

SUDAKOV LOG'S in ELECTROWEAK PROCESSES

J. Kühn, Karlsruhe

NNLL in Four-Fermion Processes

J.H. Kühn, A.A. Penin; HEP-PH/9906545

J.H. Kühn, A.A. Penin, V.A. Smirnov; Eur. Phys. J. C17 (2000) 97

J.H. Kühn, S. Moch, A.A. Penin, V.A. Smirnov; Nucl. Phys. B616 (2001) 286

and related work by

E. Accomando, A. Denner, S. Pozzorini,

M. Ciafaloni, P. Ciafaloni, D. Comelli,

M. Melles

1) Motivation

large one-loop corrections at high energies

2) NNLL for SU(2) Toy Model

A) Form Factor

B) Four-Fermion Scattering

3) Subleading n_f Terms

4) Standard Model: W, Z, γ

5) Questions about Experimental Setup

6) Related Work

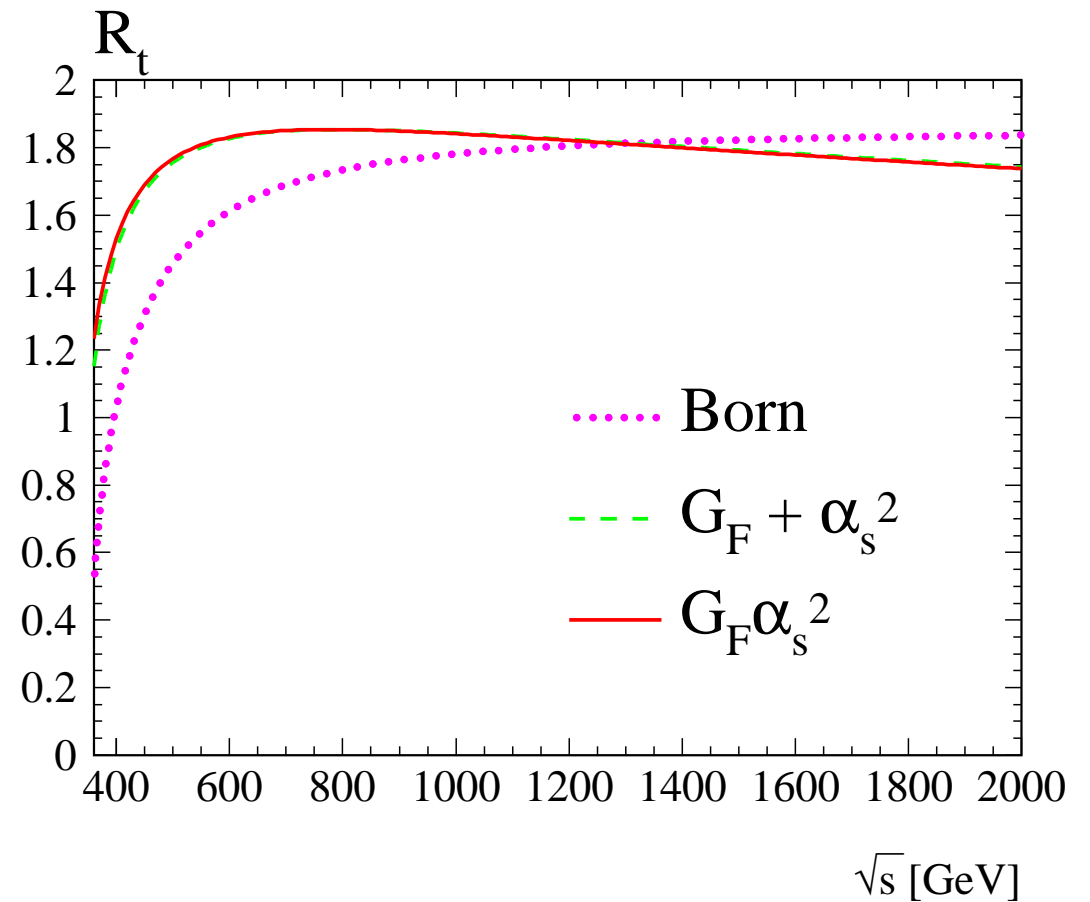
1) MOTIVATION (One-Loop!)

example: $e^+ e^- \rightarrow t\bar{t}$

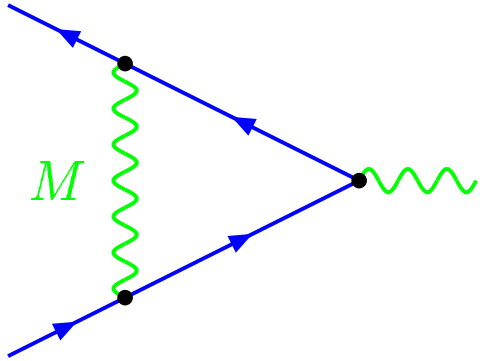
boxes + vertex corrections dominate

running coupling: $\sim \ln(s)$

boxes, vertices: $\sim \ln^2(s/M^2)$



example: massive U(1)



$$\Rightarrow \text{Born} * \left(1 - 2 \frac{\alpha}{\pi} \left(\frac{1}{4} \ln^2 \frac{s}{M^2} + \dots \right) \right)$$

magnitude ($\alpha_w/\pi = 10^{-2}$)

$\left(\frac{s}{M^2}\right)$	$-\frac{1}{4} \ln^2 \frac{s}{M^2}$	$+\frac{3}{4} \ln \frac{s}{M^2}$	$-\frac{7}{8} + \frac{\pi^2}{12}$	Σ	$*2 \frac{\alpha_w}{\pi}$
$\left(\frac{1000}{100}\right)^2$	- 5.30	+ 3.45	- 0.05	-1.9	-4%
$\left(\frac{2000}{100}\right)^2$	- 8.97	+ 4.49	- 0.05	-4.5	-9%

large negative correction, compensated by real radiation in inclusive rate!

massive boson: exclusive rate physically meaningful!

higher orders, leading log: $\left(1 - \frac{1}{2} \underbrace{\frac{g^2}{16\pi^2} \ln^2 \frac{s}{M^2}}_L\right) \Rightarrow e^{-\frac{1}{2}L}$ (per leg) in amplitude

nonabelian theory: $e^{-\frac{C_F}{2}L}$, $C_F = \frac{3}{4}$ for SU(2)

numerically: $L_{ew} = \begin{matrix} 0.07 & \text{for } 1 \text{ TeV} \\ 0.11 & \text{for } 2 \text{ TeV} \end{matrix}$

expected correction for $f_L \bar{f}_L \rightarrow f'_L \bar{f}'_L$: $\sim \underbrace{2 \cdot 4}_{\text{rate}} \cdot \underbrace{4}_{\text{legs}} \cdot \frac{3}{4} \cdot \frac{1}{2} L_{ew} \sim 20 - 30\%$

\Rightarrow large number of papers during past years:
NNLL; Yukawa coupling, external gauge bosons.

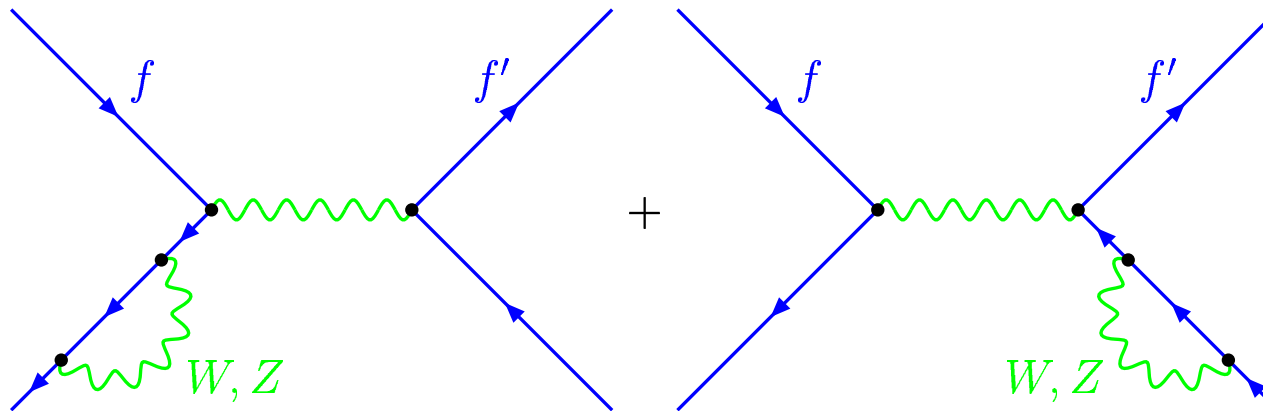
EW theory; One-Loop

$$f_L \bar{f}_L \rightarrow f'_L \bar{f}'_L : \quad \mathcal{M}_{\text{Born}} = \frac{ig^2}{s} \left(T_f^3 T_{f'}^3 + \tan^2 \Theta_w \frac{Y_f Y_{f'}}{4} \right) \quad (M_Z^2/s \rightarrow 0)$$

one-loop: explicit calculation ✓

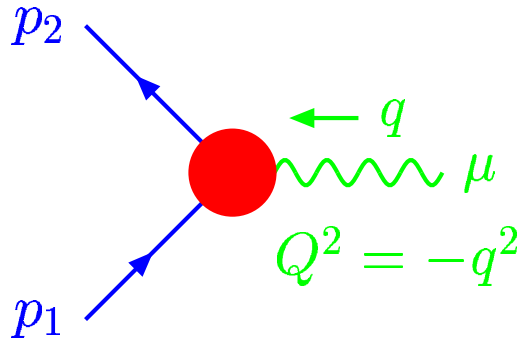
dominant correction: * $(1 - F_{L,R}^f)$ (per line) with

$$F_L^f = \left(\underbrace{\frac{1}{2}}_W + \underbrace{\frac{1}{4c_W^2} + \tan_w^2 (s_W^2 Q_f^2 - 2T_f^3 Q_f)}_Z \right) L, \quad F_R^f = \left(\underbrace{\tan_w^2 s_W^2 Q_f^2}_Z \right) L$$



2) NNLL for SU(2) Toy Model

A) Form Factor



Born:

$$\mathcal{F}_B = \bar{\psi}(p_2)\gamma_\mu\psi(p_1)$$

$$\frac{\partial}{\partial \ln Q^2} \mathcal{F} = \left[\int_{M^2}^{Q^2} \frac{dx}{x} \gamma(\alpha(x)) + \zeta(\alpha(Q^2)) + \xi(\alpha(M^2)) \right] \mathcal{F}$$

Collins, Sen

$$\Rightarrow \mathcal{F} = \mathcal{F}_B F_0(\alpha(M^2)) \exp \left\{ \int_{M^2}^{Q^2} \frac{dx}{x} \left[\int_{M^2}^x \frac{dx'}{x'} \gamma(\alpha(x')) + \zeta(\alpha(x)) + \xi(\alpha(M^2)) \right] \right\}$$

aim: NNLL \Rightarrow corresponds to all terms of the form:

$$\alpha^n \ln^{2n} \left(\frac{Q^2}{M^2} \right) + \alpha^n \ln^{2n-1} \left(\frac{Q^2}{M^2} \right) + \alpha^n \ln^{2n-2} \left(\frac{Q^2}{M^2} \right)$$

LL NLL NNLL

requires running of α and:

$$\begin{array}{ll} \zeta(\alpha), \xi(\alpha), F_0(\alpha) & \text{up to } \mathcal{O}(\alpha) \\ \gamma(\alpha) & \text{up to } \mathcal{O}(\alpha^2) ! \end{array}$$

expansion by regions:

$$\gamma^{(1)} = -2C_F, \quad \zeta^{(1)} = 3C_F, \quad \xi^{(1)} = 0, \quad F_0^{(1)} = -C_F \left(\frac{7}{2} + \frac{2\pi^2}{3} \right)$$

$$\gamma^{(2)} = -2C_F \left[\left(\frac{67}{9} - \frac{\pi^2}{3} \right) C_A - \frac{20}{9} T_F n_f \right]$$

Matsuura + ..., Kramer + ...

Higgs doublet: $\gamma^{(2)} \rightarrow + \frac{16}{9} C_F T_F$

$$\mathcal{F} = \mathcal{F}_B + \frac{\alpha}{4\pi} \mathcal{F}^{(1)} + \left(\frac{\alpha}{4\pi}\right)^2 \mathcal{F}^{(2)} + \dots$$

one-loop:

$$\mathcal{F}^{(1)} = \left(\frac{1}{2} \gamma^{(1)} \ln^2 \left(\frac{Q^2}{M^2} \right) + (\xi^{(1)} + \zeta^{(1)}) \ln \left(\frac{Q^2}{M^2} \right) + F_0^{(1)} \right) \mathcal{F}_B$$

two-loop:

$$\Rightarrow \mathcal{F}_{LL}^{(2)} = \frac{1}{8} (\gamma^{(1)})^2 \ln^4 \left(\frac{Q^2}{M^2} \right) \mathcal{F}_B$$

$$\mathcal{F}_{NLL}^{(2)} = \frac{1}{2} \left(\zeta^{(1)} - \frac{1}{3} \beta_0 \right) \gamma^{(1)} \ln^3 \left(\frac{Q^2}{M^2} \right) \mathcal{F}_B$$

$$\mathcal{F}_{NNLL}^{(2)} = \frac{1}{2} \left(\gamma^{(2)} + (\zeta^{(1)} - \beta_0) \zeta^{(1)} + F_0^{(1)} \gamma^{(1)} \right) \ln^2 \left(\frac{Q^2}{M^2} \right) \mathcal{F}_B$$

concrete numbers:

SU(2)

$$\mathcal{F}^{(1)} = \left[-\frac{3}{4} \ln^2 \left(\frac{Q^2}{M^2} \right) + \frac{9}{4} \ln \left(\frac{Q^2}{M^2} \right) - \left(\frac{21}{8} + \frac{\pi^2}{2} \right) \right] \mathcal{F}_B$$

$$\mathcal{F}^{(2)} = \left[\frac{9}{32} \ln^4 \left(\frac{Q^2}{M^2} \right) - \frac{19}{48} \ln^3 \left(\frac{Q^2}{M^2} \right) - \left(\frac{463}{48} - \frac{7\pi^2}{8} \right) \ln^2 \left(\frac{Q^2}{M^2} \right) \right] \mathcal{F}_B$$

U(1)

$$\mathcal{F}^{(1)} = \left[-\ln^2 \left(\frac{Q^2}{M^2} \right) + 3 \ln \left(\frac{Q^2}{M^2} \right) - \left(\frac{7}{2} + \frac{2\pi^2}{3} \right) \right] \mathcal{F}_B$$

$$\mathcal{F}^{(2)} = \left[\frac{1}{2} \ln^4 \left(\frac{Q^2}{M^2} \right) - \frac{52}{9} \ln^3 \left(\frac{Q^2}{M^2} \right) + \left(\frac{625}{18} + \frac{2\pi^2}{3} \right) \ln^2 \left(\frac{Q^2}{M^2} \right) \right] \mathcal{F}_B$$

Large coefficients!

B) Four-Fermion Scattering $f + f' \rightarrow f + f'$

$$\frac{ig^2}{s} (\bar{\psi}_2 t^a \gamma^\mu \psi_1) (\bar{\psi}_4 t^a \gamma_\mu \psi_3) \equiv \frac{ig^2}{s} \mathcal{A}^\lambda$$

one-loop:

$$\begin{aligned} \frac{ig^2(Q^2)}{s} \frac{\alpha}{2\pi} & \left[\left\{ -C_F \left(\ln^2 \left(\frac{-s}{M^2} \right) - 3 \ln \left(\frac{-s}{M^2} \right) \right) + \right. \right. \\ & \left. \left(-C_A \ln \left(\frac{u}{s} \right) + 2 \left(C_F - \frac{T_F}{N} \right) \ln \left(\frac{u}{t} \right) \right) \ln \left(\frac{-s}{M^2} \right) \right\} \mathcal{A}^\lambda \\ & \left. + \left\{ 2 \frac{C_F T_F}{N} \ln \left(\frac{u}{t} \right) \ln \left(\frac{-s}{M^2} \right) \right\} \mathcal{A}^d \right] + \text{non-log-terms} \end{aligned}$$

with $\mathcal{A}^d = (\bar{\psi}_2 \gamma^\mu \psi_1) (\bar{\psi}_4 \gamma_\mu \psi_3)$ diagonal!

\Rightarrow evolution for $\tilde{\mathcal{A}} = \begin{pmatrix} \tilde{\mathcal{A}}^\lambda \\ \tilde{\mathcal{A}}^d \end{pmatrix}$ ($\tilde{\mathcal{A}} = \mathcal{A} * \text{collinear logs}$)

$$\frac{\partial}{\partial \ln Q^2} \tilde{\mathcal{A}} = \chi(\alpha(Q^2)) \tilde{\mathcal{A}}$$

Sen + ..., Serman + ...

$\chi_{\lambda\lambda}, \chi_{\lambda d}$ from the one-loop terms (single-log \Rightarrow NLL, non-log \Rightarrow NNLL)
 $\chi_{d\lambda}$ analogous; $\chi_{dd} = 0$

$\tilde{\mathcal{A}}$ from complete one-loop result

diagonalize matrix χ

$$\tilde{\mathcal{A}} = \sum_i \tilde{\mathcal{A}}_{0i}(\alpha(M^2)) \exp \left[\int_{M^2}^{Q^2} \frac{dx}{x} \chi_i(\alpha(x)) \right]$$

starting values for $\tilde{\mathcal{A}}$ from one-loop result (including constant terms)

Angular dependence is important for NLL and NNLL.

result:

$$\sigma = \sigma_B + \frac{\alpha}{4\pi} \sigma^{(1)} + \left(\frac{\alpha}{4\pi}\right)^2 \sigma^{(2)} + \dots$$

$u\bar{u} \rightarrow u\bar{u}$

$$\sigma^{(1)} = \left[-3 \ln^2 \left(\frac{s}{M^2} \right) + \frac{80}{3} \ln \left(\frac{s}{M^2} \right) - \left(\frac{22}{9} + 3\pi^2 \right) \right] \sigma_B$$

$$\sigma^{(2)} = \left[\frac{9}{2} \ln^4 \left(\frac{s}{M^2} \right) - \frac{449}{6} \ln^3 \left(\frac{s}{M^2} \right) + \left(\frac{2414}{9} + \frac{37\pi^2}{3} \right) \ln^2 \left(\frac{s}{M^2} \right) \right] \sigma_B$$

$u\bar{u} \rightarrow d\bar{d}$

$$\sigma^{(1)} = \left[-3 \ln^2 \left(\frac{s}{M^2} \right) + \frac{26}{3} \ln \left(\frac{s}{M^2} \right) + \left(\frac{221}{9} - 3\pi^2 \right) \right] \sigma_B$$

$$\sigma^{(2)} = \left[\frac{9}{2} \ln^4 \left(\frac{s}{M^2} \right) - \frac{125}{6} \ln^3 \left(\frac{s}{M^2} \right) - \left(\frac{1625}{18} - \frac{37\pi^2}{3} \right) \ln^2 \left(\frac{s}{M^2} \right) \right] \sigma_B$$

U(1) theory

$$\sigma^{(1)} = \left[-4 \ln^2 \left(\frac{s}{M^2} \right) + 12 \ln \left(\frac{s}{M^2} \right) - \left(\frac{376}{9} - \frac{4\pi^2}{3} \right) \right] \sigma_B$$

$$\sigma^{(2)} = \left[8 \ln^4 \left(\frac{s}{M^2} \right) - \frac{532}{9} \ln^3 \left(\frac{s}{M^2} \right) + \left(\frac{1130}{3} + \frac{16\pi^2}{3} \right) \ln^2 \left(\frac{s}{M^2} \right) \right] \sigma_B$$

3) Subleading n_f Terms

Stability, subleading terms?

consider the full series of the n_f terms for the form factor

$$\mathcal{F} = \left\{ 1 + \left(\frac{\alpha}{4\pi} \right)^2 n_f \left[-\frac{4}{9} \ln^3 \left(\frac{Q^2}{M^2} \right) + \frac{38}{9} \ln^2 \left(\frac{Q^2}{M^2} \right) - \frac{34}{3} \ln \left(\frac{Q^2}{M^2} \right) + \frac{16\pi^2}{27} + \frac{115}{9} \right] \right\} \mathcal{F}_B$$

Feucht, Kühn, Moch

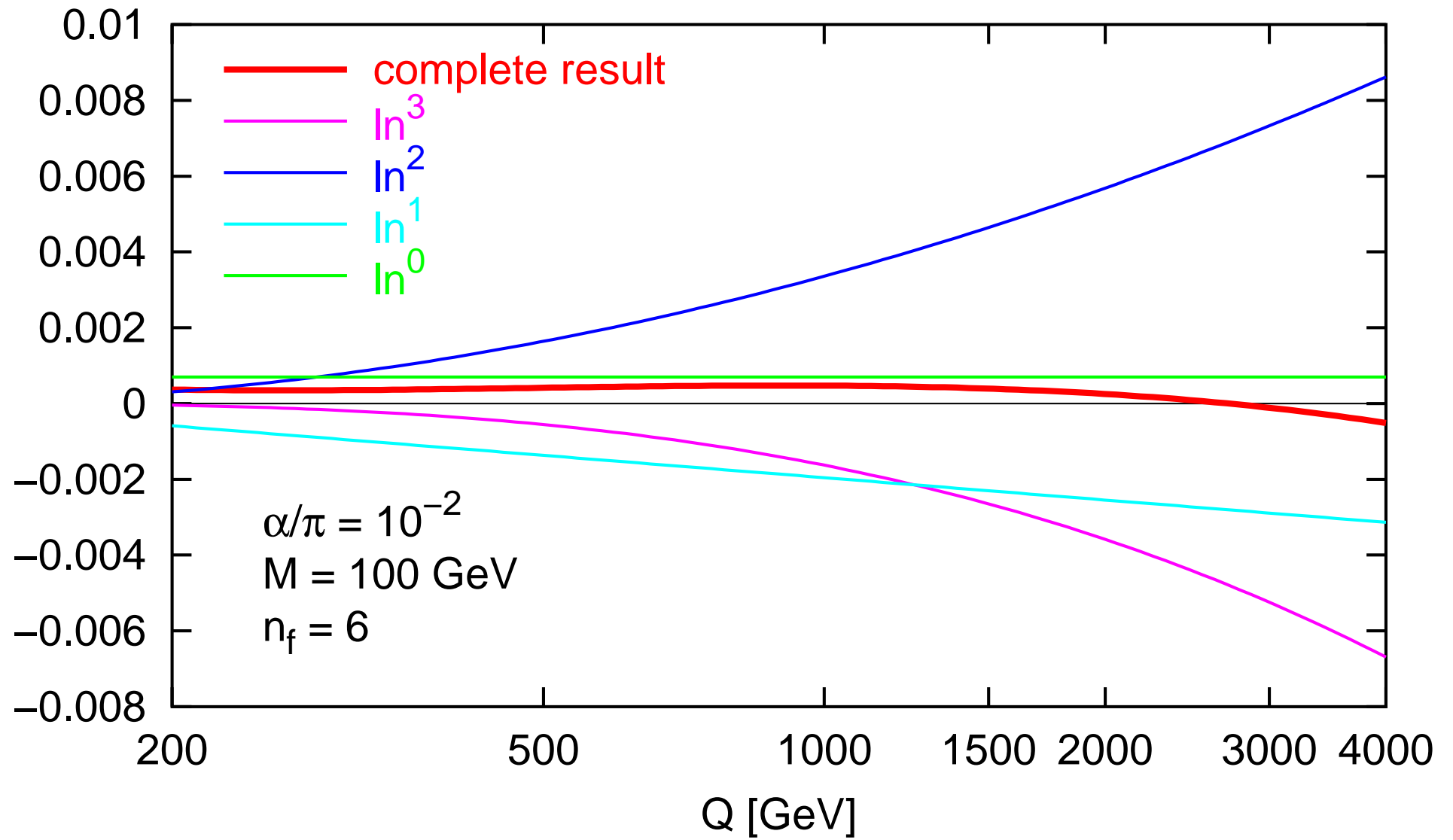
\ln^3 and \ln^2 consistent with NNLL,

\ln^1 and \ln^0 new;

alternating sign!

increasing coefficients!

energy dependence of correction:



4) Standard Model: W, Z, γ

examine $f' \bar{f}' \rightarrow f \bar{f}$

$$A_B = \frac{ig^2}{s} \sum_{I,J=L,R} \left(T_{f'}^3 T_f^3 + t_W^2 \frac{Y_{f'} Y_f}{4} \right) A_{IJ}^{f'f} \quad \text{with} \quad A_{IJ}^{f'f} = (\bar{f}'_I \gamma^\mu f'_I) (\bar{f}_J \gamma_\mu f_J)$$

corrections from photon radiation up to cutoff $\omega \ll M_{W,Z}$ must be taken into account separately (prescription of **Fadin et al.**).

define

$$L(s) = \frac{g^2}{16\pi^2} \ln^2 \left(\frac{s}{M^2} \right) = 0.07 \quad (0.11)$$

$$l(s) = \frac{g^2}{16\pi^2} \ln \left(\frac{s}{M^2} \right) = 0.014 \quad (0.017)$$

$$a = \frac{g^2}{16\pi^2} = 0.003$$

for $\sqrt{s} = 1 \text{ TeV}$ (2 TeV)

result:

$$\begin{aligned}\sigma/\sigma_B(e^+e^- \rightarrow Q\bar{Q}) &= 1 - 1.66 L(s) + 5.31 l(s) - 15.86 a \\ &\quad + 1.93 L^2(s) - 9.43 L(s)l(s) + 28.73 l^2(s)\end{aligned}$$

$$\begin{aligned}\sigma/\sigma_B(e^+e^- \rightarrow q\bar{q}) &= 1 - 2.18 L(s) + 20.58 l(s) - 36.34 a \\ &\quad + 2.79 L^2(s) - 50.06 L(s)l(s) + 295.12 l^2(s)\end{aligned}$$

$$\begin{aligned}\sigma/\sigma_B(e^+e^- \rightarrow \mu^+\mu^-) &= 1 - 1.39 L(s) + 10.12 l(s) - 31.33 a \\ &\quad + 1.42 L^2(s) - 18.43 L(s)l(s) + 99.89 l^2(s)\end{aligned}$$

similar and even larger corrections for A_{LR}

smaller corrections for A_{FB}

5) Questions about Experimental Setup

- How exclusive are “exclusive” four-fermion processes?
- Can collinear ISR of W, Z be detected at TESLA or at LHC?
- Can collinear FSR of W, Z be detected in quark jets?
If we constrain the jet mass, will we encounter QCD Sudakov logs?
- Interplay?

6) Related Work

Accomando, Denner, Pozzorini (ABS 924, hep-ph/0110114)

one-loop logarithmic corrections in $p + p \rightarrow W Z + X$
 $\rightarrow W \gamma + X$

large negative corrections (5–20%)

difference between transverse and longitudinal gauge bosons

$$\begin{aligned} \text{transverse} &\sim C_A \ln^2 \frac{s}{M^2} \\ \text{longitudinal} &\sim C_F \ln^2 \frac{s}{M^2} \end{aligned}$$

important differences between double resonant and narrow width approximations!

M. Ciafaloni, P. Ciafaloni, Comelli (ABS 15, hep-ph/0111109)

hard inclusive (!) processes in spontaneously broken gauge theory
(fermion and boson initiated)

e.g. $f\bar{f} \rightarrow f'\bar{f}' + X$

difference with respect to QCD: initial beam charge $\neq 0$

\Rightarrow different evolution equation

\Rightarrow Bloch-Nordsieck theorem violated!

(infrared cutoff provided by SSB)

Melles (ABS 26, hep-ph/0108221)

Beccaria, Melles, Renard, Verzegnassi (hep-ph/0112273)

Melles

angular dependent (one-loop) terms important for NLL analysis

(cf. KPS for fermions)

application to gauge bosons

Beccaria, Melles, Renard, Verzegnassi

SUSY-scalar production at lepton colliders: assume “light” SUSY-scalars
leading and subleading terms in one-loop LL and NLL

claim: Sudakov region starts around 1 TeV

(role of subleading logarithmic terms?)

SUMMARY

- large double logarithmic corrections
e.g. $e^+e^- \rightarrow q\bar{q}$ (2 TeV), $M = M_W$ for IR cutoff
 $\sim -24\%$ (one-loop) LL $\sim +3.4\%$ (two-loop) LL
- large compensations from subleading terms
 $\sim +35\%$ (one-loop) NLL $\sim -9.3\%$ (two-loop) NLL
- again alternating sign in NNLL order
 $\sim -10\%$ (one-loop) NNLL $\sim +8.5\%$ (two-loop) NNLL
- corresponding pattern in subleading n_f terms
- large differences between different flavours
- results for σ , A_{FB} , A_{LR} available
- rapidly evolving field