Determination of the unitarity triangle parameters

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Outline

- Introduction
- Review of the Fit methods
- Results in the SM
- Strategies for the UT determination
- Fit in the Minimal Flavour Violation models
- Conclusions

Based on the Conference papers:

657: M. Ciuchini,E. Franco,V. Lubicz, G. Martinelli, F.P., L. Silvestrini, A. Stocchi and P. Roudeau

661: A. Buras, F.P. and A. Stocchi

959: G.P. Dubois-Felsmann, G. Eigen, D.G. Hitlin and F. C. Porter

Introduction

The weak charged current interaction of quarks is parametrized by the Cabibbo-Kobayashi-Maskawa matrix:

$$\hat{V}_{\rm CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

$$= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -s_{23}c_{12} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

δ is the phase necessary for CP violation.
From experimental observation we know that s_{13} and s_{23} are small ($\mathcal{O}(10^{-3})$ and $\mathcal{O}(10^{-2})$ respectively).
The $\hat{V}_{CKM}$ can be described by the four independent parameters

$$s_{12} = |V_{us}|, \quad s_{13} = |V_{ub}|, \quad s_{23} = |V_{cb}|, \quad \delta.$$

or (using Wolfenstein param. $s_{12} = \lambda$, $s_{23} = A\lambda^2$, $s_{13}e^{-i\delta} = A\lambda^3(\rho - i\eta)$)[1]:

$$|V_{us}| = \lambda, \qquad |V_{cb}|, \qquad \bar{\varrho}, \qquad \bar{\eta}$$

[1] $\bar{\rho} = \rho(1 - \lambda^2/2)$, $\bar{\eta} = \eta(1 - \lambda^2/2)$

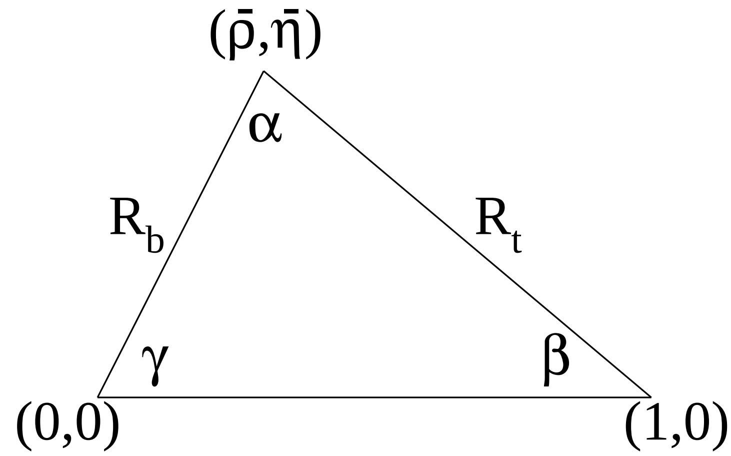
The pair $(\bar{\varrho}, \bar{\eta})$ describes the apex of the unitarity triangle (UT) representing the unitarity relation

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$$

rescaled by $|V_{cd}V_{cb}^*| = A\lambda^3 = \lambda|V_{cb}|$:

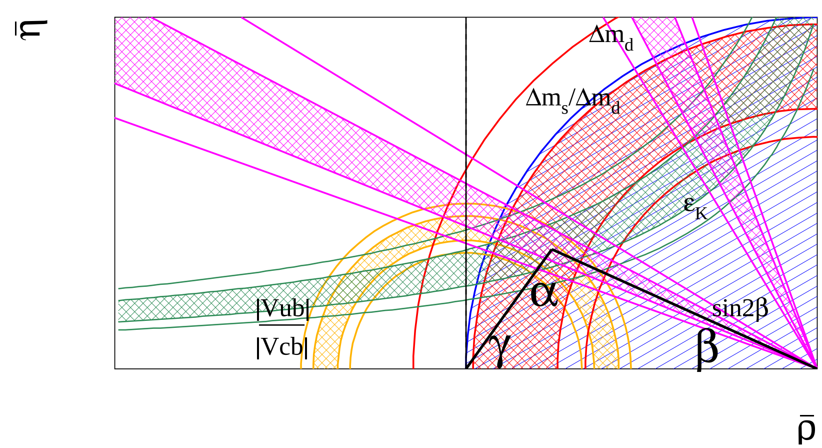
$$\bar{\varrho} + i\bar{\eta} - 1 + 1 - \bar{\varrho} - i\bar{\eta} = 0$$

Sides: R_b, R_t
Angles: α, β, $\gamma \equiv \delta$



Experimental constraints on the UT:

Meas.	$V_{CKM}\times$other	$(\bar{\rho}, \bar{\eta})$
$\frac{b\to u}{b\to c}$	$\|V_{ub}/V_{cb}\|^2$	$\bar{\rho}^2 + \bar{\eta}^2$
Δm_d	$\|V_{td}\|^2 f_{B_d}^2 B_{B_d}$	$(1-\bar{\rho})^2 + \bar{\eta}^2$
$\frac{\Delta m_d}{\Delta m_s}$	$\left\|\frac{V_{td}}{V_{ts}}\right\|^2 \xi^2$	$(1-\bar{\rho})^2 + \bar{\eta}^2$
ϵ_K	$f(A, \bar{\eta}, \bar{\rho}, B_K)$	$\propto \bar{\eta}(1-\bar{\rho})$
$A(J/\psi K^0)$	$sin2\beta$	$\frac{2\bar{\eta}(1-\bar{\rho})}{\sqrt{\bar{\eta}^2+(1-\bar{\rho})^2}}$

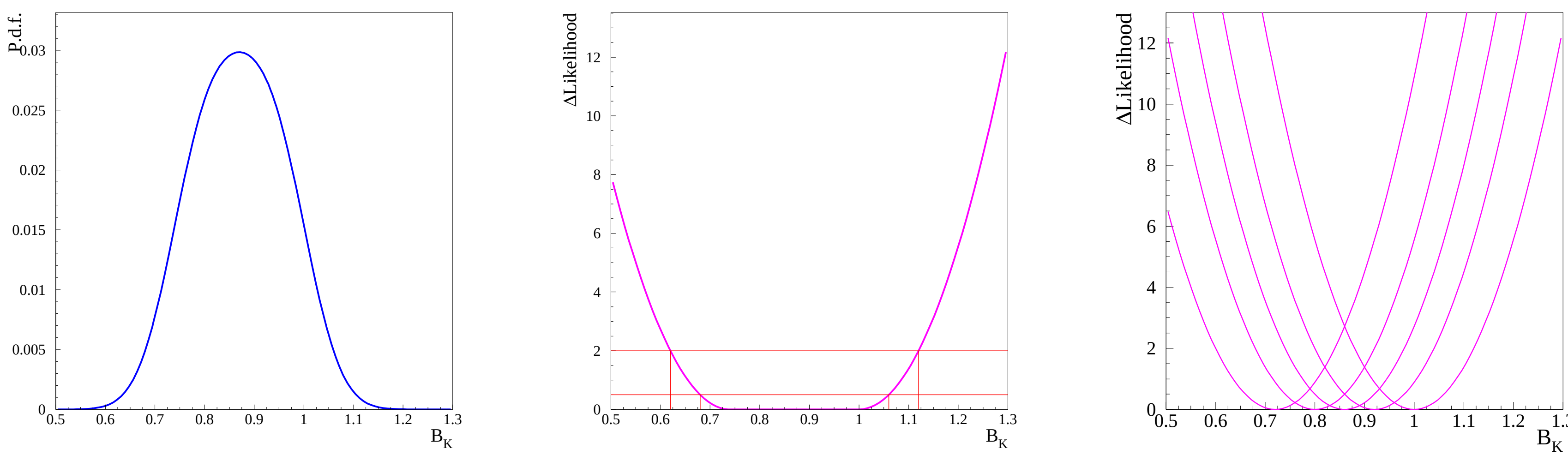


Fit Methods: Bayesian, Rfit, Scan

Main difference: treatment of the systematical error of the theoretical parameters.

	Bayes	Rfit	Scan
Input	**p.d.f. (syst.+stat.)**	**stat. likelihood + syst. ranges**	**stat. likelihood + set of theo. values**
Output	Probability regions for all the parameters	Allowed CL regions (CL=lower bound on CL)	Allowed regions for a given set of theo. param.

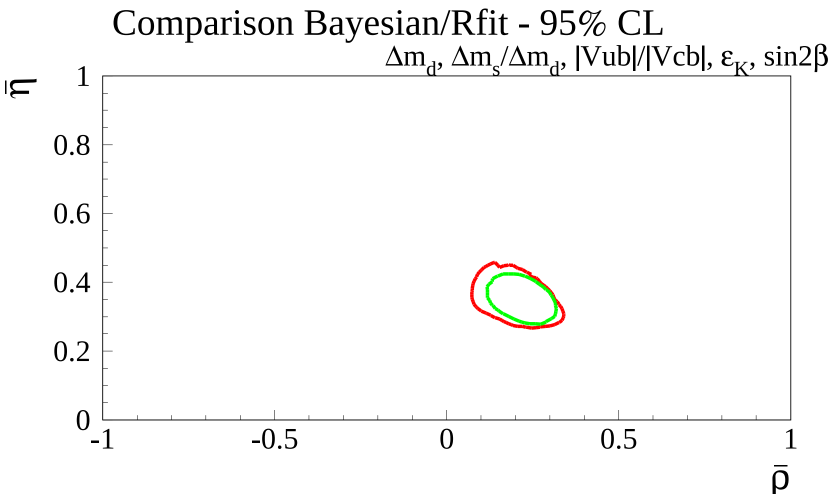
Example: $B_K = 0.86 \pm 0.06_{stat} \pm 0.14_{syst}$

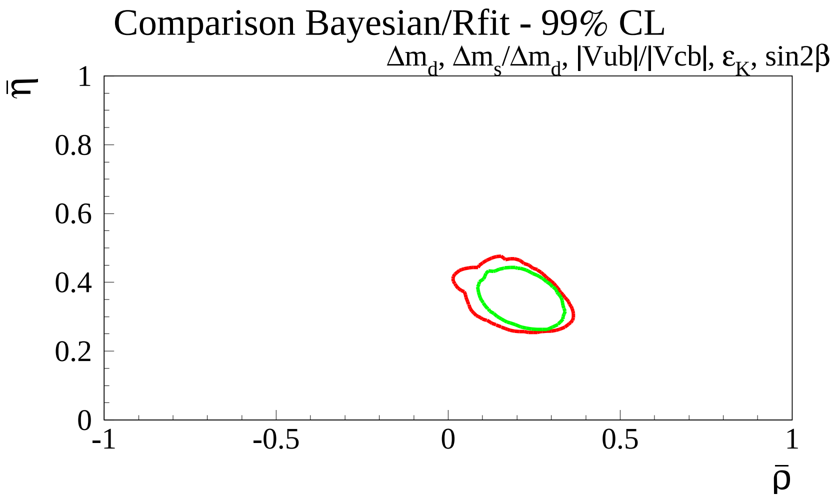


Starting from the same values the allowed regions are different (factor 1.7 at 68% C.L. between Bayesian and Rfit)

Comparison of Bayesian/Rfit

Work started at the CKM Workshop. Aim: improve our understanding of the differences between differenr fit methods comparing their results starting from the same inputs.





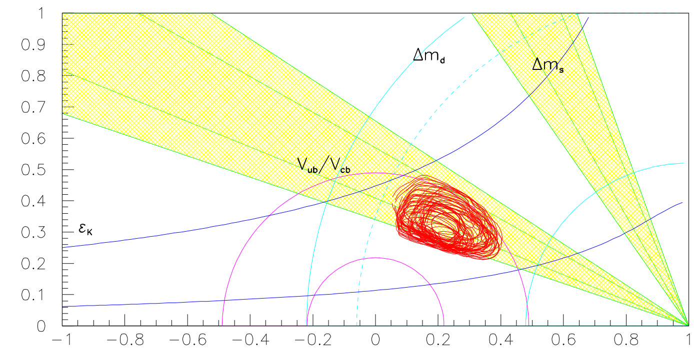
Ratio RFit/Bayesian Method			
Parameter	5% CL	1% CL	0.1% CL
$\bar{\rho}$	1.42	1.34	1.12
$\bar{\eta}$	1.18	1.12	1.05
$\sin 2\beta$	1.16	1.16	1.17
γ°	1.51	1.31	1.09

Scan method

G.P. Dubois-Felsmann, G. Eigen, D.G. Hitlin and F. C. Porter

Fit for $(\bar{\rho}, \bar{\eta})$ with fixed set of theo. parameters (a "model"):

$$\mathcal{M} = \{F_{D^*}(1), \tilde{\Gamma}_{excl}, f_{B_d}\sqrt{B_{B_d}}, B_K, \xi\}$$



V_{ub} and V_{cb}: only exclusive measurements are used.

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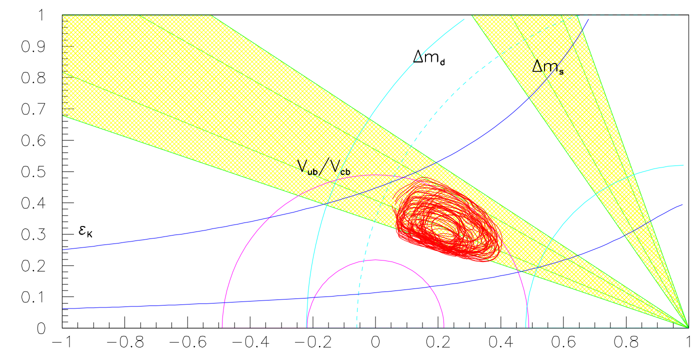
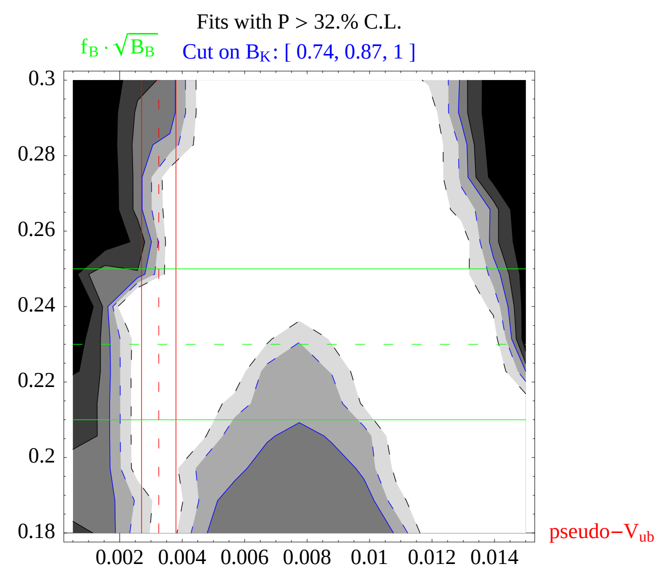


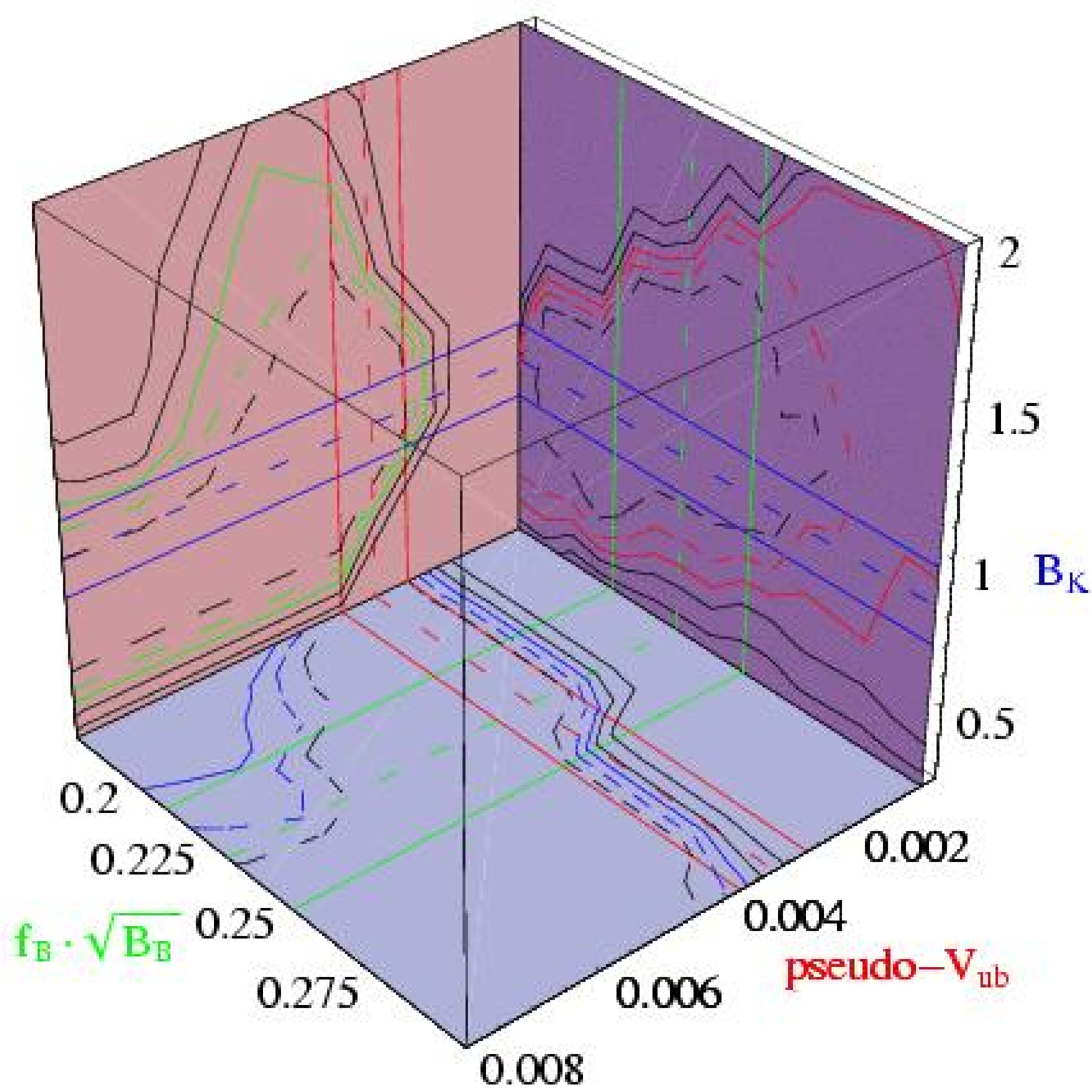
V_{ub} and V_{cb}: only exclusive measurements are used.

Display the consistency of the fit in the theoretical parameters space.

For each pair (T_1,T_2) verify if at least one fit pass a certain χ^2 cut $\longrightarrow$ draw contour
Different contours $\leftrightarrow$ different configurations for the undisplayed parameters



Cyclic permutations of the two variables give 3D plots



Inputs for the CKM fit

Standard set:

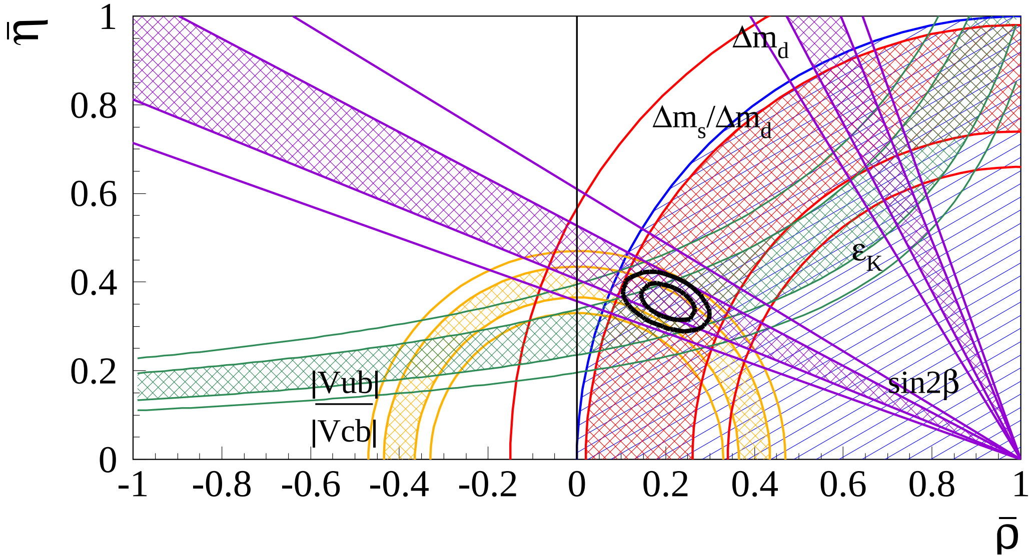
Parameter	Value	Gaussian σ	Uniform half-width
λ	0.2210	0.0020	-
$\|V_{cb}\|$(excl.)	42.1×10^{-3}	2.1×10^{-3}	-
$\|V_{cb}\|$(incl.)	40.4×10^{-3}	0.7×10^{-3}	0.8×10^{-3}
$\|V_{cb}\|$(ave.)	40.6×10^{-3}	0.8×10^{-3} *	
$\|V_{ub}\|$(excl.)	32.5×10^{-4}	2.9×10^{-4}	5.5×10^{-4}
$\|V_{ub}\|$(incl.)	40.9×10^{-4}	4.6×10^{-4}	3.6×10^{-4}
$\|V_{ub}\|$(ave.)	36.3×10^{-4}	3.2×10^{-4} *	
$\|V_{ub}\|/\|V_{cb}\|$(ave.)	0.089	0.008*	
ΔM_d	0.503 ps^{-1}	0.006 ps^{-1}	–
ΔM_s	$>$ 14.4 ps^{-1} at 95% C.L.	sensitivity 19.2 ps^{-1}	
m_t	$167\ GeV$	$5\ GeV$	–
sin 2β	0.762	0.064	-
$\hat{B}_K$	0.86	0.06	0.14
$f_{B_d}\sqrt{\hat{B}_{B_d}}$	$230\ MeV$	$30\ MeV$	$15\ MeV$
ξ	1.18	0.03	0.04

New lattice QCD parameters with "chiral logarithms"

$f_{B_d}\sqrt{\hat{B}_{B_d}}$	$235\ MeV$	$33\ MeV$	${}^{+0}_{-24}\ MeV$
$\xi =$	1.18	0.04	${}^{+12}_{-0}$

Fit Results

M. Ciuchini,E. Franco,V. Lubicz, G. Martinelli, F.P., L. Silvestrini, A. Stocchi and P. Roudeau



$$
\begin{aligned}
V_{cb} &= (40.43 \pm 0.74)10^{-3} \\
\bar{\rho} &= (0.203 \pm 0.040) \\
\bar{\eta} &= (0.355 \pm 0.027)
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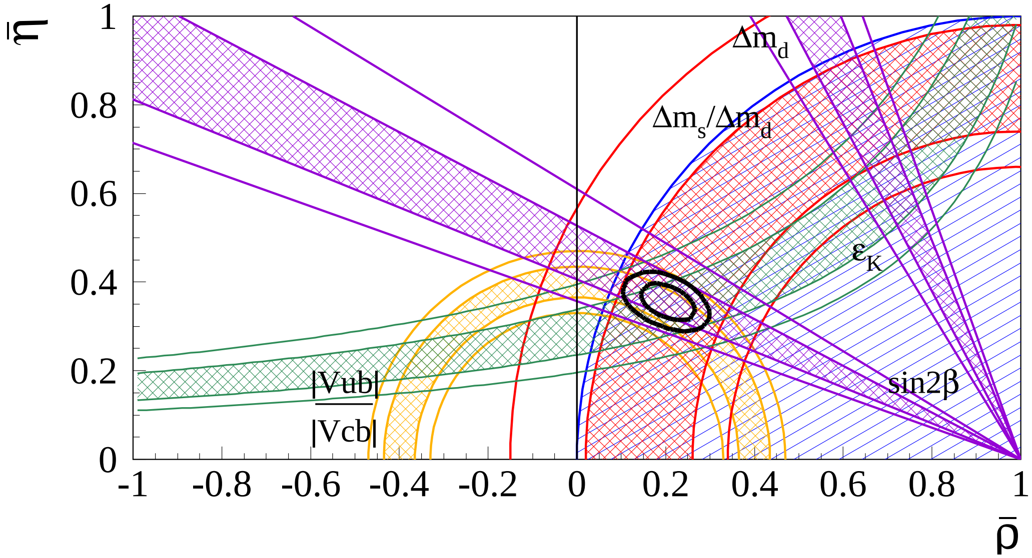

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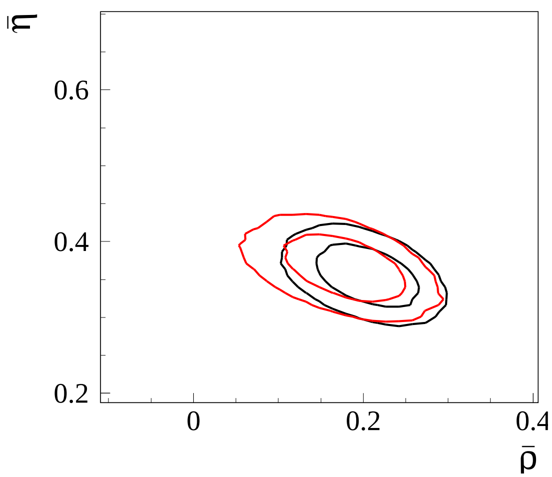
\end{aligned}
$$

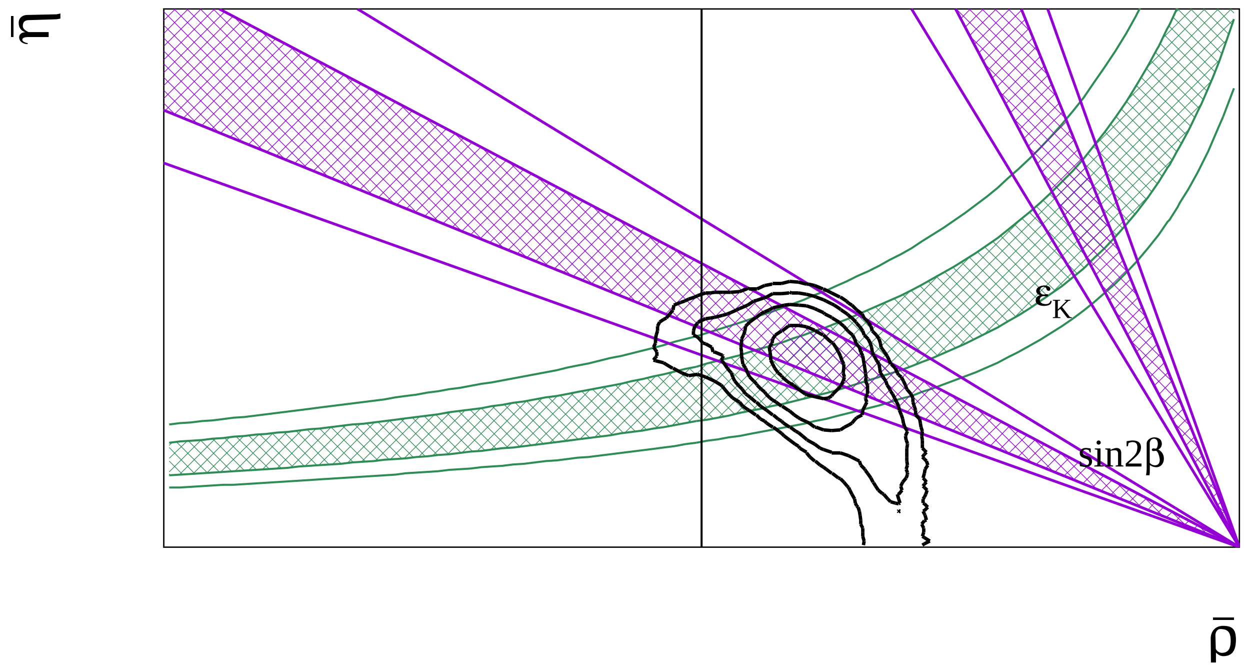
Fit including "chiral logs" syst. in ξ and $f_{B_d}\sqrt{B_{B_d}}$:

$$
\begin{aligned}
\bar{\rho} &= (0.177^{+0.047}_{-0.044}) \\
\bar{\eta} &= (0.365 \pm 0.028)
\end{aligned}
$$



Comparison between sides and angles

Comparing CP violating measurements (ϵ_K, $sin2\beta$) with measurements without CP-information (Δm_d, Δm_s, $|V_{ub}/V_{cb}|$)

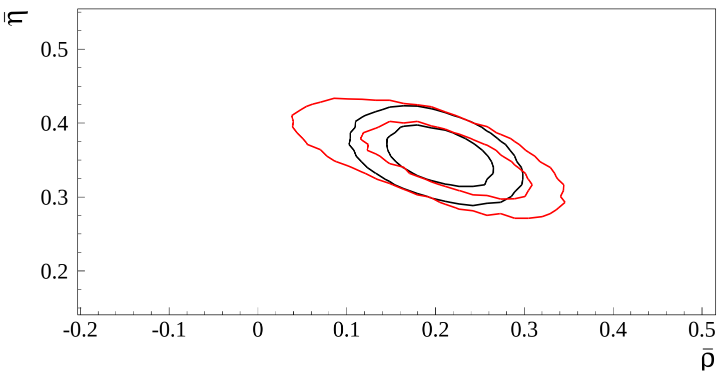


Coherent CP picture in the SM !

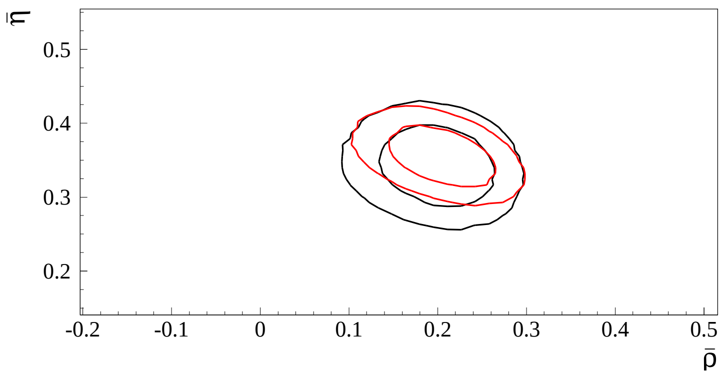
Consistency checks and predictions

Fit overconstrained: remove constraints one by one to check their impact and the global consistency

Impact of Δm_s



Impact of $sin2\beta$



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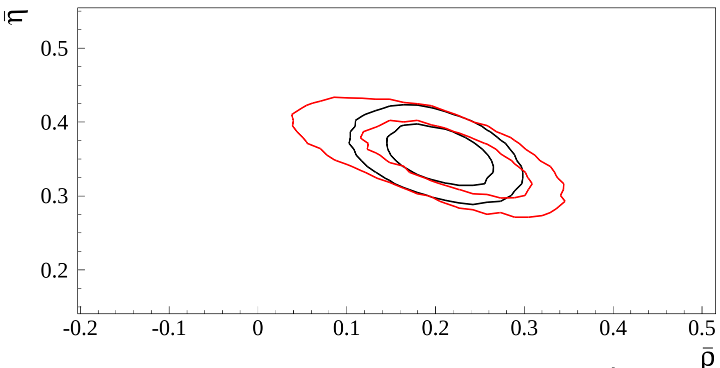

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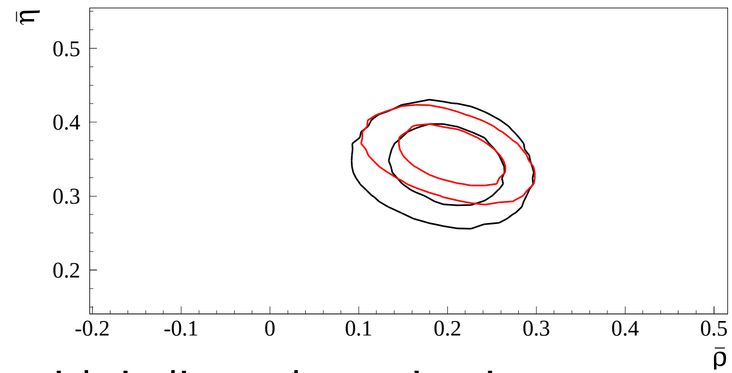


$sin2\beta$: compare direct measurements (BaBar[2], Belle,..) with indirect determination:

$$sin2\beta_{WA} = (0.762 \pm 0.064)$$
$$sin2\beta_{indirect} = (0.715^{+0.055}_{-0.045})$$

The two determinations are well in agreement and have similar precisions.

Note: real pre-diction !

CKM fits have predicted this value (with slightly larger error) already in 1997.

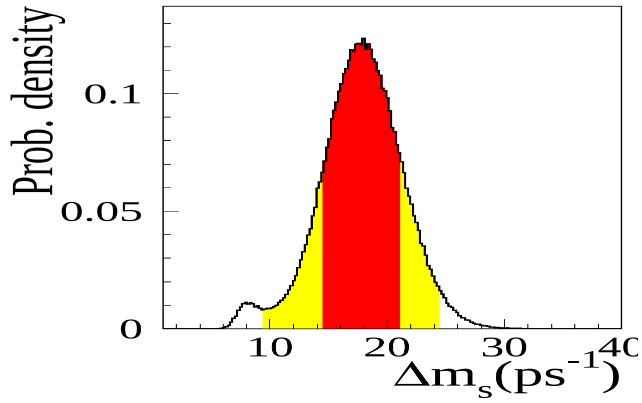
Including the $sin2\beta$ constraint the fit gives:

$$sin2\beta = (0.734^{+0.045}_{-0.034})$$

[2] This average contains the latest measurement from BaBar: $sin2\beta = 0.741 \pm 0.067 \pm 0.033$

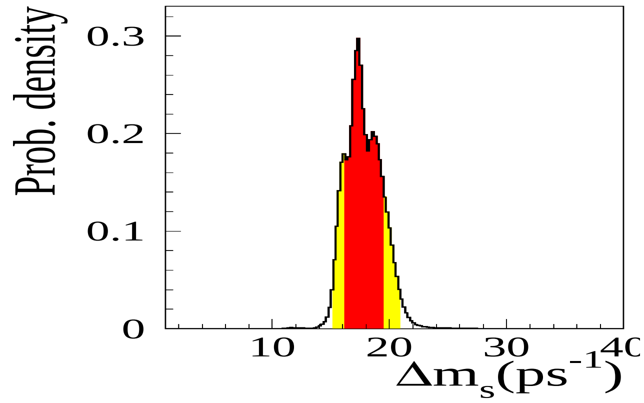
Next to come: Δm_s, γ and α

Δm_s p.d.f. without the Δm_s constraint



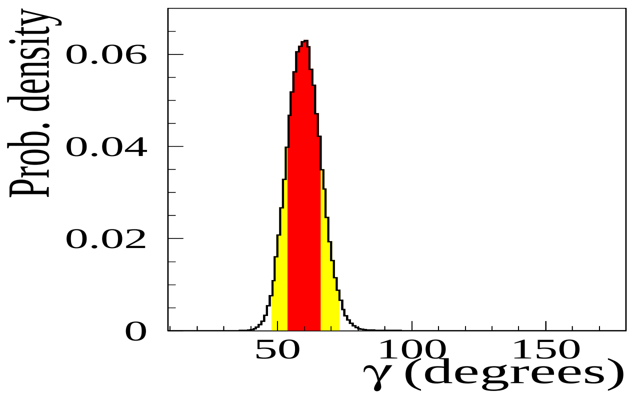
$\Delta m_s = (17.8^{+3.4}_{-3.2})\ ps^{-1}$
$[9.4 - 24.4]\ ps^{-1}$at 95% CL

Δm_s p.d.f. with the Δm_s constraint



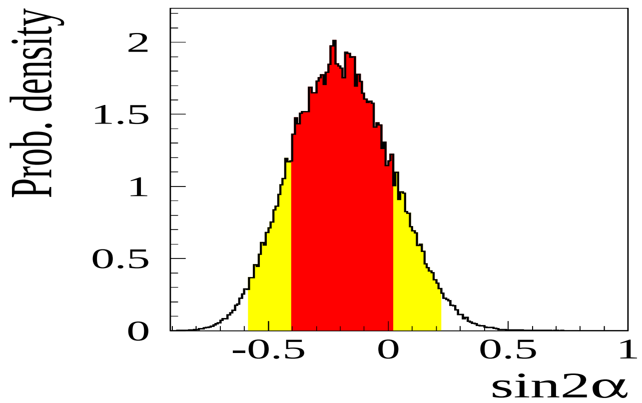
$\Delta m_s = (17.6^{+2.0}_{-1.3})\ ps^{-1}$
$[15.2 - 20.9]\ ps^{-1}$at 95% CL

γ p.d.f.



$\gamma = (59.5^{+6.5}_{-5.5})$
$[47 - 72]$at 95% CL

$sin2\alpha$ p.d.f



$sin2\alpha = (-0.20^{+0.23}_{-0.20})$
$[-0.58 - 0.22]$at 95% CL

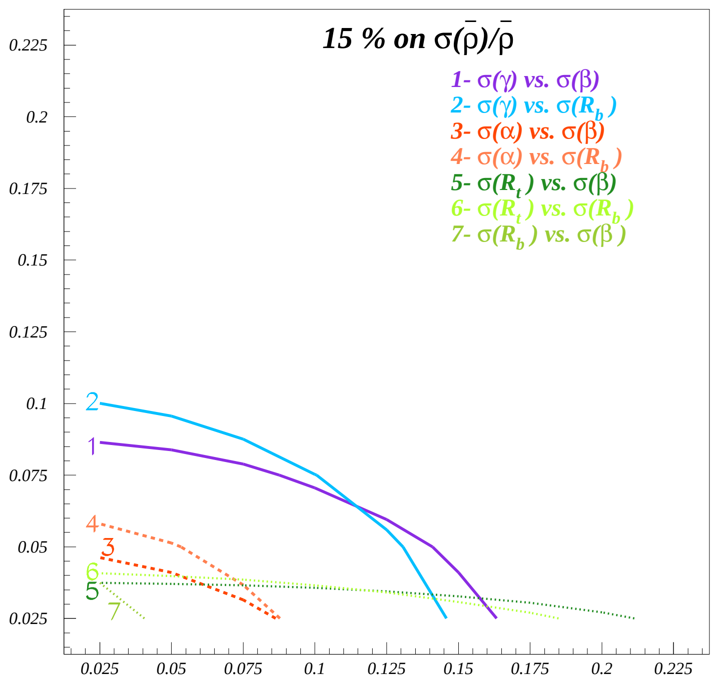
Strategies for the UT

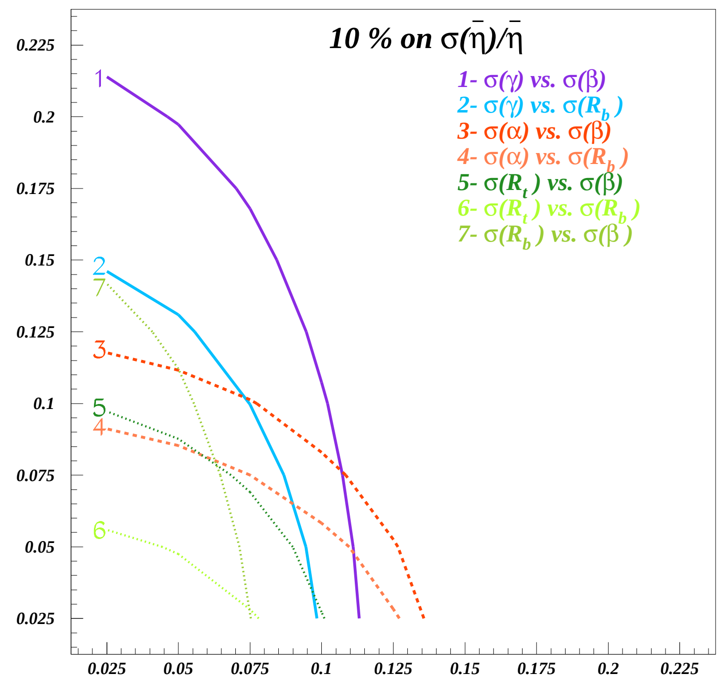
A. Buras, F.P. and A. Stocchi

The determination of $(\bar{\rho}, \bar{\eta})$ only requires two indipendent measurements.

Questions: fixing the same relative precision which are the most effective pairs of variables among $R_b, R_t, \alpha, \beta, \gamma$?

First divide the variables in two groups: (R_b, β) (R_t, α, γ)



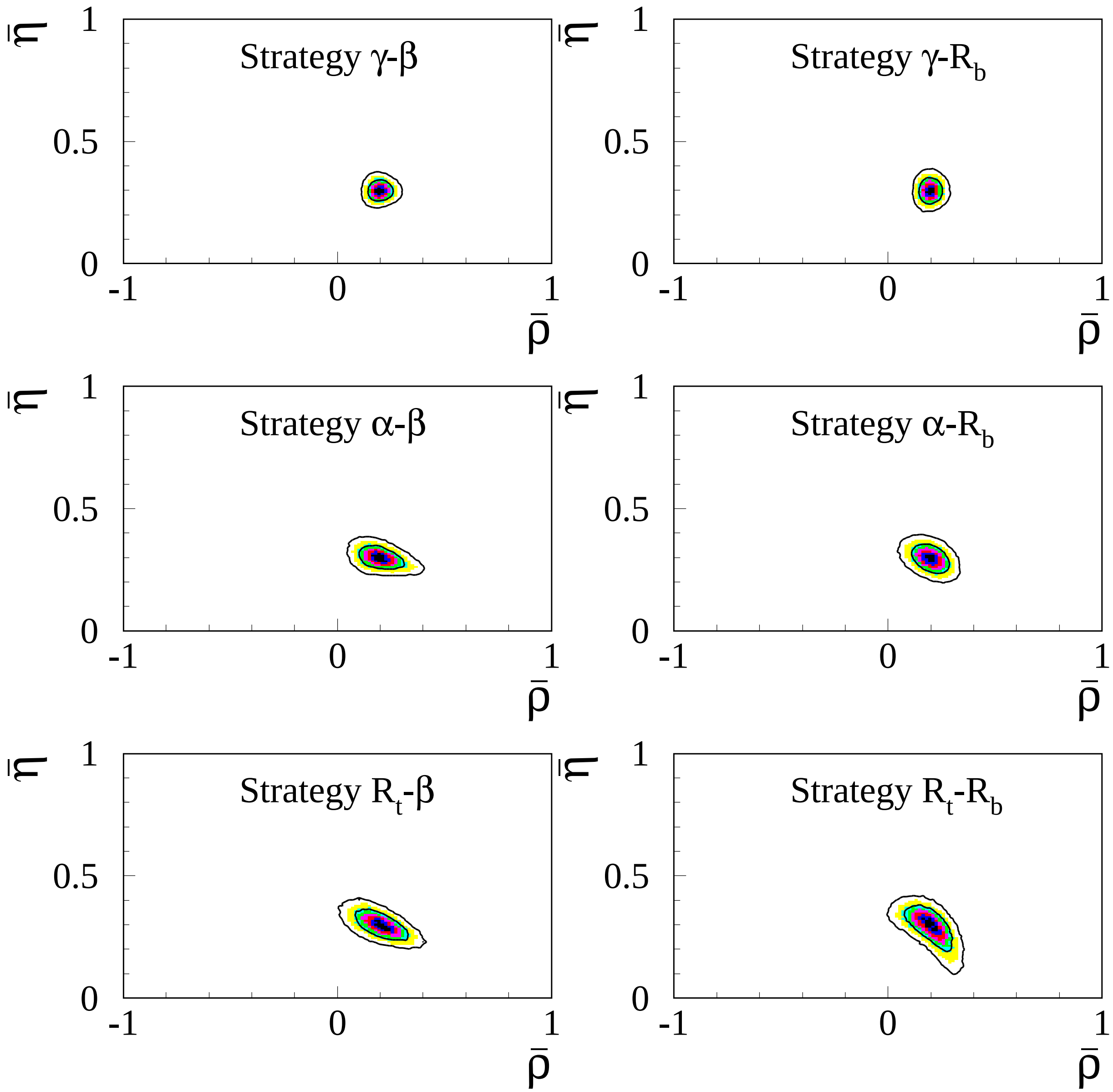


Ranking:

(γ, β) (γ, R_b)

(α, β) (α, R_b)

(R_t, β) (R_t, R_b) (R_b, β) available at present


Strategy γ-β
Strategy γ-R_b
Strategy α-β
Strategy α-R_b
Strategy R_t-β
Strategy R_t-R_b
η̄
ρ̄
0
0.5
1
-1

Alternative Set of Parameters

Parameters in Electroweak Gauge Sector

$$\alpha_{QED},\ G_F,\ sin^2\theta_W$$

$$\Downarrow$$

$$\alpha_{QED},\ G_F,\ M_Z$$

$$\Downarrow$$

$$\alpha_{QED},\ M_W,\ M_Z$$

Flavour Sector

Until 2001

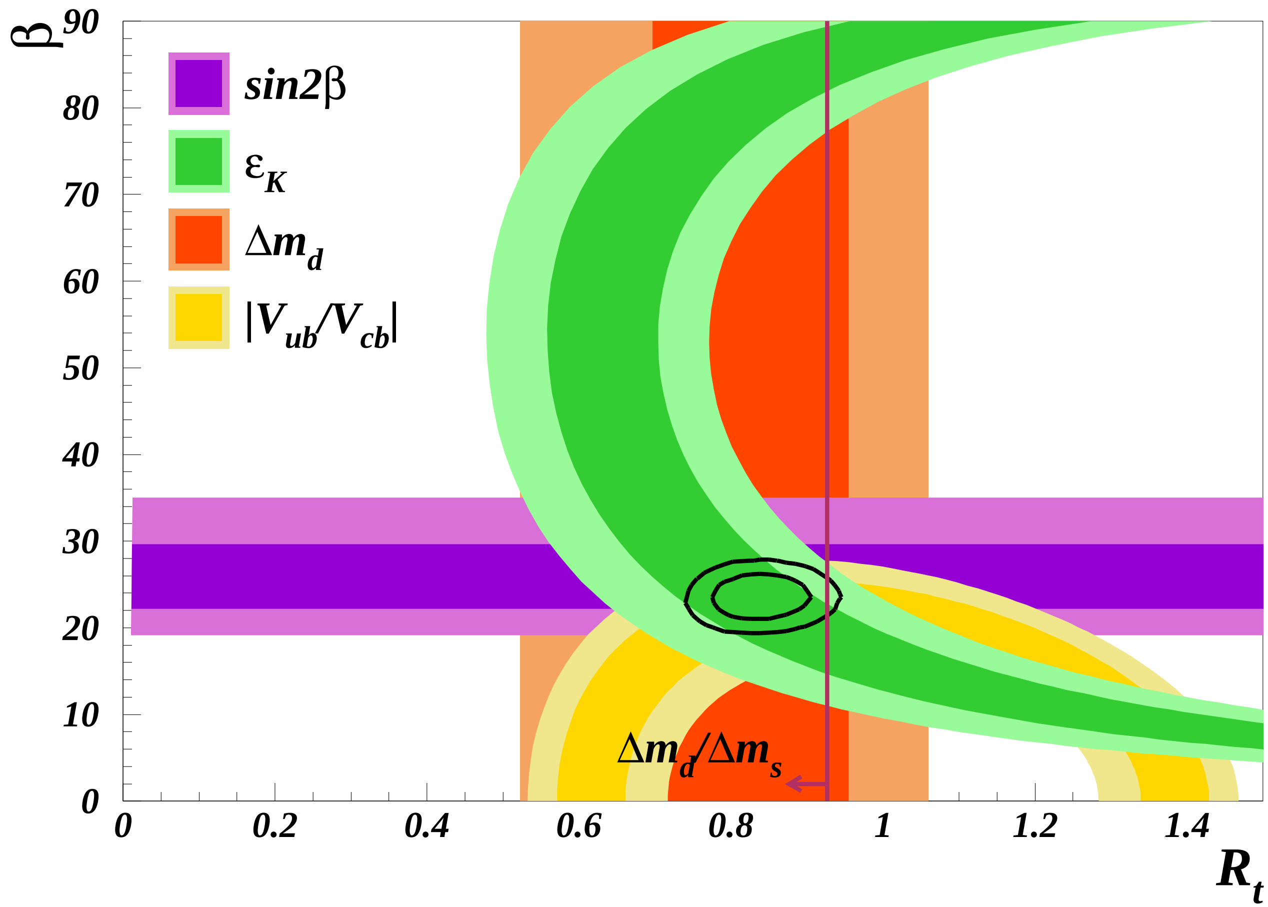
$$|V_{us}|,\ |V_{cb}|,\ \bar{\rho},\ \bar{\eta}$$

No measurements of $\bar{\rho}$ and $\bar{\eta}$ are available

Taking into account experimental feasibility and theoretical cleanness

$$|V_{us}|,\ |V_{cb}|,\ R_t,\ \beta$$

appears as a better choice



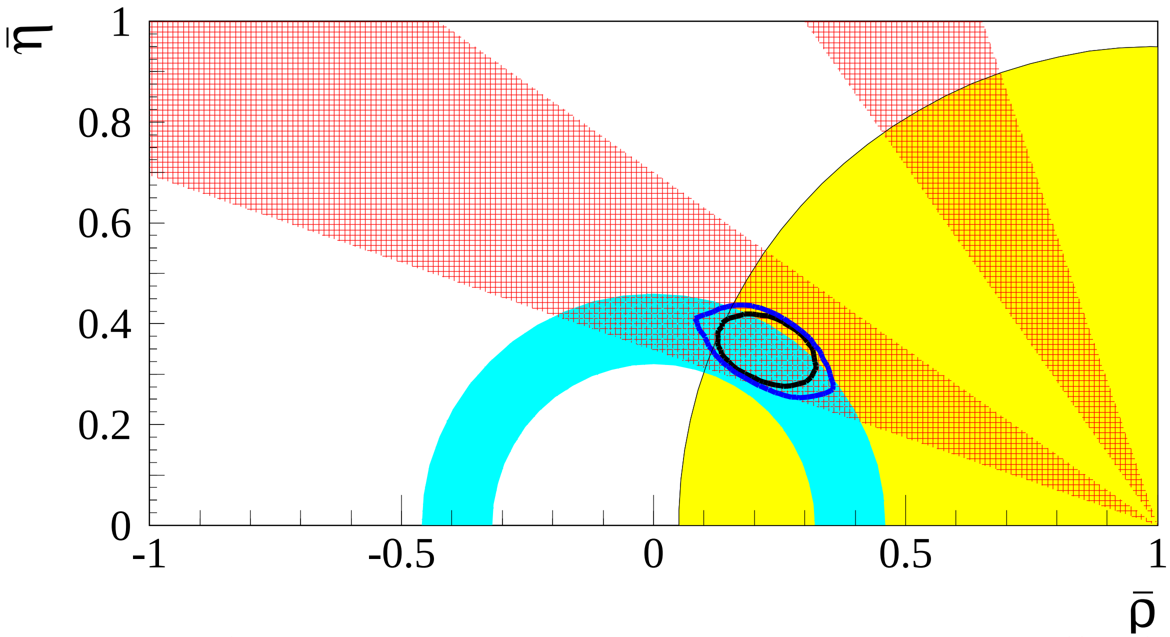
Present impact of this strategy on the standard parameters:

	$\bar{\rho}$	$\bar{\eta}$
$(R_t(\Delta m_d, \Delta m_s), \beta(sin2\beta))$	0.241 $\pm$ 0.050	0.363 $^{+0.043}_{-0.040}$
	[0.139-0.344]	[0.282-0.449]
All the constraints	0.203 $\pm$ 0.040	0.355 $\pm$0.027
	[0.124-0.278]	[0.302-0.410]

UT Fit in MFV models

Minimal Flavour Violation models: flavour violation only in V_{CKM}, new physics in the loops.
Virtue: all the effects of new physics parametrized in the function F_{tt} (entering in Δm_d and ϵ_K)

A Universal Unitarity Triangle for MFV can be constructed using only measurements that do not depend on F_{tt}: $|V_{ub}/V_{cb}|$, $\Delta m_d/\Delta m_s$ and $sin2\beta$.



Little room for MFV models that, in their prediction, differ from SM.

Adding Δm_d and ϵ_K one can fit F_{tt}:

$$F_{tt} \in [1.6, 4.1] \quad at \quad 95\%\ CL$$

(to be compared with $F_{tt} = (2.39 \pm 0.12)$ in the SM)

Conclusions

- Different fit methods on the market.
  Groups are collaborating trying to understand/quantify the differences.
  From the present study the numerical differences in the physics output are small.
- Precise determination of the UT parameters

$$\bar{\rho} \; = \; (0.203 \pm 0.040) \qquad \bar{\eta} \; = \; (0.355 \pm 0.027)$$
$$sin2\beta = (0.734^{+0.045}_{-0.034}) \quad sin2\alpha = (-0.20^{+0.23}_{-0.20}) \quad \gamma = (59.5^{+6.5}_{-5.5})$$

- CP violation picture in the SM is working well !
  - ▷ agreement between CP violating measurements and measurements without CP violation information
  - ▷ perfect agreement between the direct and the indirect determination of $sin2\beta$.
- Next to come: Δm_s.
  Expected in the range $[15.2 - 20.9]\ ps^{-1}$at 95% CL

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...Hoping for surprises !

Backup

Standard set

$\lvert V_{cb}\rvert$	$(40.43 \pm 0.74)10^{-3}$ $(39\text{-}41.9)10^{-3}$
$\bar{\rho}$	0.203 ± 0.040 (0.124-0.278)
$\bar{\eta}$	0.355 ± 0.027 (0.302-0.410)
$\sin 2\beta$	$0.734\ ^{+0.045}_{-0.034}$ (0.67-0.81)
$\sin 2\alpha$	$-0.20\ ^{+0.23}_{-0.20}$ (-0.58-0.22)
γ (degrees)	$59.5\ ^{+6.5}_{-5.5}$ (49-72)
Δm_s	$17.6\ ^{+1.9}_{-1.3}\ ps^{-1}$ (15.2-20.9) ps^{-1}

Chiral logs

$\lvert V_{cb}\rvert$	$(40.43 \pm 0.74)10^{-3}$ $(39\text{-}41.9)10^{-3}$
$\bar{\rho}$	$0.177\ ^{+0.047}_{-0.044}$ (0.082-0.266)
$\bar{\eta}$	0.365 ± 0.028 (0.31-0.42)
$\sin 2\beta$	$0.734\ ^{+0.045}_{-0.034}$ (0.67-0.81)
$\sin 2\alpha$	$-0.08\ ^{+0.25}_{-0.22}$ (-0.52-0.40)
γ (degrees)	$63.5\ ^{+7.5}_{-6.5}$ (51-78)
Δm_s	$18.0\ ^{+1.7}_{-1.5}\ ps^{-1}$ (15.4-21.6) ps^{-1}

Impact of the present constraint from $K \to \pi\nu\nu$