The $B \to D^* \ell \nu$ Form Factor at Zero Recoil and the Determination of $|V_{cb}|$

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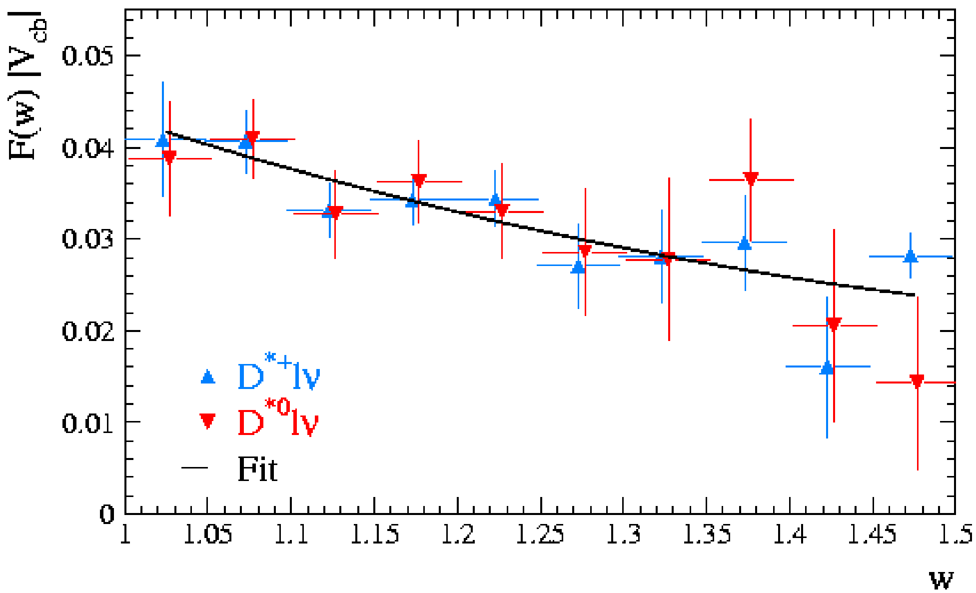
$|\mathrm{V_{cb}}|$ and the $\mathrm{B} \rightarrow \mathrm{D}^* \ell \nu$ Decay Rate

In the zero recoil limit,

$$\lim_{\omega \to 1} \frac{1}{(\omega^2 - 1)^{1/2}} \frac{d\Gamma(B \to D^* \ell \nu)}{d\omega} = \begin{pmatrix} \text{known} \\ \text{factors} \end{pmatrix} |V_{cb}|^2 \; |h_{A_1}(1)|^2$$

[CLEO CLNS 01-1773]



$$|V_{cb}| \; h_{A_1}(1) \;\; = \;\; (38.3 \pm 0.5 \pm 0.9) \times 10^{-3}$$ [World Avg.: Artuso & Barberio]

Average of ALEPH, DELPHI, OPAL, Belle and CLEO

$\mathrm{B} \to \mathrm{D}^*\ell\nu$ in HQET

Anatomy of power law corrections [Falk & Neubert]:

$$h_{A_1}(1) = \eta_A \left[1 - \frac{\ell_V}{(2m_c)^2} + \frac{2\ell_A}{4m_c m_b} - \frac{\ell_P}{(2m_b)^2} + \mathcal{O}(1/m_Q^3) \right]$$

Radiative correction η_A known to two-loop level [Czarnecki & Melnikov].

Three unknown long-distance coefficients: ℓ_V, ℓ_A and ℓ_P. **Use latttice**!

Three constraints: the matrix elements [Hashimoto]

$$\langle\, D^* \mid \mathcal{A}_j \mid B \,\rangle \propto \quad h_{A_1}(1) \quad \Longrightarrow \quad \ell_V,\ \ell_A \text{ and } \ell_P$$

$$\langle\, D \mid \mathcal{V}_0 \mid B \,\rangle \propto \quad h_+(1) \quad = \eta_V \left[1 - \ell_P \left(\frac{1}{2m_c} - \frac{1}{2m_b} \right)^2 + \ldots \right]$$

$$\langle\, D^* \mid \mathcal{V}_0 \mid B^* \rangle \propto \quad h_1(1) \quad = \eta_V \left[1 - \ell_V \left(\frac{1}{2m_c} - \frac{1}{2m_b} \right)^2 + \ldots \right]$$

Double Ratios of 3-pt Correlators

- Bulk of statistical and systematic errors cancel.
- Large part of renormalzation captured nonperturbatively.

$$\mathcal{R}_+ = \frac{\langle\, D \,|\, \mathcal{V}_0 \,|\, B \,\rangle \; \langle\, B \,|\, \mathcal{V}_0 \,|\, D \,\rangle}{\langle\, D \,|\, \mathcal{V}_0 \,|\, D \,\rangle \; \langle\, B \,|\, \mathcal{V}_0 \,|\, B \,\rangle} \Rightarrow |h_+(1)|^2$$

form factor for $B \longrightarrow D$

$$\mathcal{R}_1 = \frac{\langle\, D^* \,|\, \mathcal{V}_0 \,|\, B^* \,\rangle \; \langle\, B^* \,|\, \mathcal{V}_0 \,|\, D^* \,\rangle}{\langle\, D^* \,|\, \mathcal{V}_0 \,|\, D^* \,\rangle \; \langle\, B^* \,|\, \mathcal{V}_0 \,|\, B^* \,\rangle} \Rightarrow |h_1(1)|^2$$

form factor for $B^* \longrightarrow D^*$

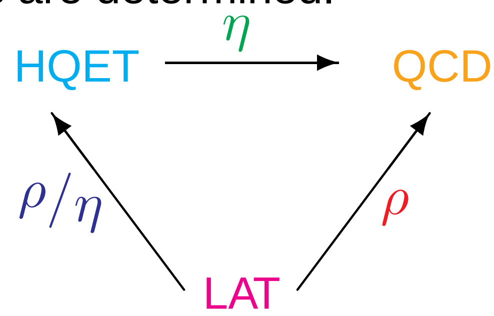
$$\mathcal{R}_{A_1} = \frac{\langle\, D^* \,|\, \mathcal{A}_1 \,|\, B \,\rangle \; \langle\, B^* \,|\, \mathcal{A}_1 \,|\, D \,\rangle}{\langle\, D^* \,|\, \mathcal{A}_1 \,|\, D \,\rangle \; \langle\, B^* \,|\, \mathcal{A}_1 \,|\, B \,\rangle} \Rightarrow |H_{A_1}(1)|^2$$

Note $H_{A_1}(1)$ is not the desired form factor $h_{A_1}(1)$ for $B \longrightarrow D^* \ell\nu$.

$$H_{A_1}(1) = \hat{\eta}_A \left[1 - \ell_A \left(\frac{1}{2m_c} - \frac{1}{2m_b} \right)^2 + \ldots \right]$$

Perturbative Matching

Lattice to QCD matching proceeds through intermediate HQET scheme in which the long-distance ℓ coefficients are determined.

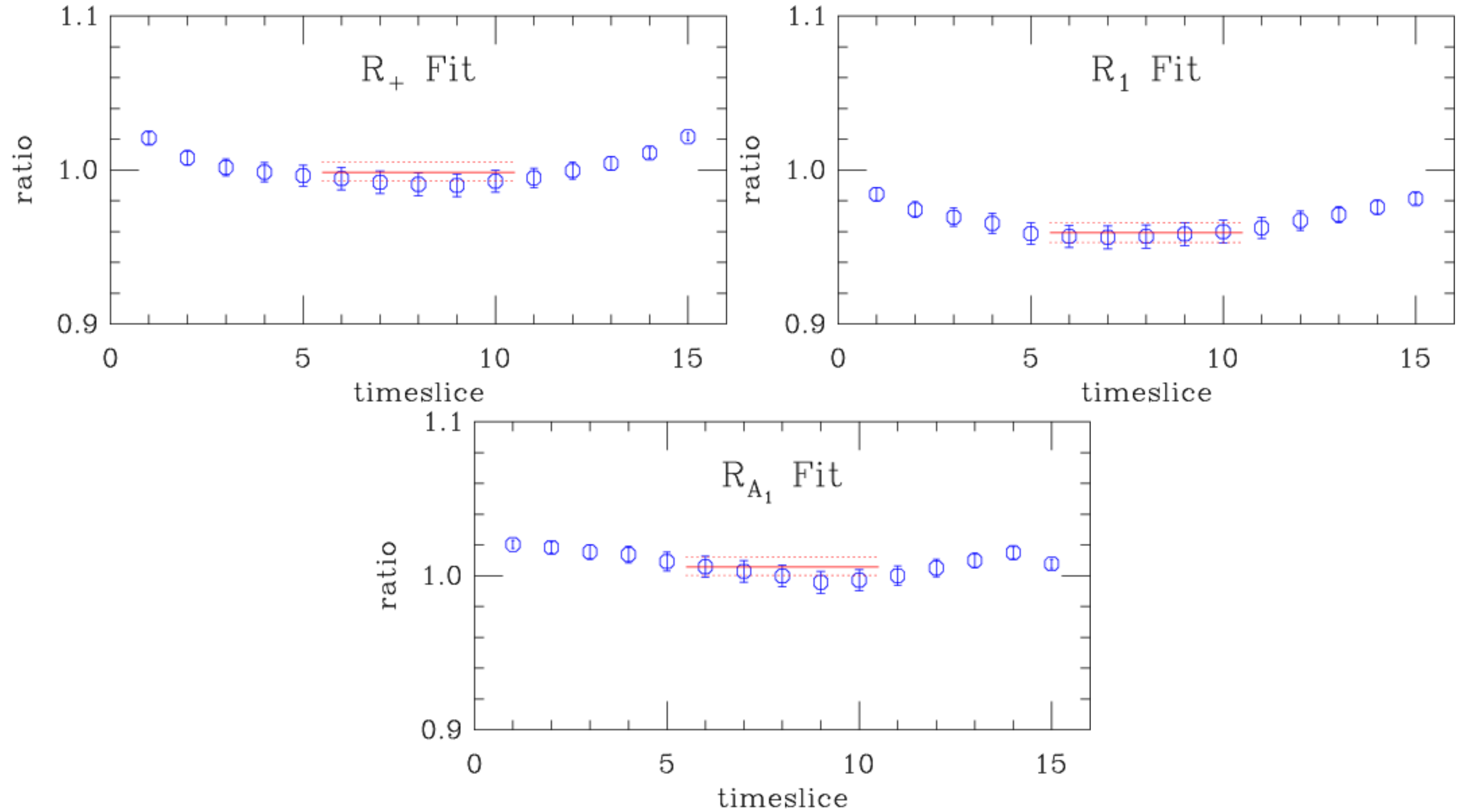


- ρ's known to one-loop order [Harada, Kronfeld, Hashimoto & Onogi].
- η's know to two loops [Czarnecki & Melnikov].
- BLM-improved matching performed to one-loop order in the V-scheme.
- For $\beta = 5.9$, $\eta_A = 0.9724$, and e.g.

m_{0b}, m_{0c} GeV	ρ_V/η_V	$\hat{\rho}_A/\hat{\eta}_A$
$6.03, 0.83$	1.0015	0.9868
$4.36, 1.16$	1.0016	0.9944
$3.06, 2.02$	1.0003	0.9990

Double Ratio Plateaus

Example fits. Mesons created (annihilated) at $t = 0$ ($t = 16$). Double ratio values as a function of time when vector or axial-vector current is applied.



Reasonable χ^2 obtained for all ratio averages.

$h_+(1)$, $h_1(1)$ and $H_{A_1}(1)$ Mass Dependence

Mass dependence is of the form *e.g.*

$$\frac{1 - h_1(1)/\eta_V}{\Delta^2} = a^2 \ell_V - a^3 \ell_V^{[3]} \left(\frac{1}{2am_c} + \frac{1}{2am_b} \right) + \ldots$$

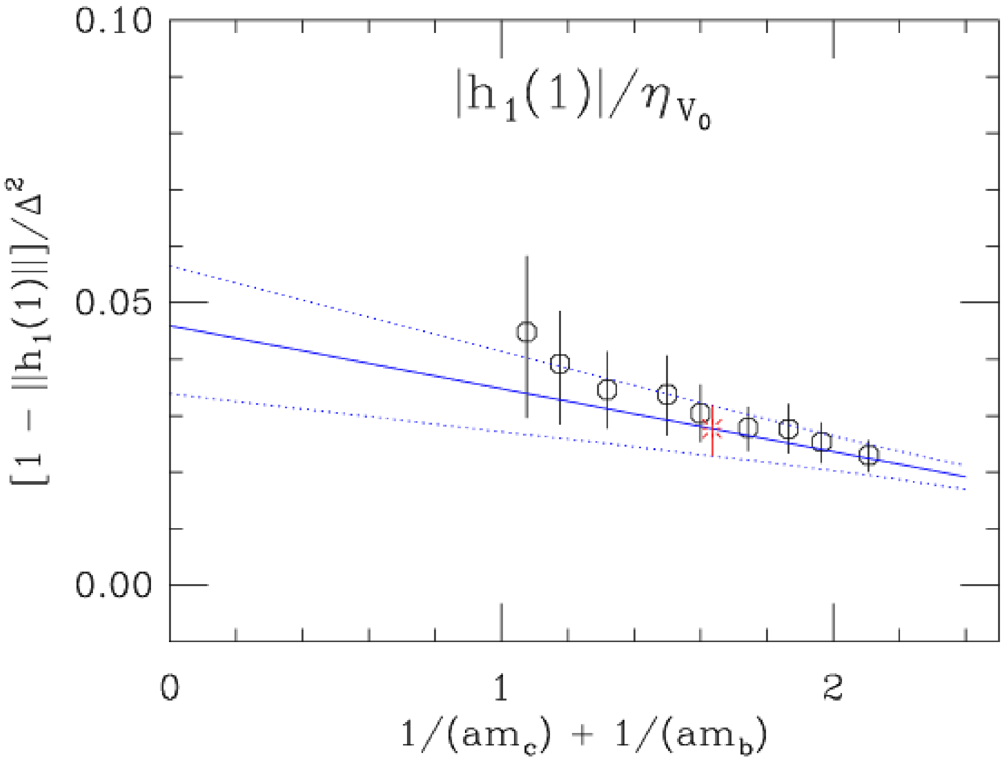
where $\Delta = (\frac{1}{2am_c} - \frac{1}{2am_b})$.

$a^2 \ell_V = 1.84 \pm 0.48 \times 10^{-1}$

$a^3 \ell_V^{[3]} = 0.89 \pm 0.36 \times 10^{-1}$

Need $\ell_V^{[3]}$ term to describe data.

$\ell_V^{[3]}$ and ℓ_V highly correlated.



Determining $\mathbf{h_{A_1}(1)}$

Recall,

$$h_{A_1}(1) = \eta_A \left[1 - \frac{\ell_V}{(2m_c)^2} + \frac{2\ell_A}{4m_c m_b} - \frac{\ell_P}{(2m_b)^2} + \mathcal{O}(1/m_Q^3)\right]$$

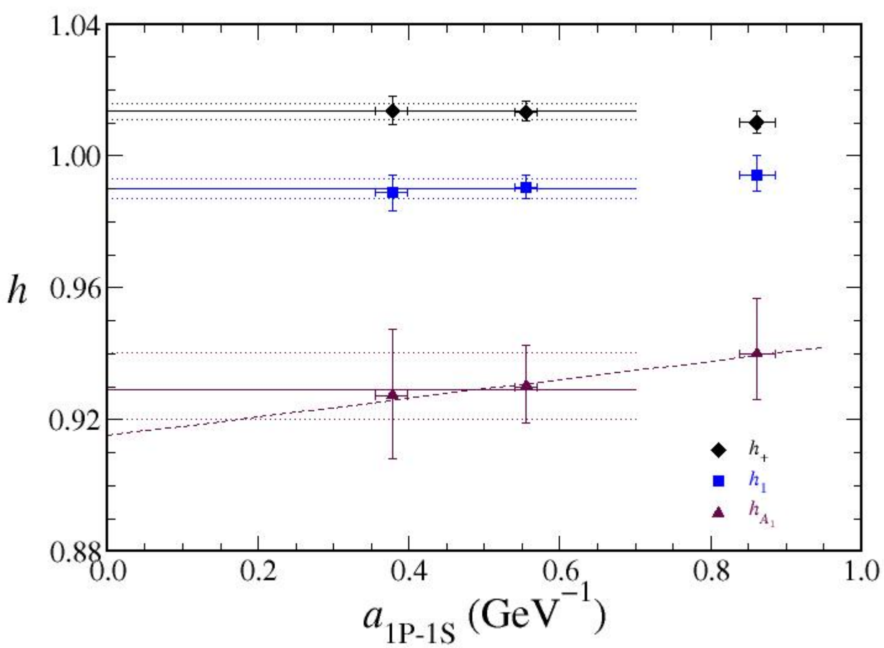
- HQ spin symmetry relates $B \to D^*$ to $B^* \to D^*$ in the limit $m_b \to \infty$.
- Conclude that the replacement $\ell_V^{eff} = \ell_V - \ell_V^{[3]}/(2m_c)$ in the formula reproduces the $1/(m_c^3)$ correction to $h_{A_1}(1)$.
- We include the analogous $1/(m_b^3)$ correction from $\ell_P^{[3]}$.
- The replacement $\ell_A^{eff} = \ell_A + \frac{1}{4}\ell_A^{[3]}\left(\frac{1}{m_c} + \frac{1}{m_b}\right)$ yields much of the $1/(m_c^2 m_b)$ correction.
- Remaining leading uncertainty is an $\mathcal{O}\left(\frac{\bar{\Lambda}}{8m_c m_b}\left[\frac{1}{m_c} - \frac{1}{m_b}\right]\right)$ correction to $h_{A_1}(1)$. Estimate $\delta h_{A_1} \approx \pm 0.0017$

h_{A_1} Cutoff Dependence

- Mild cutoff dependence relative to statistical errors.
- Average the $\beta = 6.1$ and 5.9 determinations for strange spectator quark.
- $h_{A_1}(1) = 0.9293^{+110}_{-\ 92}$ with combined statistical and fit uncertainties.
- Use coarsest ($\beta = 5.7$) result to bound a dependence: $+0.0032\ -0.0141$

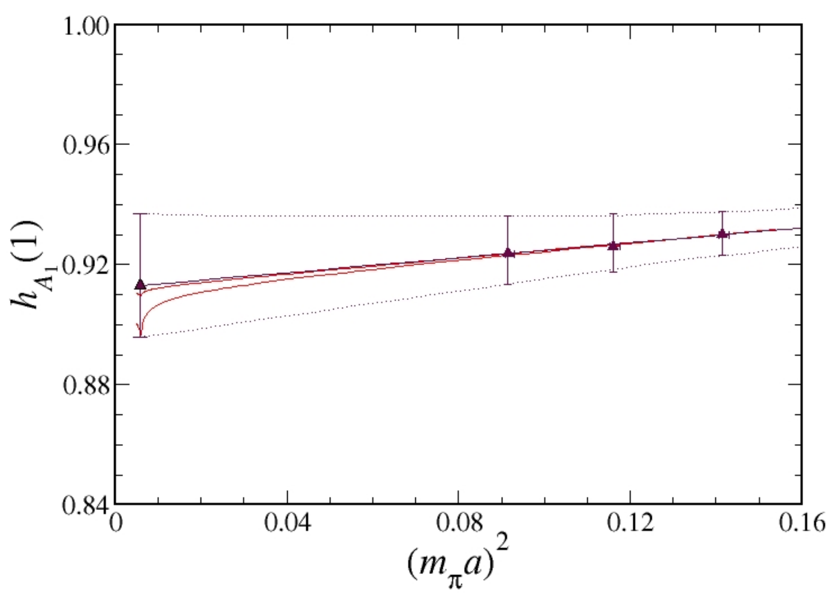
β	$h_{A_1}(1)$
6.1	0.9274^{+163}_{-148}
5.9	$0.9300^{+\ 76}_{-\ 68}$
5.7	0.9400^{+152}_{-135}

Statistical and fit uncertainties added in quadrature.



Spectator mass dependence

- extrapolate (in m_π^2) to the physical mass
- linear extrapolation for $\beta = 5.9$ results (black curve with uncert. contours)
- $h_{A_1}(1) = 0.9130^{+0.0283}_{-0.0173}$ value shifts down; statistical errors increase
- red curves are expectation in one loop χPT [Randall & Wise]
- vary $g_{D^* D\pi}$: systematic uncertainty of ${}^{+0.0000}_{-0.0163}$ in h_{A_1}.



Results

$$h_{A_1}(1) = 0.9130^{+0.0238}_{-0.0173}$$

Additional uncertainties that must be added to this result include:

source of uncertainty	$\delta \mathbf{h_{A_1}(1)}$	remedy
spectator quark mass dependence	$^{+0.0000}_{-0.0163}$	smaller m_π
tuning of m_c and m_b	$^{+0.0066}_{-0.0068}$	improve $\mathcal{S}$
terms of order $\alpha_s(\bar{\Lambda}/2m_c)^2$	± 0.0114	improve J_μ, PT
radiative corrections $\geq$ 2-loop order	± 0.0082	2-loop PT
term of order $\bar{\Lambda}/(8m_c^2 m_b)$	± 0.0017	extend method
residual a dependence	$^{+0.0032}_{-0.0141}$	improve $\mathcal{S}$, J_μ

The quenched approximation affects the **deviation of $\mathbf{h_{A_1}(1)}$ from unity**.

Guided by the quenched uncertainty in f_B (long dist.) and the running of quenched α_s (short dist.) we estimate the quenching uncertainty to be $^{+0.0061}_{-0.0143}$.