B_s}^2 \hat{B}_{B_s}} \lambda^2 [(1-\bar{\rho})^2 + \bar{\eta}^2]$$

$$\left|\frac{V_{ub}}{V_{cb}}\right| = \frac{\lambda}{1-\lambda^2/2}\sqrt{\bar{\rho}^2 + \bar{\eta}^2}$$

(see talks by A.Stocchi and by F.Parodi)

Must provide a model independent determination of the hadronic quantities

$$\widehat{B}_K \qquad f_{B_d}\sqrt{B_{B_d}} \qquad \xi = \frac{f_{B_s}}{f_{B_d}}\sqrt{\frac{B_{B_s}}{B_{B_d}}}$$

Line-up of the "back-up" quantities relevant for the CKM analyses:

- $|V_{cb}|$ from the exclusive semileptonic $B \to D^{(*)}$ decay modes *(talk by J.Simone)*
- $|V_{ub}|$ from $B \to \pi(\rho)\ell\nu$
- $|V_{ts}|$ and/or non-SM physics through $B \to K^{(*)}\ell^+\ell^-$
- matrix elements that might help solving(?) the $K \to \pi\pi$ puzzle (ε'/ε)
- quantities necessary for the modern studies of the factorisation in the non-leptonic decays (moments of the LCwf, decay constants, hadronic couplings such as $B^*B\pi$, $D^*D\pi$, ...)

That's where the lattice QCD enters the stage and proposes a complete non-perturbative first principle method to compute the above quantities to arbitrary accuracy.

Lattice QCD

In the Euclidean metric

$$Z_{QCD} = \int \mathcal{D}A_\mu e^{-S_g} \prod_q \det(\not{D} + m_q)$$

○ First principles:
The only parameters entering the computations are those which appear in the QCD lagrangian, namely

$$m_q \ (\kappa_q) \text{ and } g_0^2 \ (\beta)$$

○ Arbitrary accuracy:
Integral handled by using the Monte Carlo methods (the SU(3) gauge field configurations generated)

$$\Rightarrow \langle O \rangle = \frac{1}{Z_{QCD}} \int \mathcal{D}A_\mu e^{-S_{QCD}} O(x_1, \ldots, x_n) \simeq \frac{1}{N} \sum_i^N \{O_i\}$$

stat.errors $\propto 1/\sqrt{N}$ (central limit theorem)

Nowdays the stat.errors are at the level of a few % for almost all the quantities of phenomenological interest.

To make a problem solvable by a computer, significant approximations are needed :-(

- **Discretization effects:** finite lattice spacing "a"

$$\mathcal{F}(a) = \mathcal{F}(0) + a\mathcal{F}'(0) + \ldots$$

  (i) use OPE in "a" and improve the theory
  (get rid of $\mathcal{O}(a^n)$ effects) *K.Symanzik 1983*
  (ii)work at several (small) lattice spacings and go to $a \to 0$

- **Renormalization and Matching:**
  "a" hard cut-off : renormalization perturbative and non-perturbative (NPR in RI/MOM and SF schemes)
  G.Martinelli et al. 1995, M.Lüscher et al. 1996

- **Quenching errors:**
  dynamical quark loops left out ($n_f = 0$):

$$\det(\not{D} + m_q) = \text{const.}$$

  (nowdays start probing the $n_f = 2$ world)

- **Physical quark masses:** current lattices not as fine as to resolve m_b, nor the lattice sizes are large enough to accomodate very light pseudoscalar mesons

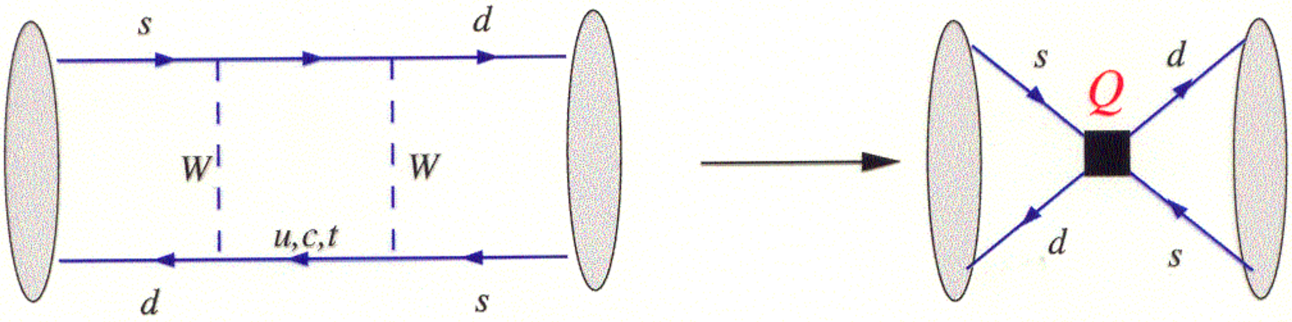
All systematic uncertainties improvable: you only need to be strong and clever

⊗ TFlop computing resources: CP-PACS, QCDOC, APE-NeXt

$K^0 - \overline{K}^0$ Mixing Amplitude

$$\varepsilon_K \sim \langle \bar{K}^0 | \mathcal{H}_{\rm eff}^{\Delta S=2} | K^0 \rangle$$

$$= C(\mu) \cdot \langle \bar{K}^0 | \underbrace{\bar{s}\gamma_\mu(1-\gamma_5)d\, \bar{s}\gamma_\mu(1-\gamma_5)d}_{Q_1(\mu)} | K^0 \rangle$$



a) $C(\mu)$ info on the high energy dynamics: **Perturbation Theory**

see compendium by A.Buras et al. 1995, U.Nierste, 1995

b) low energy QCD dynamics: **Lattice QCD**

$$\langle \bar{K}^0 | Q_1(\mu) | K^0 \rangle = \frac{8}{3} f_K^2 m_K^2 B_K(\mu)$$

(*V.S.A. in M.K.Gaillard, B.W.Lee, 1974*)

★ ★ ★

Scale dependence in $B_K(\mu)$ and $C(\mu)$ cancel against each other. AD coefficients known at NLO ⇒ define

$$\hat{B}_K = \alpha_s(\mu)^{-\frac{\gamma_0}{2\beta_0}} \left(1 + \frac{\alpha_s(\mu)}{4\pi} J\right) B_K(\mu)$$

(G.Altarelli et al, 1981; A.Buras at al, 1990)[in the $\overline{\rm MS}$ scheme]

$\hat{B}_K$ from lattice QCD

story about the light quarks on the lattice

When discretising $\mathbf{D} = \gamma_\mu D_\mu + m_q$ naively → quark propagator contains two distinct poles in one Brillouin zone.

To kill the doublers ⇒ sacrifice a symmetry (restored in $a \to 0$)!

- ♣ **Staggered fermions**, preserve chiral symmetry but the flavor symmetry lost (Q_1^{latt} way too complicated)

  $$\hat{B}_K = 0.86(6)$$

  JLQCD (S.Aoki et al.) 1998

  Unquench, *G.Kilcup et al. 1997*: 0% (n_F=2) & +5% (n_F=4)

- ♣ **Wilson fermions**, break chiral symmetry

  $$\hat{B}_K = 0.94(10)$$

  JLQCD (S.Aoki et al.) 1999

  Unquench, *APE (D.B. et al.) 2001*: n_F=2 ≈ quenched

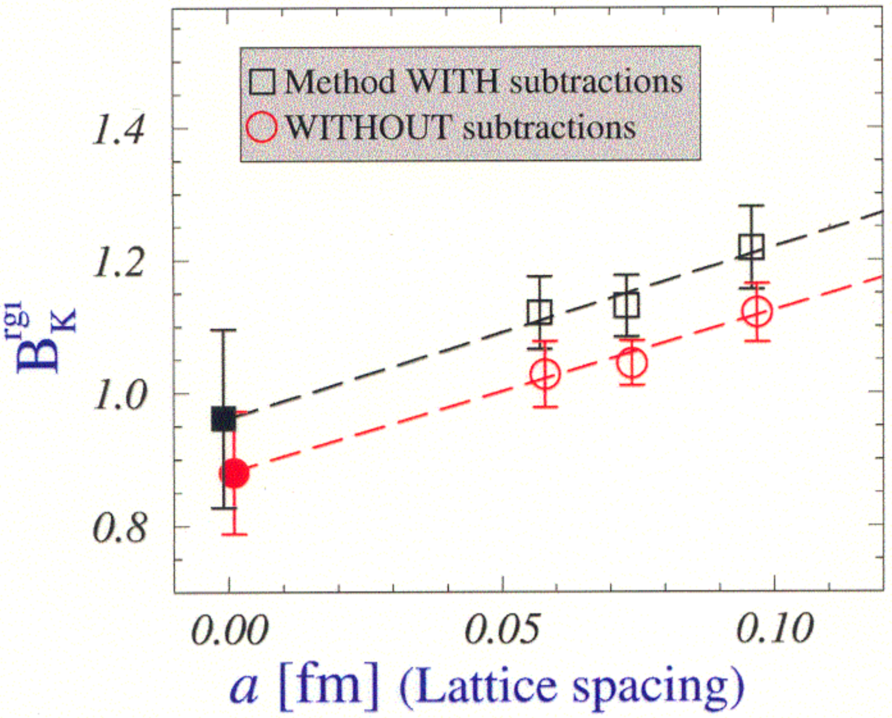
- ♣ **Domain wall fermions**, preserve chirality by satisfying the Ginsparg-Wilson relation

  $$\gamma_5 D + D\gamma_5 = \frac{1}{\rho} D\gamma_5 D$$

  $$\hat{B}_K = 0.79(3) \text{ and } 0.73(2)$$

  CP-PACS (A.Ali Khan et al.) 2001; RBC (T.Blum et al.) 2001

  N.B. $m_{res} \to 0$ as $L_5 \to \infty$



♣ **This year $SPQ_{cd}R$, new results:**

(a) High statistics lattice study with Wilson fermions

(b) 3 values of lattice spacings (extrapolated to the continuum)

(c) avoided spurious subtractions with other (parity even) 4f-operators (*D.Becirevic et al, 2000*)

(d) renormalisation non-perturbative in the RI/MOM scheme, (*A.Donini et al, 1999*)

(e) conversion to the RGI form at NLO in perturbation theory (*Ciuchini et al, 1998; Buras at al, 2000*)

$$\hat{B}_K = 0.88(9) \quad \text{and} \quad \hat{B}_K = 0.96(13)$$

lead to

$$\hat{B}_K = 0.91(7)$$

♣ **This year first study with overlap fermions:**

[chirality guaranteed by working in 4-dimensions!]

(i) Boston-Marseille (N.Garron et al.) $\hat{B}_K = 0.84(10)(??)$

(ii) T.DeGrand, $\hat{B}_K = 0.90(5)(??)$

Current W.A. (*see talk by L. Lellouch*)

$$\hat{B}_K = 0.86(6)(14)$$

Comments and perspectives:

- ALPHA collab.is performing the high statistics calculation of $\hat{B}_K$ by using the tmQCD (*Frezzotti et al. 2000*); results expected next year
- More studies with DW and overlap fermions
- Systematic errors are large due to unquenching which -IMHO- is overestimated. Unquenched studies mandatory! ○ Unquenched studies mandatory (SPQcdR): $\sin(2\beta)$ and ε_K are constraining quite impressively $\hat{B}_K$ *(see Parodi's talk)*.

f_B from the lattice

story about the heavy quarks on the lattice

m_b is [redacted] too [redacted] heavy for the presently accessible lattices

$$1/m_b < a \lesssim 1/m_c$$

4 ways of treating the heavy quarks on the lattice,

4 ways to compute $\langle 0|\bar{b}\gamma_\mu\gamma_5 q|B_q\rangle = ip_\mu f_{B_q}$

- ♠ **Lattice QCD** with fully relativistic quarks that are accessible: extrapolate to $1/m_B$ by using the heavy quark scaling laws (*APE, UKQCD*)
- ♠ **HQET** (static limit) $m_b \to \infty$: $\mathcal{L}_{\text{HQET}} = Q^\dagger D_4 Q$
  - bad signal/noise : need huge statistics
  - problems with renormalisation of the axial current (solved)

  (*ALPHA, SPQcdR*)
- ♠ **NRQCD** (static limit + $1/m_b$ terms which are cut-off as $m_Q v \ll m_Q$) : $\mathcal{L}_{\text{NRQCD}} = Q^\dagger \left(D_4 - (\vec{D}^2 + \vec{\sigma}\cdot\vec{B})/2m_Q\right) Q$
  - expansion in $1/(am_Q) \Rightarrow$ no continuum limit
  - problems in including terms $\propto 1/m_Q$ in renormalisation

  (*CP-PACS, JLQCD, GLOK*)
- ♠ **FNAL approach**: use the full QCD Wilson action and go over the cut-off; redefine the mass and reinterpret the theory in terms of $1/m_Q$ expansion; in some cases in "renormalon shadows" (*FNAL, CP-PACS, MILC*)

Novelties and results

Quenched values: steady the same

Raporteurs at the Lattice conferences

f_B[MeV] =	175±25	*(Flynn, 1996)*
	163 ±23	*(Onogi, 1997)*
	165 ±20	*(Draper, 1998)*
	170 ±20	*(Hashimoto, 1999)*
	175 ±20	*(Bernard, 2000)*
	173 ±23	*(Ryan, 2001)*
	173 ±23	*(Yamada, 2002)*

New study by TOV group *(M.Guagnelli et al,2002)*: use the step scaling function ⇒ precision QCD computation of f_B. Preliminary quenched result: f_B=170(11)(??) MeV.

Unquenched values: higher than quenched

Extensive study after including n_F=2 this year by MILC. First study with n_F=3 by MILC *(S.Gottlieb et al,2002)* get

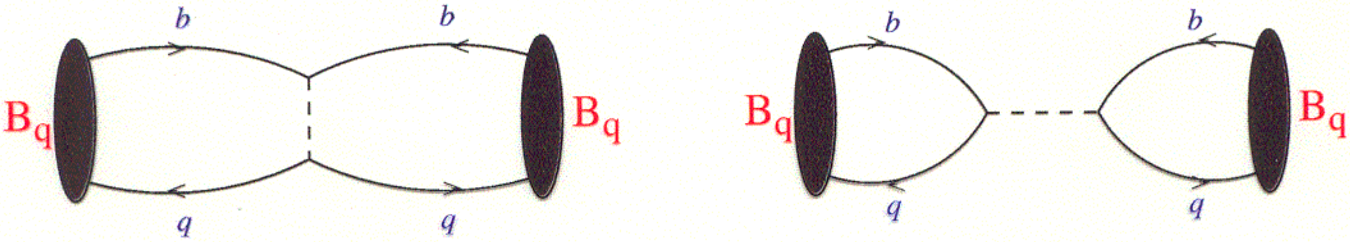
$$\frac{f_B^{(n_F=3)}}{f_B^{(n_F=0)}} = 1.18(1)\left({}^{+4}_{-1}\right)$$

They also show that f_{B_s}/f_{B_d} remains stable when $n_F = 0 \to 2 \to 3$

B_B-parameter from the lattice

$$\langle \bar{B}^0 | Q(m_b) | B^0 \rangle = \frac{8}{3} f_B^2 m_B^2 B_B(m_b)$$

Results presented in the $\overline{\text{MS}}$ scheme @ $\mu = m_b$



⊗ **HQET** : Static limit $m_b \to \infty$ (heavy quark decoupled!) pert. matching (@ NLO) to the operator $Q(\mu)$ renormalized in the continuum (*V.Giménez, J.Reyes, 2000*)

$$B_B(m_b) = 0.83(5)(6) \qquad B_{B_s}/B_B \approx 1$$

⊗ **NRQCD** : $1/m_b$ corrections included in the NRQCD action and partly in 4f-operator. Matching to the continuum, perturbative. *Hi-KEK (S.Hashimoto et al), 2000*

$$B_B(m_b) = 0.85(3)(11) \qquad B_{B_s}/B_B = 1.01(3)$$

⊗ **QCD** : Heavy quark treated relativisticaly, but $m_c \lesssim m_Q < m_b$ Extrapolate to m_b, guided by HQS (scaling laws).
Non-perturbative matching *APE (D.Becirevic et al), 2000*

$$B_B(m_b) = 0.93(8)^{+0}_{-8} \qquad B_{B_s}/B_B = 0.98(5)$$

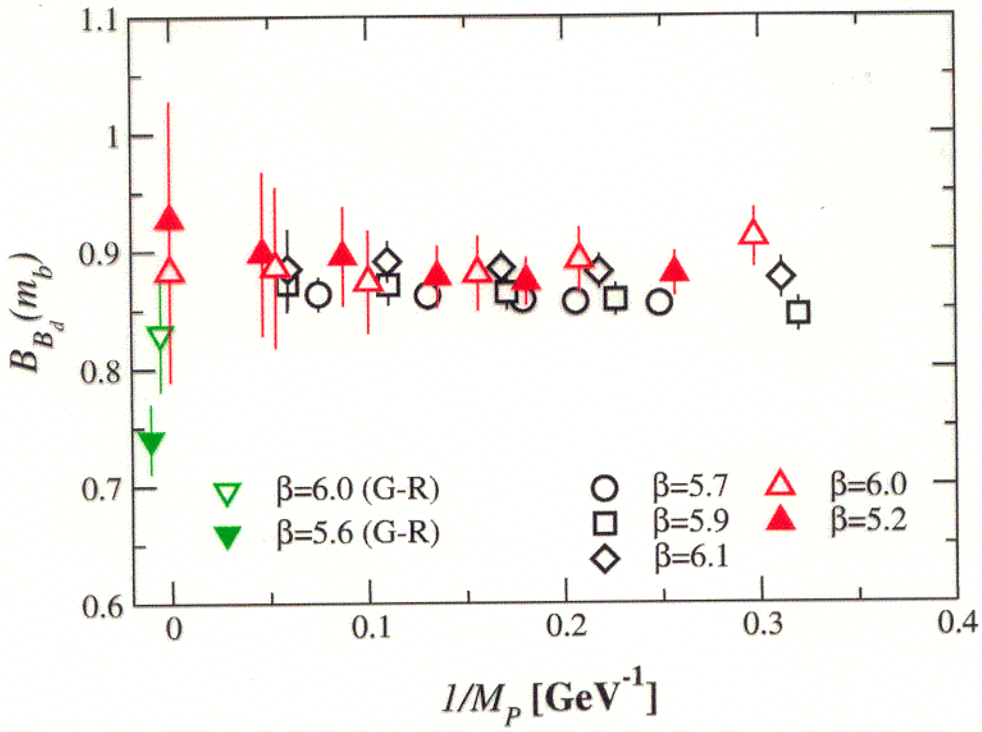
Perturbative matching *UKQCD (Lellouch, Lin), 2000*

$$B_B(m_b) = 0.91(4)^{+4}_{-0} \qquad B_{B_s}/B_B = 0.98(2)^{+0}_{-2}$$

Novelties: JLQCD & S.P.Qcd R.

JLQCD (S. Hashimoto, N. Yamada), 2001-2002

Unquenched simulation within the NRQCD approach



New results:

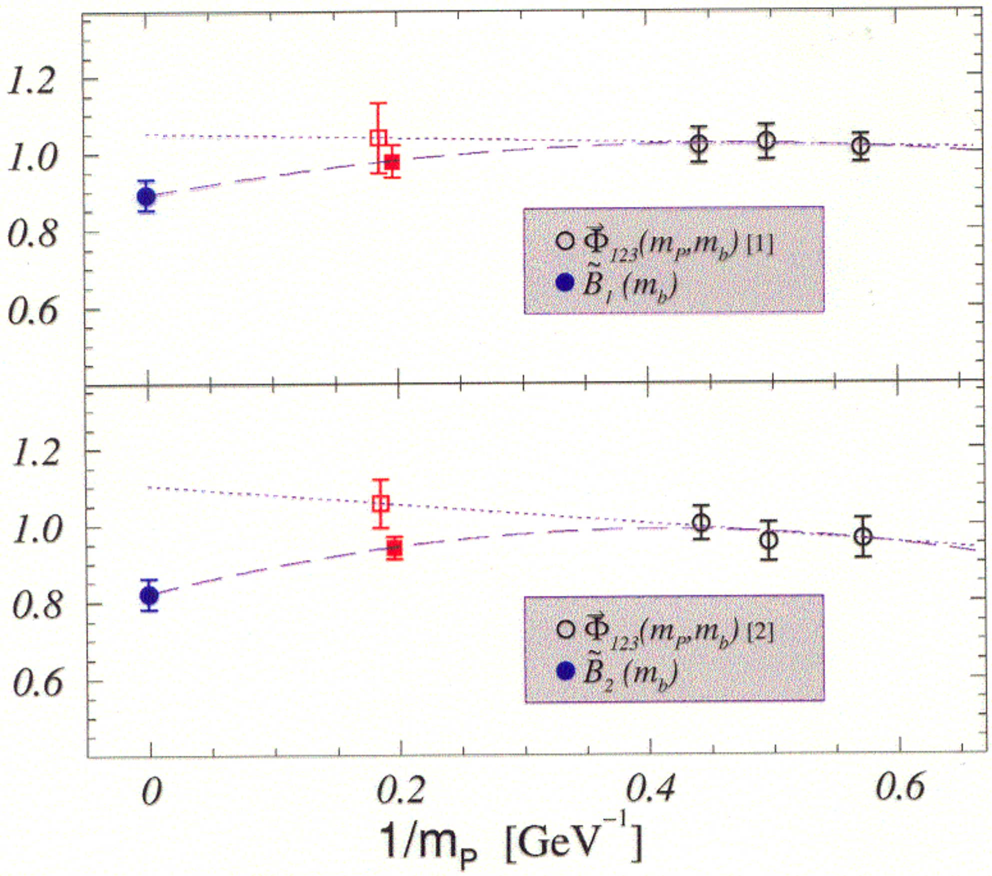
$$B_B(m_b) = \begin{cases} 0.85(2)(8) & (n_F = 0) \\ 0.83(3)(8) & (n_F = 2) \end{cases}$$

No sea quark effects!

S.P.Qcd R. (D. Becirevic, et al), 2001

- Combine the static HQET results for B-parameters with the relativistic lattice QCD ones
  $\Rightarrow$ extrapolation $\rightarrow$ "interpolation"

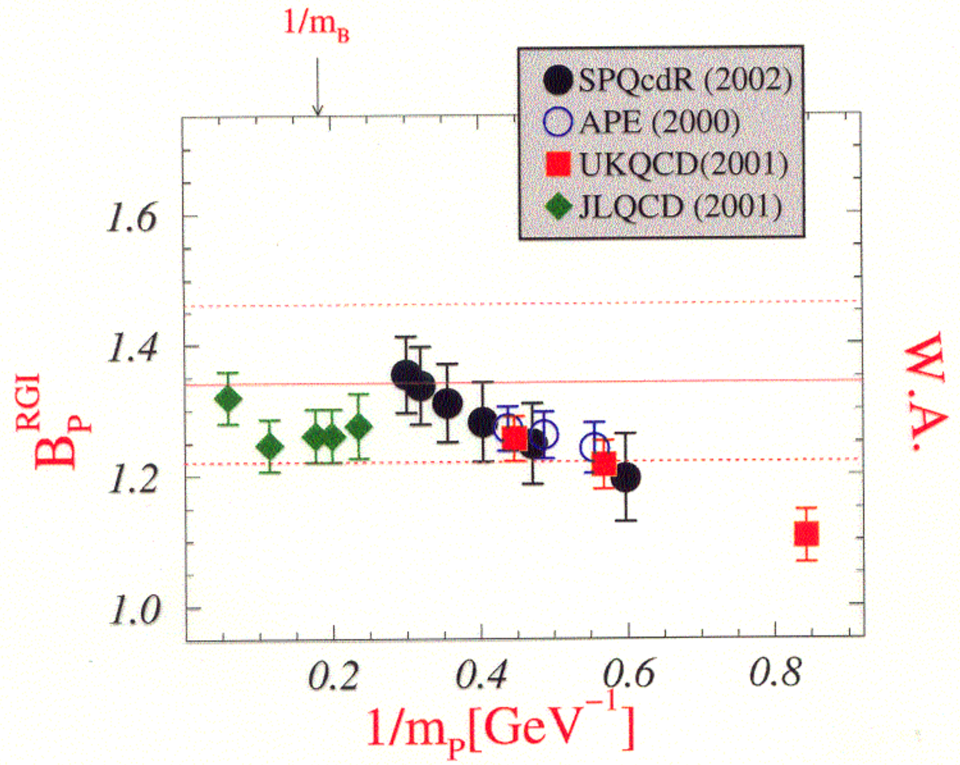
- Perturbative matching of the anomalous dimensions of 4-f QCD and HQET operators made @ NLO in perturbation theory!



Combined result (quenched!):

$$B_B(m_b) = 0.87(4)(5)$$

Landscape of lattice estimates (converted to RGI @ NLO)



Note new (preliminary) results by SPQcdR [very fine lattice spacing (3.9 GeV)]

New W.A. (*see talk by L. Lellouch*)

$$\widehat{B}_{B_d} = 1.34(12)$$

$$\widehat{B}_{B_s}/\widehat{B}_{B_d} = 1.00(4)$$

$$f_{B_d}\sqrt{\widehat{B}_{B_d}} = 235(33)\left({}^{+00}_{-24}\right)\ \text{MeV}$$

$$\xi = 1.18(4)\left({}^{+12}_{-0}\right)$$

In unquenched studies: chiral extrapolation might be affected by the functional form used to reach the physical "d" quark mass (*Sharpe & Zhang 1996, Kronfeld & Ryan 2002*).

Lattice data for heavy-light decay constants typically show the linear dependence in $m_q \lesssim m_s$. If the chiral log is included with the coefficient predicted by ChPT $\Rightarrow$ "baroque" fit : interpolation linear, extrapolation explodes.
Assessing the syst.error due to the chiral extrapolation (if possible) is. . . difficult (more details in Lellouch's talk).

New W.A.'s

$$f_{B_d} = 203(27)\left({}^{+00}_{-20}\right)\ \text{MeV}$$

$$f_{B_s} = 238(31)\ \text{MeV}$$

$$f_{B_s}/f_{B_d} = 1.18(3)\left({}^{+12}_{-0}\right)$$

3^{rd} error is the Lellouch's estimate of the impact of the chiral log on the chiral extrapolation (based on the data obtainend by JLQCD, presented at Latt2002).

A new estimate of the $B \to K^*\gamma$ amplitude from Lattice QCD

SPQcdR Collaboration

$b \to s\gamma$ theory Vs. experiment

Inclusive decays:

⊗ experimentally difficult

$$BR_{exp.}(B \to X_s\gamma) = \begin{cases} (3.21 \pm 0.43^{+0.32}_{-0.29}) \cdot 10^{-4} & \text{CLEO, 2001} \\ (3.36 \pm 0.53 \pm 0.68) \cdot 10^{-4} & \text{Belle, 2001} \end{cases}$$

⊗ theoretically rather clean

(QCD calculation at NLO completed in *A.Buras et al,2002*)

$$BR_{th.}(B \to X_s\gamma)_{E_\gamma > 1.6\text{GeV}} = (3.57 \pm 0.30) \cdot 10^{-4}$$

Exclusive decays:

⊗ experimentally easier

$$BR_{exp.}(B^+ \to K^{*+}\gamma) = \begin{cases} (3.79 \pm 0.86 \pm 0.28) \cdot 10^{-5} & \text{CLEO, 2000} \\ (3.83 \pm 0.62 \pm 0.22) \cdot 10^{-5} & \text{BaBar, 2002} \\ (4.97 \pm 0.56 \pm 0.38) \cdot 10^{-5} & \text{Belle, 2002} \end{cases}$$

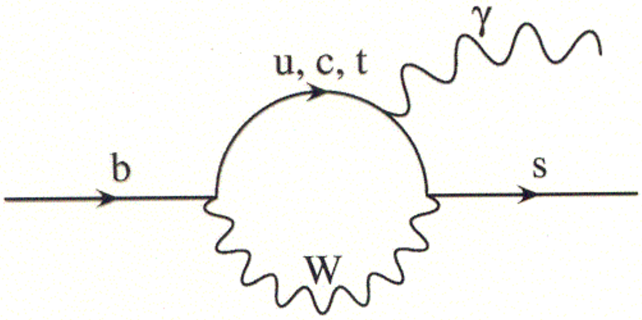
Large data samples from BaBar and Belle arriving ⇒ Errors will go down!

⊗ theoretically not clean

Large hadronic uncertainties:

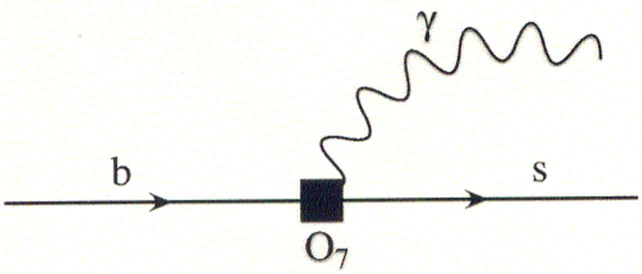
Lattice QCD may help!

Theoretical expression for $B \to K^*\gamma$ is derived by applying the OPE (expansion in $1/m_W^2$)



$\rightarrow$

$C_7 \times$



$+ \mathcal{O}(1/m_W^2)$

$$\mathcal{H}_{eff}^{b\to s\gamma} = -\frac{G_F}{\sqrt{2}} V_{tb}^* V_{ts} C_7(\mu) \mathcal{O}_7(\mu)$$

- $C_7(\mu)$ `Wilson coefficient`

information on the short distance physics (stuff in the loops) [use perturbative QCD]

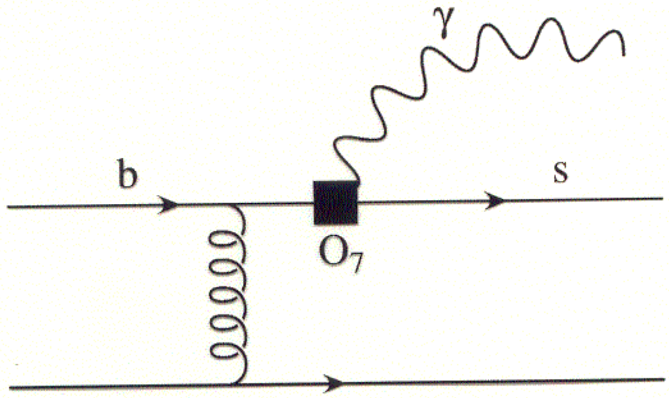
- $\mathcal{O}_7(\mu)$ `EM penguin operator`

$$\mathcal{O}_7 = -\frac{e m_b}{8\pi^2} \bar{s} \sigma_{\mu\nu} (1+\gamma_5) b F^{\mu\nu}$$

soft physics in $\langle K^* | \mathcal{O}_7 | B \rangle$ [need non-perturbative QCD]

(N.B. $\mu \sim m_b$)

★ Tacit assumption that the factorisation works. Very recently the hard spectator effects have been included in $C_7(m_b)$. E.g.



$$|C_7(m_b)|^2 = 0.17(2)$$

M. Beneke, T. Feldmann, D. Seidel (2001), and A. Ali, A. Ya. Parkhomenko (2001)

★ $B \to K^*\gamma$ **matrix element**

$$\langle K^*(p', e_\lambda)|\bar{s}\sigma_{\mu\nu}(1+\gamma_5)b|B(p)\rangle = c^{(1)}_{\mu\nu}T_1(q^2) + c^{(2)}_{\mu\nu}T_2(q^2) + c^{(3)}_{\mu\nu}T_3(q^2)$$

$c^{(1,2,3)}$-known functions of the kinematical variables

$$(p, p', e_\lambda, m_{K^*}, m_B)$$

$T_{1,2,3}(q^2)$ - unknown form factors relevant for

$$B \to K^*\ell^+\ell^-$$

For the on-shell photon ($q^2 = 0$): $c^{(3)} = 0$ and $T_1(0) = T_2(0)$

Brief history of the lattice computations of $T_{1,2}(q^2)$

- 4 lattice studies made so far
- the most recent lattice results: 6-7 years old!
- all results obtained by using Wilson fermions
- all worked with heavy quarks of masses near m_c, and then extrapolated to m_b
- difficulties with the simultaneous extrapolations $q^2 \to 0$ and to $m_Q \to m_b$

⋆ ⋆ ⋆

♣ C.Bernard, P. Hsieh, A. Soni (1994)

[Wilson, 35 cfgs @ $\beta = 6.0$, 20 cfgs @ $\beta = 6.3$
small lattices; 2 and 3 values of κ_Q and 1 value of κ_q]

♣ UKQCD (1995)

[tree level impr. Wilson, 60 cfgs @ $\beta = 6.2$
size $24^3 \times 48$, with 4 values of κ_Q and 3 value of κ_q]

♣ APE (1996)

[tree level impr. Wilson, 180 cfgs @ $\beta = 6.0$
size $18^3 \times 64$, with 4 values of κ_Q and 3 value of κ_q]

♣ LANL (1995)

[Wilson, 100 cfgs @ $\beta = 6.0$
size $32^3 \times 64$, with 2 values of κ_Q and 3 value of κ_q]

♣ S.P.Qcd R. (2002) **NEW!**

[non-pert.impr. Wilson 200 cfgs @ $\beta = 6.2$
size $24^3 \times 64$, with 4 values of κ_Q and 3 values of κ_q]
[non-pert.impr. Wilson 100 cfgs @ $\beta = 6.45$
size $32^3 \times 70$, with 6 values of κ_Q and 4 values of κ_q]

To what q^2-form can we fit the data?

- Kinematical region large $[0 \leq q^2 \leq (m_B - m_{K^*})^2]$. The nearest pole at $q^2 = m^2_{B_s^*}$ influences $T_1(q^2)$. Its position known! $T_2(q^2)$ couples to 1^+ states which are farther away from q^2_{max}.

- HQET (HQS) in heavy → light decays helps with the scaling laws applicable for small recoils $q^2 \simeq q^2_{max}$ (N.Isgur, M.Wise, 1990):

$$T_1(q^2 \simeq q^2_{max}, M) \sim \sqrt{M} \qquad T_2(q^2 \simeq q^2_{max}, M) \sim 1/\sqrt{M}$$

- LEET: all heavy → light vector meson form factors can be expressed in terms of two universal functions (J.Charles et al, 1999). $T_{1,2}(q^2)$ both related to a single function

$$T_1(q^2) = \zeta_\perp(M, E) \qquad T_2(q^2) = \frac{2E}{M}\zeta_\perp(M, E)$$

$$T_{1,2}(q^2 \approx 0) \simeq T_2(q^2 \approx 0) \sim \sqrt{E}/M^2 \sim M^{-3/2}$$

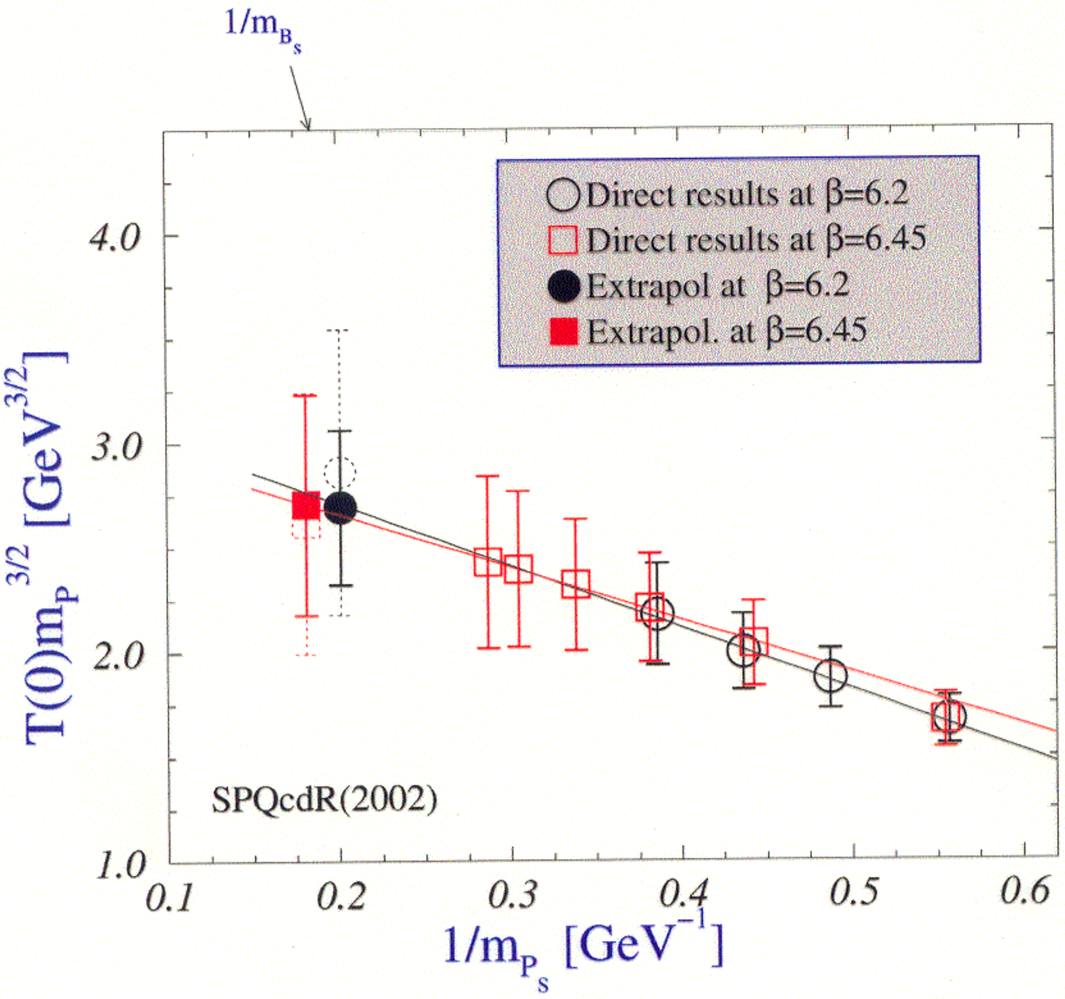
applicable for $q^2 \simeq 0$ (explicitely verified by LCSR (P.Ball, V.Braun, 1998)); LEET sick for it hasn't the same IR properties as QCD ⇒ SCET (Ch.Bauer et al. 2000), in which the conclusions by Charles et al. remain true (M.Beneke et al. 2002).

A form satisfying all mentioned constraints is (D.Becirevic, A.Kaidalov, 2000)

$$T_1(q^2) = \frac{C(1-\alpha)}{(1-q^2/M_V^2)(1-\alpha q^2/M_V^2)}$$
$$T_2(q^2) = \frac{C(1-\alpha)}{1-\beta q^2/M_V^2}$$

The extrapolation to $1/M = 1/m_{B_s}$

$$T(0, M_d) m_{P_d}^{3/2} = a_0 + a_1/m_{P_s} + a_2/m_{P_s}^2$$



Result-I:

$$T^{B\to K^*}(0) = 0.24(5)^{+0}_{-2}\Big|_{\beta=6.2}, 0.22(5)^{+1}_{-0}\Big|_{\beta=6.45}$$

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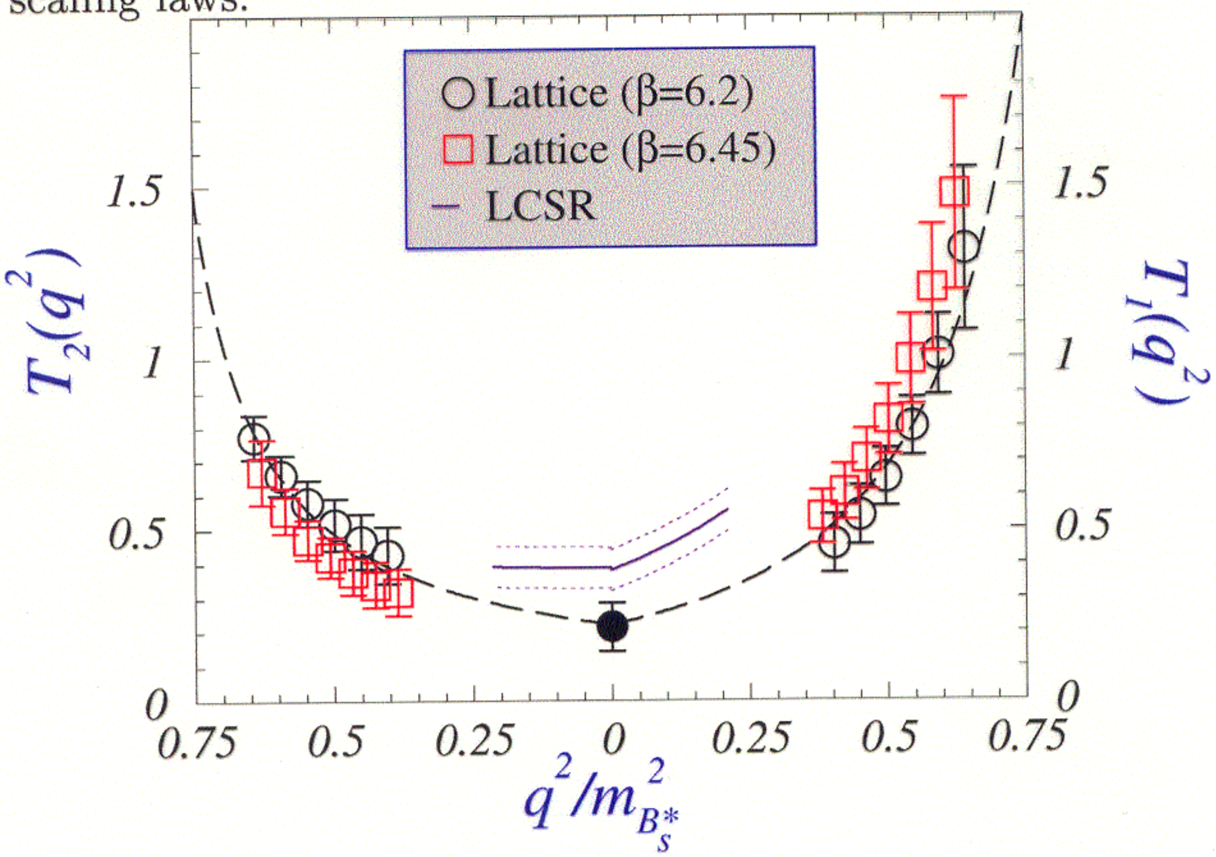
Alternatively use the UKQCD strategy and interpolate to K^*-meson at fixed q^2 (UKQCD, 1999) and then for a fixed $v \cdot p$ extrapolate in inverse heavy quark mass by asumming the HQET scaling laws.



Result-II:

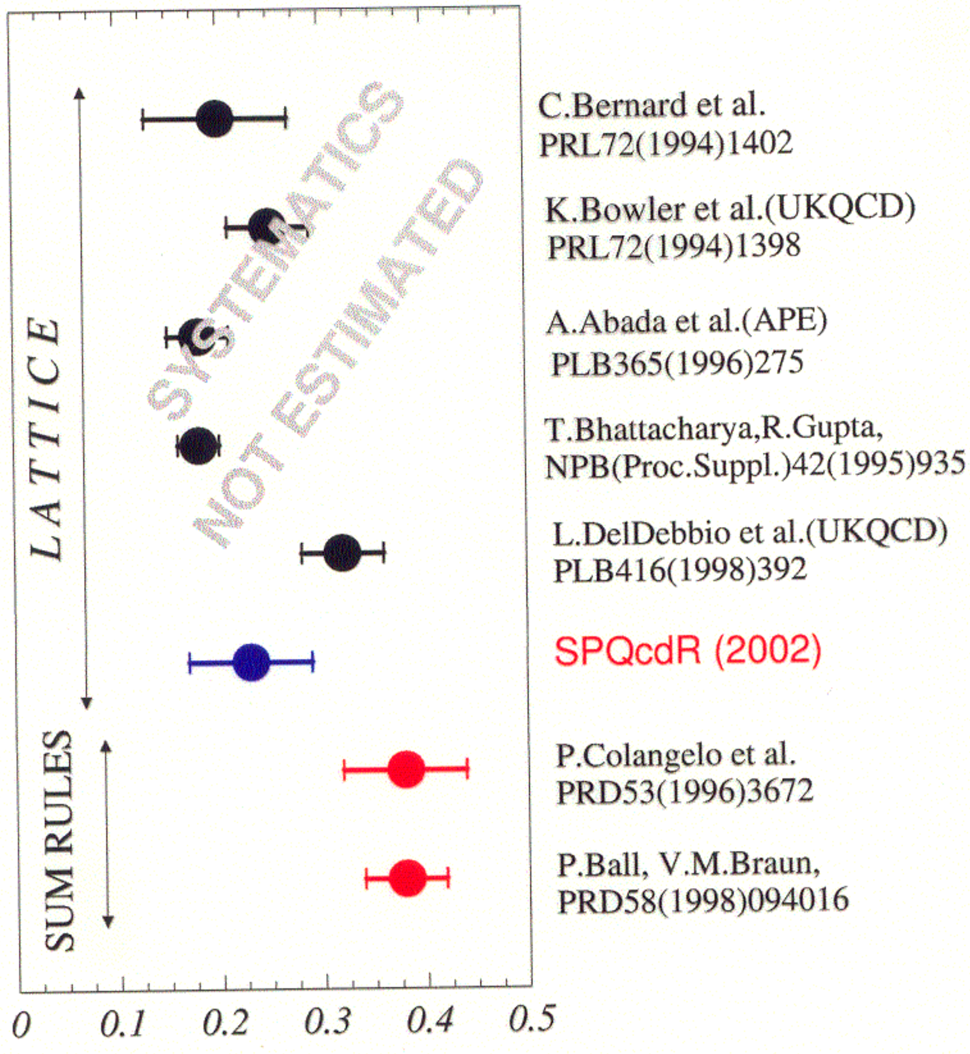
$$T^{B\to K^*}(0) = 0.23(6)^{+1}_{-2}\Big|_{\beta=6.2}, 0.21(4)^{+1}_{-2}\Big|_{\beta=6.45}$$

As our final result we quote

$$T^{B\to K^*}(0) = 0.23(5)^{+2}_{-3}$$

We conclude:

$$BR(\bar{B}^0 \to \bar{K}^{*0}\gamma) = (2.89 \pm 1.63)\cdot 10^{-5}$$



Exclusive semileptonic decays

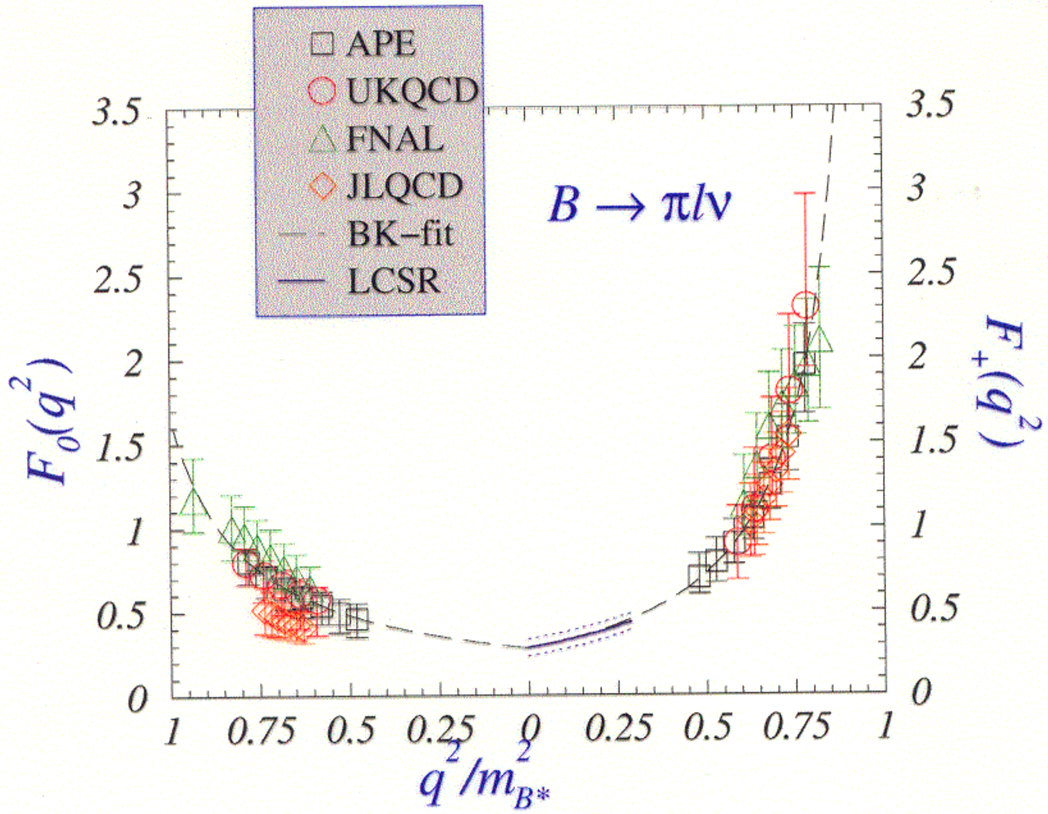
Road to $|V_{ub}|$

$B \to \pi \ell \nu$

Similar constraints to those enumerated for $B \to K^*$ apply in this case too

$$\langle \pi | V_\mu^{bu} | B \rangle = c_1^{kin} F_+(q^2) + c_2^{kin} F_0(q^2)$$

Last 3 years many lattice computations:
all three ways to treat the heavy quark were used



Last week exploratory study of B→ π on anisotropic lattice by HPQCD appeared. Their results agree well with the other groups.

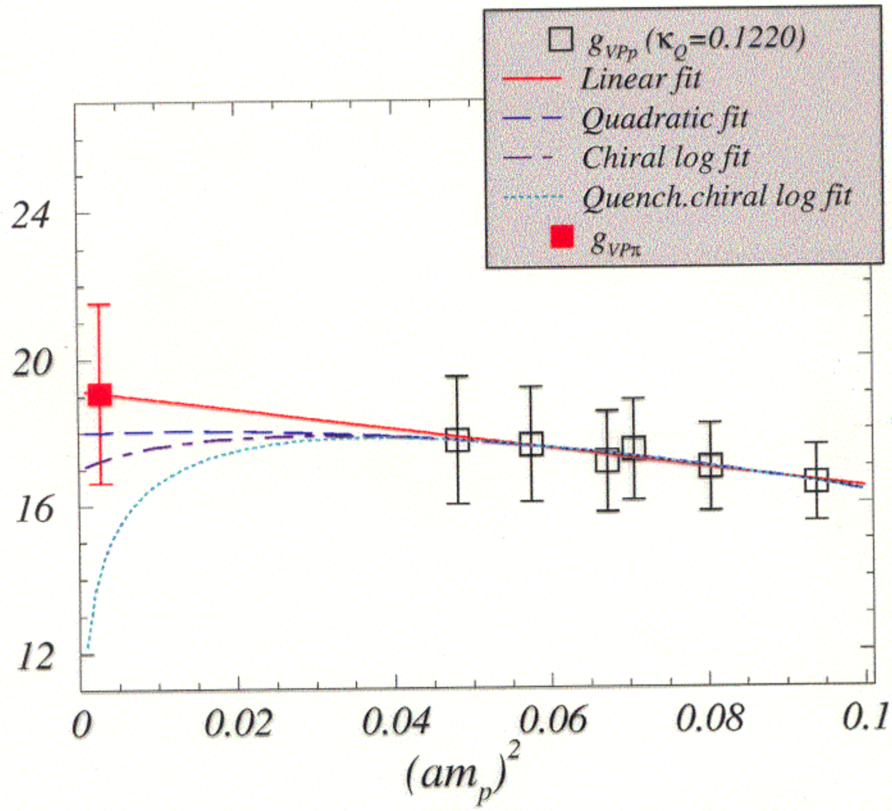
Agreement among various approaches and with the LCSR *(A. Khodjamirian et al, 2000)* **quite impressive.**
All studies quenched! Comparing the quenched and standard ChPT, it was shown that $F_{0,+}$ suffer from the quenched pathology (singular quenched chiral logs), i.e. the chiral limit is not defined *(D.B, S.Prelovsek, J.Zupan, to appear).*

For example $g_{H^*H\pi} = 2m_H\hat{g}/f_\pi$:

$$g^{\mathrm{QQCD}} = g\left(1 - c_Q \log(m_P^2/\mu^2) - c_L m_P^2 \log(m_P^2/\mu^2) + c_0 + c_1 m_P^2 + \ldots\right)$$

$$c_Q^{\mathrm{QChPT}} = (4/3)g^2 m_0^2/(4\pi f)^2$$

$$g^{\mathrm{QCD}} = g\left(1 - d_L m_P^2 \log(m_P^2/\mu^2) + d_0 + d_1 m_P^2 + \ldots\right)$$



First lattice QCD computation of $g_{D^*D\pi}$ (A. Abada et al, 2002)

(a) For $g_{D^*D\pi}$ neither quenched nor unquenched logarithms are observed

(b) Linear fit gets very close to the measured value of $g_{D^*D\pi}$ (CLEO, 2001)

$$g_{D^*D\pi}^{phys} = 17.9 \pm 2.1 \text{ Vs. } g_{D^*D\pi}^{latt(lin.)} = 18.8 \pm 2.3$$

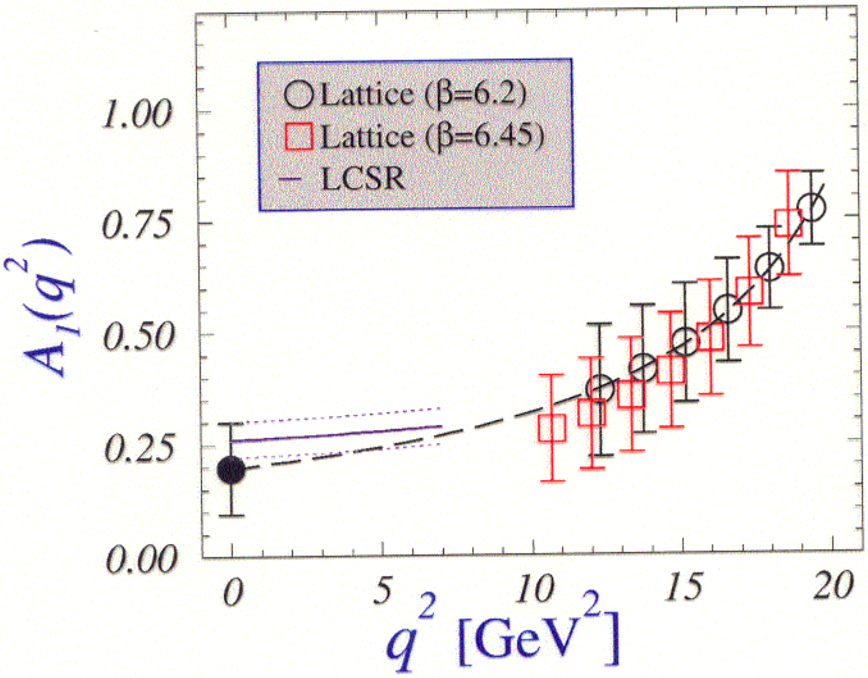
IMHO, to assess the systematic uncertainty due to the chiral extrapolation for any quantity and with the pion masses with which we are working [400, 700) MeV], one should fit to the form obtained in chiral expansion (to NLO), check the results of extrapolation with/without the log terms, but WITHOUT fixing the coefficients to the ChPT prediction!

$B \to \rho l \nu$

7 years after the benchmark lattice computation of the $B \to \rho$ form factors by UKQCD, this year SPQcdR made such a study

$$\langle \rho | V_\mu^{bu} | B \rangle = c_V^{kin} V(q^2) + c_1^{kin} A_1(q^2) + c_2^{kin} A_2(q^2)$$

$A_1(q^2)$ dominant form factor ($A_1(0) = 0.20(9)$):



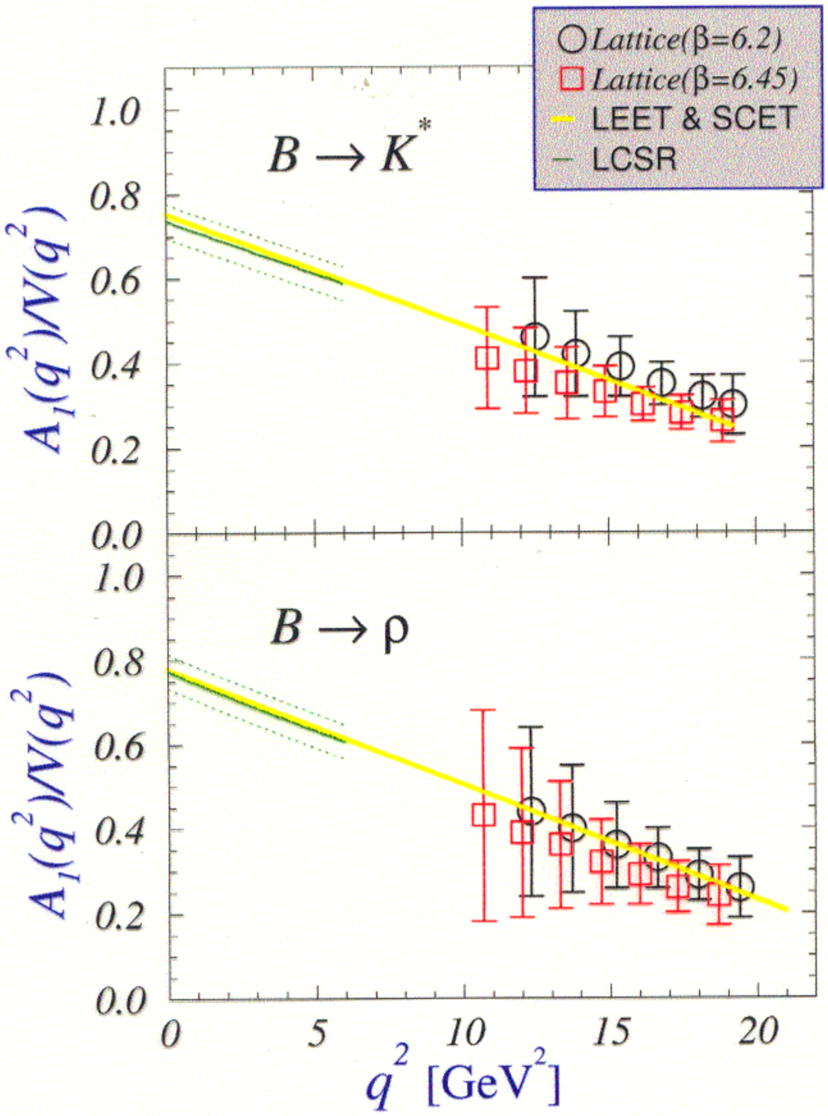
SCET [LEET] prediction

$$\frac{V(q^2)}{A_1(q^2)} = \frac{(m_B + m_V)^2}{2E_V m_B}$$

This is the ratio of the transverse form factors which does not even receives α_s corrections in the LEL

(Charles et al. 1999, Burdman & Hiller 2001, Beneke et al 2002)

Quite impressively!



As it could be expected, LEL relation for $D \to K^*$ gives $r_V = V(0)/A_1(0) = 1.78$, while the new lattice result is

$$r_V = 1.58(14)_{\beta=6.2},\ 1.55(11)_{\beta=6.45}$$

Exp.: FOCUS released the new results 4 days ago

$$r_V = 1.504(87)$$

impressive accuracy, BUT they assume both V and A_1 to be of the pole form, which IMO is not a good thing to do. Had they left the pole masses as free parameters, the error on r_V would roughly get doubled (Jim Wiss, priv. communication)

SUMMARY AND PERSPECTIVE

(1) EXTRAORDINARY PROGRESS MADE IN THE LAST DECADE

- HIGH STATISTICS STUDIES
- NEW GENERATION OF PARALLEL COMPUTERS (EVER MORE SYSTEMATICS UNDER CONTROL)
- WILSON FERMIONS. NOW $O(a^2)$ ONLY.
- NONPERTURBATIVE RENORMALISATION IMPLEMENTED

(2) UNQUENCHING IS A MUST!

- CHALLENGING TECHICALLY
- $n_f = 3$ MAYBE / $n_F = 2$!!!

(3) NONLEPTONIC DECAYS ARE NOT "TERRA INCOGNITA" ANY MORE
(HOW TO IMPLEMENT THE IDEAS AND MAKE ACCURATE COMPUTATIONS?)