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The SM prediction of g-2 of the muon

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I. $(g-2)_\mu$: QFT at the highest precision level

- The different contributions to g-2 in the SM
- The recent BNL measurement: is there a sign of non-SM-physics in g-2?
- Why a reanalysis of the LO hadronic (vac.-pol.) contr.?

II. A reanalysis of the LO hadr. (vac.-pol.) contr.

- Processing the hadronic data: Clustering
- Results. The puzzle exclusive vs. inclusive
- Comparison w. other analyses. Full result.

III. Report on Abstr. 893 (Y. Srivastava and A. Widom):

Electric field corrections: are they overdone?

IV. Summary/Outlook

I. $(g-2)_\mu$: QFT at the highest precision level

The different contributions to $a_\mu \equiv (g-2)/2$ in the SM

Source	contr. to $a_\mu \times 10^{11}$	remarks
QED	116584705.7 ± 2.9	up to 5-loop! (Laporta+Remiddi, Kinoshita et al.)
EW	152 ± 4	number from Knecht et al., `hep-ph/0205102` (error to include higher order estimates)
LO hadronic	6924 ± 62	Davier+Höcker(98b) (e^+e^-, τ and pQCD) used by BNL, but see below!
NLO hadronic	-100 ± 6	A+D+H '98 in agreement with Krause '97
Light-by-Light !changed from	80 ± 40 -85 ± 25	number from A. Nyffeler, `hep-ph/0203243` used by D+H(98b) and BNL
$\sum$	116591761.7 ± 74.2	(with old LbL: 116591596.7 ± 67.3)

The recent BNL measurement, a sign of 'new physics'?

- Situation today (next Wednesday?): (in units 10^{-11})

Brookhaven Exp.:	116592023 ± 151	PRL86(2001)2227
$a_\mu^{exp} - a_\mu^{theory} =$	426 ± 165	$\sim 2.6\sigma$ w. wrong LbL
turned into	261 ± 168	$\sim 1.6\sigma$ w. correct LbL
	but can be	$1.x\sigma$, depending on a_μ^{had}

Why a reanalysis of the LO hadr. (vac.-pol.) contributions?

- ▶ Both EXP and TH will improve, hence more potential!
- ▶ AND: 'No Lose': either find sign for *New Physics* or *constrain* models beyond the SM.

Note: The biggest contr. to the TH error comes from the LO hadronic vacuum-polarization. To improve this, we need both:

⋆ new, more precise data as input in the dispersion integral

$$\boxed{a_\mu^{had,LO} = \frac{1}{4\pi^3}\int_{4m_\pi^2}^{\infty} \mathrm{d}s\, \sigma_{had}^0(s) K(s)\,,}$$

with $K(s) = \frac{m_\mu^2}{3s} \cdot (0.63\ldots 1)$, and:

⋆ convergence of the many different TH analyses...
$\longrightarrow$ careful, data-driven reanalysis to be compared with

see Table ▶

and to address the 'puzzles' data $\leftrightarrow$ pQCD and inclusive $\leftrightarrow$ exclusive (shown up already in $\Delta\alpha_{QED}$), see below.

Evaluations of the LO hadronic contr. $a_\mu^{\rm had,LO}$:

authors(year)[Ref.]	$a_\mu^{\rm had,LO} \times 10^{10}$	comments
Barkov *et al.*(85)[1]	684.0 ± 11.0	primarily e^+e^- data
Kinoshita *et al.*(85)[2]	707.0 ± 18.0	primarily e^+e^- data
Casas *et al.*(85)[3]	710.0 ± 11.5	QCD, theory and some e^+e^-
Martinovič+Dubnička(90)[4]	704.8 ± 11.5	e^+e^- and QCD
Martinovič+Dubnička(90)[4]	705.2 ± 7.6	e^+e^- and QCD
Eidelman+Jegerlehner(95)[5]	702.4 ± 15.3	primarily e^+e^- data
Adel+Ynduráin(95)[6]	711.3 ± 10.3	QCD, theory and some e^+e^-
Brown+Worstell(96)[7]	702.6 ± 16.0	primarily e^+e^- data
Alemany *et al.*(98)[8]	695.0 ± 15.0	primarily e^+e^- data
Alemany *et al.*(98)[8]	701.1 ± 9.4	$e^+e^- + \tau$ data
Davier+Höcker(98)[9]	695.1 ± 7.5	e^+e^-, τ and pQCD
Davier+Höcker(98)[10]	692.4 ± 6.2	e^+e^-, τ, and QCD sum rules
Eidelman+Jegerlehner(98)[11]	696.7 ± 11.9	(update of their result in '95)
Jegerlehner(01)[12]	697.4 ± 10.5	primarily e^+e^- data
Narison(01)[13]	702.1 ± 7.6	e^+e^-, τ and QCD sum rules
de Trocóniz+Ynduráin(01)[14]	690.9 ± 6.4	e^+e^- and τ data
Cvetič *et al.*(01)[15]	702.9 ± 9.6	QCD sum rules
Cvetič *et al.*(01)[15]	699.5 ± 9.1	QCD sum rules
Hoefer+Gluza+Jegerl.(01)[16]	697.4 ± 10.5	addr. rad. corrs. for $\pi\pi(\gamma)$
HMNT [this analysis]	689.50 ± 5.85	primarily e^+e^- data; 'excl.'
	683.51 ± 5.70	'incl.'

Martinovic and Dubnicka have got two values "depending on the way in which the experimental systematic errors of the dominant two-pion contribution to $a_\mu^{\rm vac}$ are taken into account".

Cvetič *et al.* cites two values depending on the values of the gluon condensate.

II. A reanalysis of $a_\mu^{had,LO(vac.-pol.)}$

Processing the hadronic data: Clustering

- Need to combine data from different experiments (for the same channel) in one way or another:

  $\longrightarrow$ $\overline{\sum}_{exp} \int \ldots$ leads in general to an overestimate of the errors.

  $\longrightarrow$ Data-points with small errors should dominate nearby data of poor quality, but sparse high quality data should not suppress shape information from other experiments with larger errors.

- Aim: preserve as much exp. information as possible and impose as few theor. constraints as possible on the shape of R (data vs. pQCD) ('minimum bias').

▶ Our fit-model: piecewise constant R within a *Cluster* of a given (min.) size.

▶ Realization: Non-linear χ^2-minimalization of:

$$\chi^2(\bar{R}_m, f_k) = \sum_{k=1}^{\#ofExp.} \left[(1 - f_k)\,/\mathrm{d}f_k\right]^2$$

$$+ \sum_{m=1}^{\#ofCl.} \sum_{i=1}^{N_{\{k,m\}}} \left[\left(R_i^{\{k,m\}} - f_k \bar{R}_m\right)/\mathrm{d}R_i^{\{k,m\}}\right]^2 .$$

Advantages:

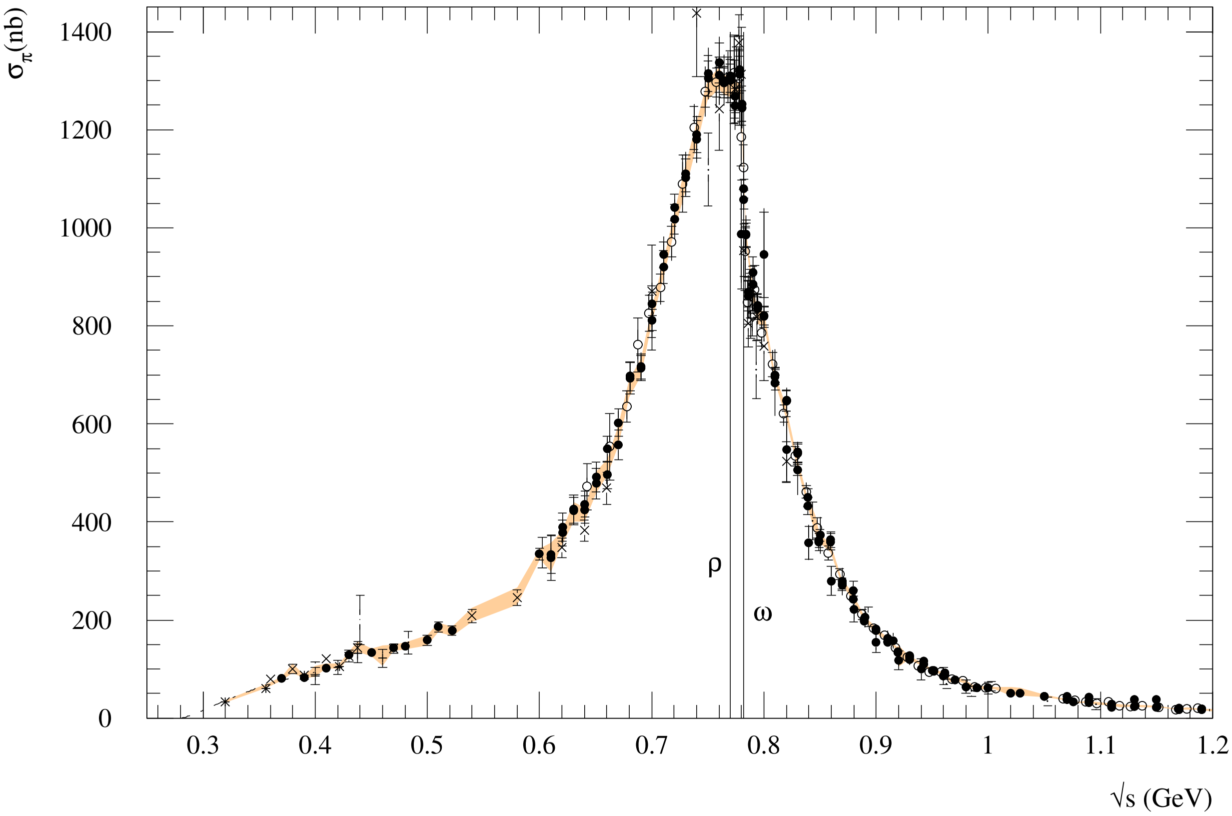
- All data contribute, weighted by their uncertainty^{-1}, hence no loss of information.
- Error estimate using the complete covariance matrix, taking into account statistical and systematic (p.t.p. and overall) errors from all different experiments
  $\longrightarrow$ correlations over different energies taken into account!
- Data-sets with a large (overall-) normalization uncertainty can still contribute shape information without leading to artificial pile-up of (asymmetric) fluctuations.
- Easy check of data-consistency and fit-quality via χ^2, and:

▶ we always find a stable $a_\mu^{had,LO}$ with stable error TOGETHER with a minimal χ^2 when varying the cluster-size.

How does it work? A few examples:

$\longrightarrow$

The most important channel: $\pi^+\pi^-$



Cluster-size: min. 5 MeV. $\rho - \omega$ interference: zoom $\longrightarrow$

Contribution to a_μ if integrated from 0.32 up to 2 GeV: $(490.94 \pm 4.75) \cdot 10^{-10}$
Further improvement will be possible soon:

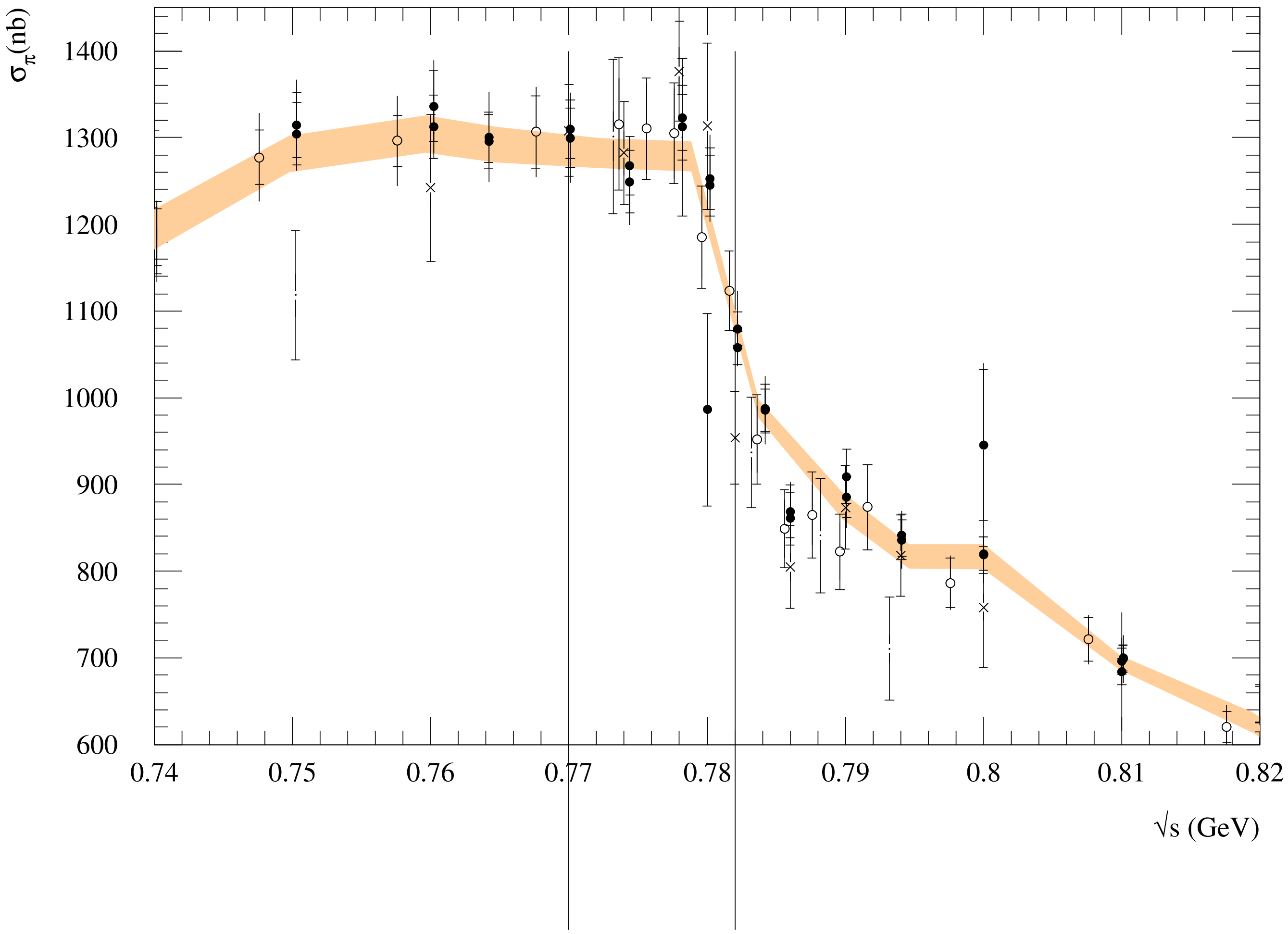
• Radiative return analysis at KLOE aims at an accuracy comparable with the recent CMD-2 measurement from Novosibirsk (but probably cannot near threshold).

• In the tail below 0.6 GeV the data is still poor, but the analysis of CMD-2 is underway, from which alone we estimate[1] an improvement of the error of $a_\mu^{had,LO}$ of about $1 \cdot 10^{-10}$!

[1] using preliminary CMD-2 data with an estimated systematic error of 2% included already in the plot for illustration

$\rho - \omega$ interference:

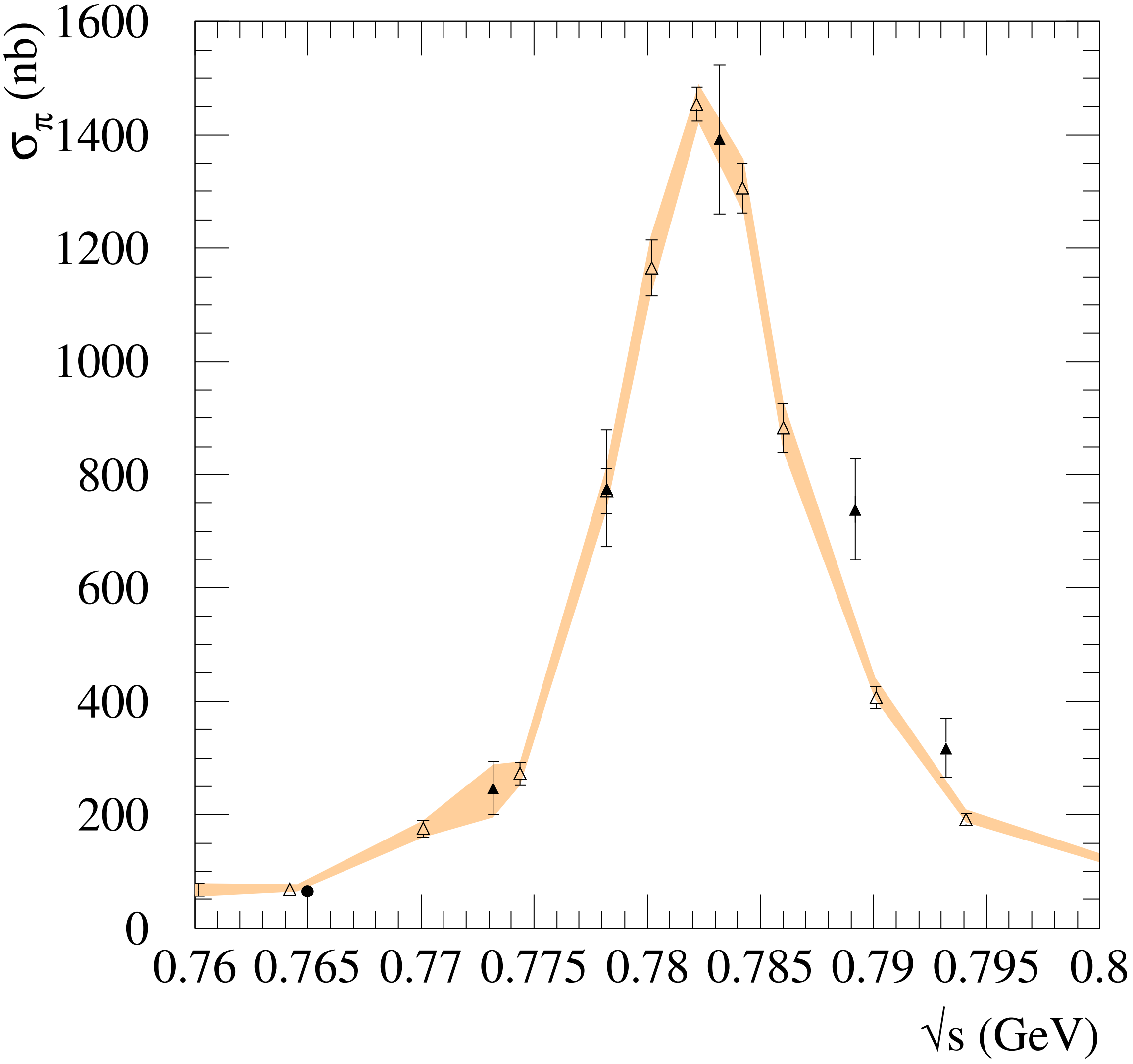
The result for a_μ and its error are stable w.r.t. the variation of the clustering size; no significant improvement of $\chi^2/d.o.f. \approx 1$ of the overall-fit of the $\pi^+\pi^-$ data for clustering finer than 5 MeV.



ω in the 3π channel:

For the (narrow!) ω and ϕ the same data-driven approach is applied: integration of the $e^+e^- \rightarrow hadrons$ data without using parametrizations of the resonance-shapes.

Clustering size: 0.5 MeV. Note the energy scale!



Results. The puzzle exclusive vs. inclusive

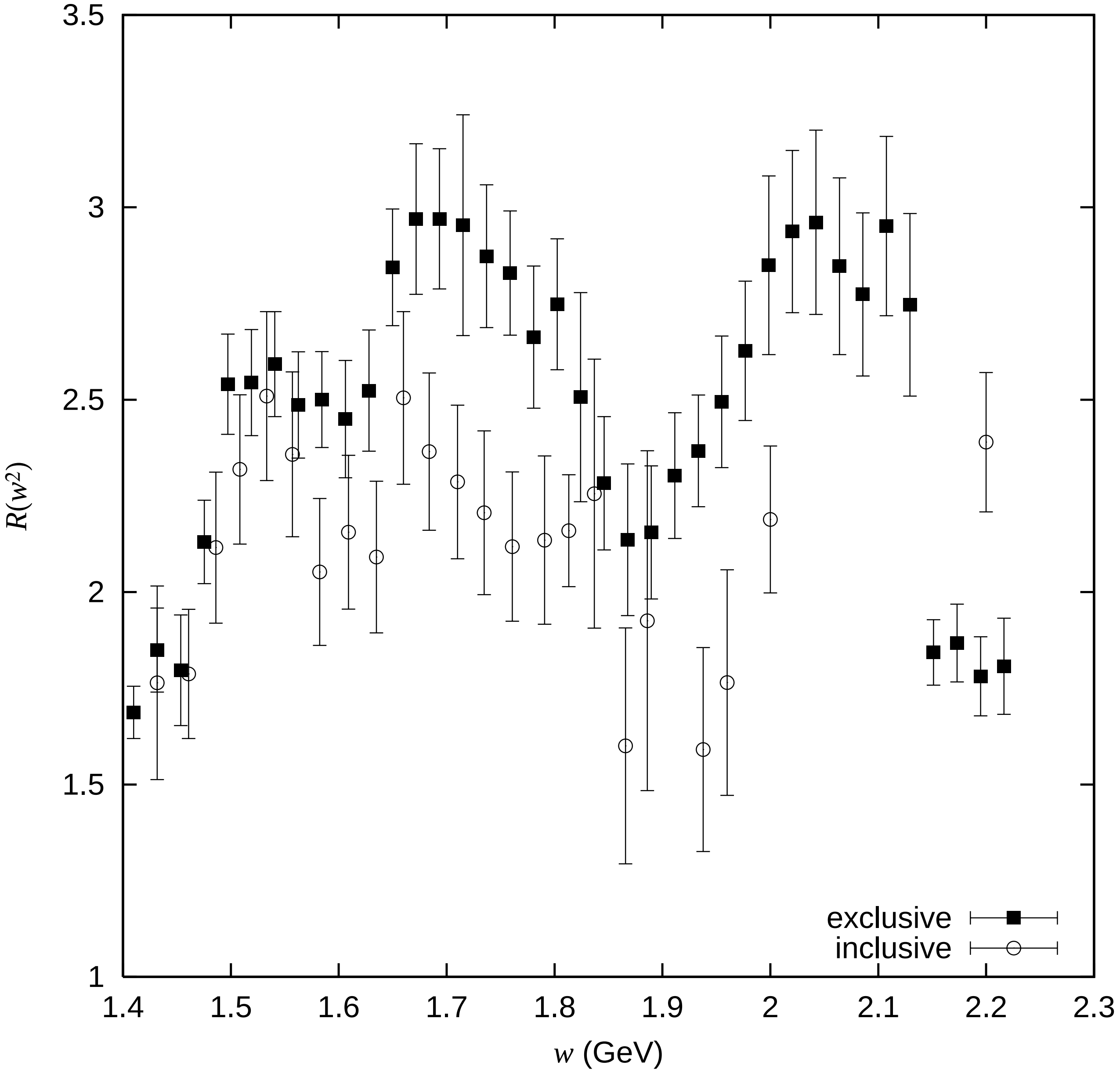
energy range	comments	$a_\mu^{\mathrm{had,LO}} \times 10^{10}$
$2m_\pi \ldots 0.32$	chiral PT	2.30 ± 0.05
$0.32 \ldots 1.43$	excl. only	597.55 ± 4.97
$1.43 \ldots 2.00$	excl. only	38.14 ± 1.68
	incl. only	32.15 ± 2.46
$2.00 \ldots 11.09$	incl. only	42.06 ± 1.25
J/ψ and $\psi(2S)$	NW	7.31 ± 0.43
$\Upsilon(1-6S)$	NW	0.10 ± 0.00
$11.09 \ldots \infty$	pQCD	2.03 ± 0.01
$\sum$ of all	ex-ex-in	689.50 ± 5.85
	ex-in-in	683.51 ± 5.70

⋆ Note: pure data-integration without using parametrizations over the ρ, ω and ϕ resonances, hence no problem with double counting, tails, interferences!

♢ Extended use of pQCD from $3.00 \ldots 3.73$ and $5.50 \ldots \infty$ gives the slightly (but not significantly) smaller values 688.52 ± 5.79 (682.53 ± 5.64).

♢ 'Puzzle' exclusive vs. inclusive analysis! Are there 'missing' radiative corrections (e.g. from vac.-pol. or extra γ emission)? $\longrightarrow$

Compilation of inclusive and $\sum$ over exclusive data after 'clustering':



Preliminary results from a QCD $\sum$-rule analysis select a value inbetween inclusive and exclusive, but closer to the inclusive result – more detailed analysis to follow.

Comparison with selected other analyses

Group	Comments	$a_\mu^{\mathrm{had,LO}} \times 10^{10}$
this analysis	ex-ex-in	689.50 ± 5.85
	ex-in-in	683.51 ± 5.70
E+J '95		702.4 ± 15.3
Jegerlehner '01	(update of E+J '98)	697.4 ± 10.5
J. (Marseille, March '02)	(prel., w. new CMD-2 data)	688.9 ± 5.8
this analysis	with their E-'binning' etc.	683.38 ± 6.06
A+D+H '98	primarily e^+e^- data	695.0 ± 15.0
D+H '98	e^+e^-, τ and pQCD	695.1 ± 7.5
D+H '98	e^+e^-, τ and QCD $\sum$-rules	692.4 ± 6.2
this analysis	with their E-'binning' etc.	690.68 ± 5.91

Note: comparing single channels/energy regimes the biggest discrepancy comes from the $\pi^+\pi^-$ channel.

▶ Here we use the latest (published) results from CMD-2 for $\sigma^0_{\pi\pi(\gamma)}$ (not π-formfactor values, sometimes not suitably defined for the use in the dispersion integral), which include a *corrected* treatment of radiative corrections, very good systematics (0.6%!) and an improved data-analysis. This is partly responsible for the reduction of the contribution to a_μ in this channel.

- Important improvement through new data from Beijing (BES, inclusive) and from Novosibirsk (CMD-2 and SND, exclusive channels).
- ? Newer experiments $\longrightarrow$ smaller x-sections ?!

The full result compared to EXP:

LO hadronic	6895 ± 59	this analysis, 'excl.'
	6835 ± 57	this analysis, 'incl.'
full a_μ	116591732.7 ± 72	'excl.'
	116591672.7 ± 70	'incl.'
Brookhaven Exp.	116592023 ± 151	PRL86(2001)2227
EXP-TH	290 ± 167	$\sim 1.7\sigma$, 'excl.'
	350 ± 166	$\sim 2.1\sigma$, 'incl.'

III. Electric field corrections

Claim of Y. Srivastava and A. Widom
hep-ph/0109020, hep-ph/0111350:

'The experimental value for g-2 contains unjustified electric field corrections.'

$\longrightarrow$

IV. Summary/Outlook

- The biggest uncertainty in a_μ still comes from the LO hadronic (vac.-pol.) contributions and is about $\pm 60 \cdot 10^{-11}$, but the error from LbL is conservatively estimated to be as high as $\pm 40 \cdot 10^{-11}$.
- We have presented a new, *data-driven* analysis for the LO hadr. contr., including new data from BES, CMD-2, SND.
- 'Non-linear Clustering' allows for a thorough treatment of stat. and sys. errors in the combination of data from different experiments.
- (New) Data in the ω and ϕ resonance regions are integrated over on the same footing (no resonance-parametrizations). Distortions from underlying background are therefore included automatically.
- τ data are not used as it is hard to quantify the associated uncertainty.

▶ With our analysis we get: $(a_\mu^{exp} - a_\mu^{theory}) \cdot 10^{11} =$

350 ± 166 (290 ± 167) for 'incl.' ('excl.').

(Preliminary QCD sum rule analysis favours 'incl.')

▶ For the future:

- ⋆ Further improved treatment of radiative corrections; inclusion of correlations between different experiments and/or channels.
- ⋆ Reanalysis of $\Delta\alpha_{\mathrm{QED}}$ using the same data-driven approach.
- ⋆ Use of QCD $\sum$-rules to disentangle the pQCD $\leftrightarrow$ incl. $\leftrightarrow$ excl. puzzle.
- ♢ Waiting for more precise data from direct and radiative return measurements (KLOE, BABAR, BELLE).
- ♢ Settle the controversial issue of Electric Field Corrections.
- ♣ Compare to the new g-2 measurement; ultimate goal at BNL: error of $\pm 40 \cdot 10^{-11}$! $\longrightarrow$ talk of Y. Semertzidis (BNL)

Thanks to Simon Eidelman for numerous help and advise with data!

Calculations of $a_\mu^{had,LO}$:

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