Heavy-to-light decays at large recoil: Systematic treatment of short- and long-distance QCD effects

Markus Diehl (RWTH Aachen, Germany)

1. Introduction
2. Effective theory for soft and collinear quarks and gluons
3. The effective heavy-to-light current
4. From quarks to hadrons: heavy-to-light form factors
5. Summary

Based on work with M. Beneke, A. Chapovsky, Th. Feldmann [hep-ph/0206152]

1. Introduction

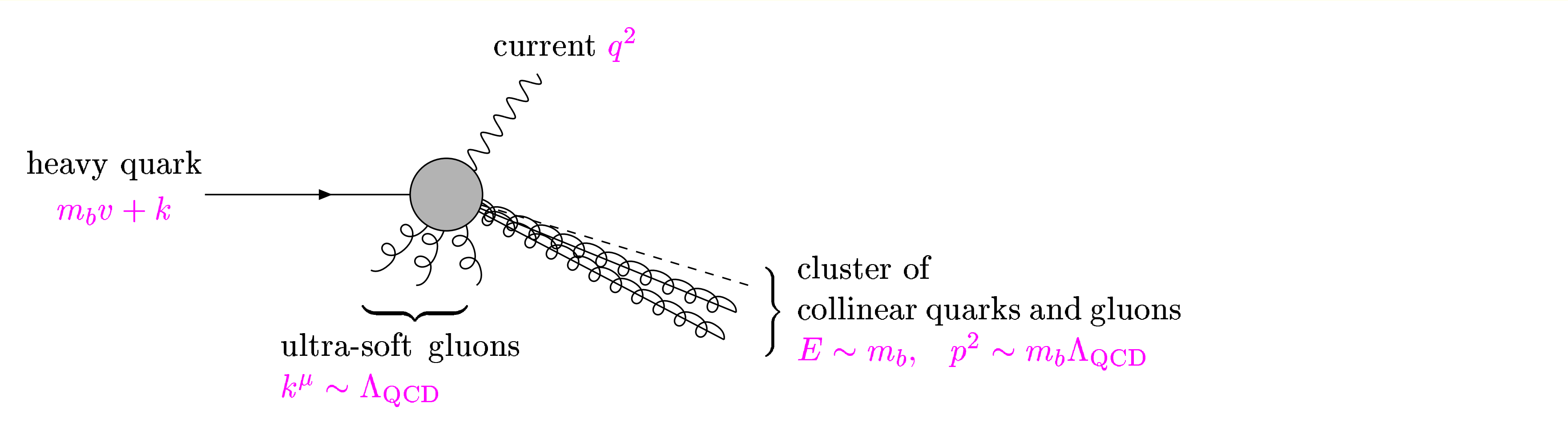
- general idea of factorization in QCD:
  - ⋆ separate dynamics at different scales
  - ⋆ explicitly evaluate perturbative physics at large momentum scales
    $\longrightarrow$ left with soft hadronic matrix elements (dynamics at smaller scales)

- exclusive B decays into light mesons (e.g. $B \to \pi\, \ell\nu$, $B \to K^* \ell^+ \ell^-$, $B \to \pi K$) in kinematics where energy E of light meson large ($\sim m_b$)
  - ⋆ small expansion parameter: $\Lambda_{\mathrm{QCD}}/m_b$

- different approaches:
  - ⋆ analyze Feynman diagrams, e.g. QCD factorization [Beneke et al., '99–'01]
  - ⋆ effective field theory: deal with fields and operators
    - ∗ make dynamical symmetries explicit
    - ∗ general framework: can use in many different processes
    - ∗ systematic treatment of power corrections (here: stay at leading order α_s)

Effective field theory approach

- identify relevant momentum regions in physical process



- retain momentum modes (nearly on-shell):

  ⋆ heavy quarks $\longrightarrow$ HQET

  ⋆ "collinear" and "ultrasoft" quarks/gluons
  $\longrightarrow$ SoftCollinearEffectiveTheory [Bauer et al., '00, '01]

- "integrate out" hard modes with $p^2 \sim m_b^2$
  (and "soft" ones with $p^\mu \sim \sqrt{m_b \Lambda_{\rm QCD}}$)

Setting up the effective theory

- introduce a separate field for each momentum mode
  $A_c^\mu(x)$, $A_{us}^\mu(x)$, ...

- eff. theory = Lagrangian for these fields that reproduces QCD results to a given accuracy in $\lambda = \sqrt{\Lambda_{\mathrm{QCD}}/m_b}$

- for fermions: project out and keep "large" spinor components
  (eliminate "small" components)

  ⋆ heavy quark: $h_v(x) = e^{-im_b vx}\, \frac{1}{2}(1 + \not{v})\, Q(x)$

  $\hookrightarrow$ also split off large phase

  ⋆ collinear quark: $\xi(x) = \frac{1}{4} \not{n}_- \not{n}_+\, q(x)$

  light-like vectors: $n_- \sim$ collinear momenta, n_+ in opposite direction

  ⋆ ultrasoft quark: keep all spinor components: $q_{us}(x)$

2. Effective theory for soft and collinear quarks and gluons

- determine power counting for
  - ⋆ momentum components: $n_+ p_c \sim 1, \;\; p_{c\perp} \sim \lambda, \ldots$
  - ⋆ fields: $n_+ A_c \sim 1, \;\; A_{c\perp} \sim \lambda, \;\; \ldots$
  - ⋆ derivatives: $(n_+\partial)\, \xi \sim \xi, \;\; (n_+\partial)\, q_{us} \sim \lambda^2 q_{us}, \;\; \ldots$
- eff. theory "inherits" gauge invariance from QCD
  $\longrightarrow$ define gauge transformations for fields ξ, q_{us}, A_c, A_{us}
- derive effective Lagrangian
  - ⋆ start with QCD Lagrangian for light quarks $\mathcal{L} = \bar{q}\, i \not{D} q$
  - ⋆ integrate out small Dirac components of collinear quark field (e.g. using path integral formalism)
  - ⋆ order operators in powers of λ

"SCET lite": without ultrasoft quarks

$$\rightarrow \quad \mathcal{L}_c = \bar{\xi}\, in_- D\, \frac{\not{n}_+}{2}\, \xi + \bar{\xi}\, i\not{D}_\perp\, W \frac{1}{in_+\partial} W^\dagger i\not{D}_\perp \frac{\not{n}_+}{2}\, \xi$$

- inverse operator $(n_+\partial)^{-1}$ → nonlocal action

- sum of gluon fields $A = A_c + A_{us}$ in covariant derivative $D = \partial - igA$

  and Wilson line $W(x) = P \exp\left\{ig \int_{-\infty}^{0} ds\, n_+A(x + sn_+)\right\}$

  ⇒ explicitly gauge invariant

- must still expand in λ:

  ⋆ A_c and A_{us} have different scaling in λ → Taylor expand D and W
  ⋆ $A_{us}(x)$ varies more slowly than collinear fields → Taylor expand in x

- read off vertices, e.g. $\bar{\xi}\,(n_-A_{us})\,\xi,\ \ \bar{\xi}\, A_{c\perp} A_{c\perp}\xi,\ \ \bar{\xi}\,(n_+A_c)^n\,\xi$

SCET including ultrasoft quarks

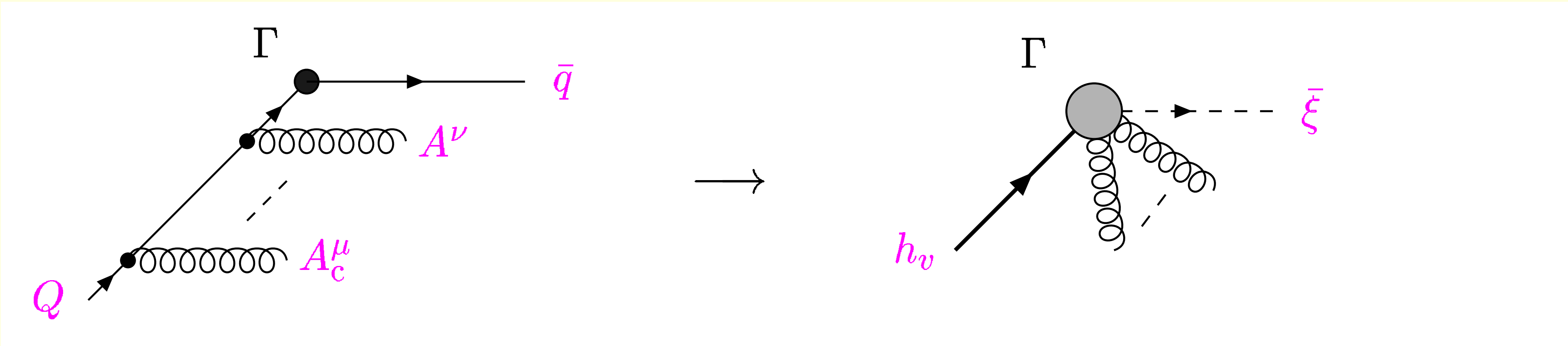
- same procedure as before
- at leading order in λ get only $\mathcal{L}_{us} = \bar{q}_{us}\, i\not{D}_{us}\, q_{us}$
- calculated corrections of $O(\lambda)$ and $O(\lambda^2)$ in $\mathcal{L}$
  $\longrightarrow$ couplings between ξ, q_{us}, and gluons

  ⋆ result involves "ultrasoft Wilson line" $Z = W|_{A_c=0}$ in addition to W

- no intrinsic scale $\Rightarrow$ no loop corrections to $\mathcal{L}$
  i.e. all Wilson coefficients $= 1$
  (different from HQET, where have scale m_b)

3. The effective heavy-to-light current

- heavy quark goes off-shell after radiation of collinear gluon $\longrightarrow$ effective vertex



- match $Q_{\rm QCD} \rightarrow e^{-im_b vx}\, Q_{\rm eff}(A_c, A_{us}, h_v)$
  two approaches:
  - ⋆ diagrammatic (radiation of 1, 2, 3, $\ldots$ gluons $\rightarrow$ geometric series)
  - ⋆ solve Dirac equation for $Q_{\rm eff}$ in external field $A = A_c + A_{us}$
- match $q_{\rm QCD} \rightarrow q_{\rm eff}(A_c, A_{us}, \xi)$ using results from SCET
- current $[\,\overline{q}\,\Gamma Q\,]_{\rm QCD} \rightarrow e^{-im_b vx}\, \overline{q}_{\rm eff}\, \Gamma\, Q_{\rm eff}$

Effective current: Results

- can write $Q_{\rm eff}$ in manifestly reparametrization invariant form:

$$Q_{\rm eff} \;=\; WZ^\dagger Q_v - \frac{1}{\mathcal{V}^2-1}\Big(\not{\mathcal{V}}\not{v}\, WZ^\dagger - WZ^\dagger \not{v}_{\rm us}\not{\mathcal{V}}_{\rm us}\Big) Q_v + O(\lambda^3 Q_v).$$

with $\mathcal{V}^\mu = v^\mu + \dfrac{iD^\mu}{m_b}$ and $Q_v = \Big(1 + \dfrac{i\not{D}_{us}}{m_b} + \dots\Big)\, h_v$ defined as in HQET

- result for effective current including corrections of $O(\lambda)$ and $O(\lambda^2)$

- Taylor expansion in λ as for eff. Lagrangian

4. From quarks to hadrons: heavy-to-light form factors

- here: consider only contributions where spectator quark remains soft (neglect hard spectator interactions $\sim \alpha_s$)

- match QCD matrix elements $\langle L|\,\overline{q}\,\Gamma\, Q\,|B\rangle$ onto $\langle L|\,\overline{q}_{\,\mathrm{eff}}\Gamma\, Q_{\mathrm{eff}}\,|B\rangle$ $(L = \pi, \rho, K^*, \ldots)$

- project $\overline{q}_{\,\mathrm{eff}}(\overline{\xi}, A_c, A_{us})$ and $Q_{\mathrm{eff}}(A_c, A_{us}, h_v)$ on large and small components
  - ⋆ leading order in λ: only large components for both fields

    $\Rightarrow$ form factor relations

L pseudoscalar:	$3 \rightarrow 1$ independent f.f.
L vector:	$7 \rightarrow 2$ independent f.f.s

  - ⋆ corrections of $O(\lambda)$: also small components of either $\overline{q}_{\,\mathrm{eff}}$ or Q_{eff}

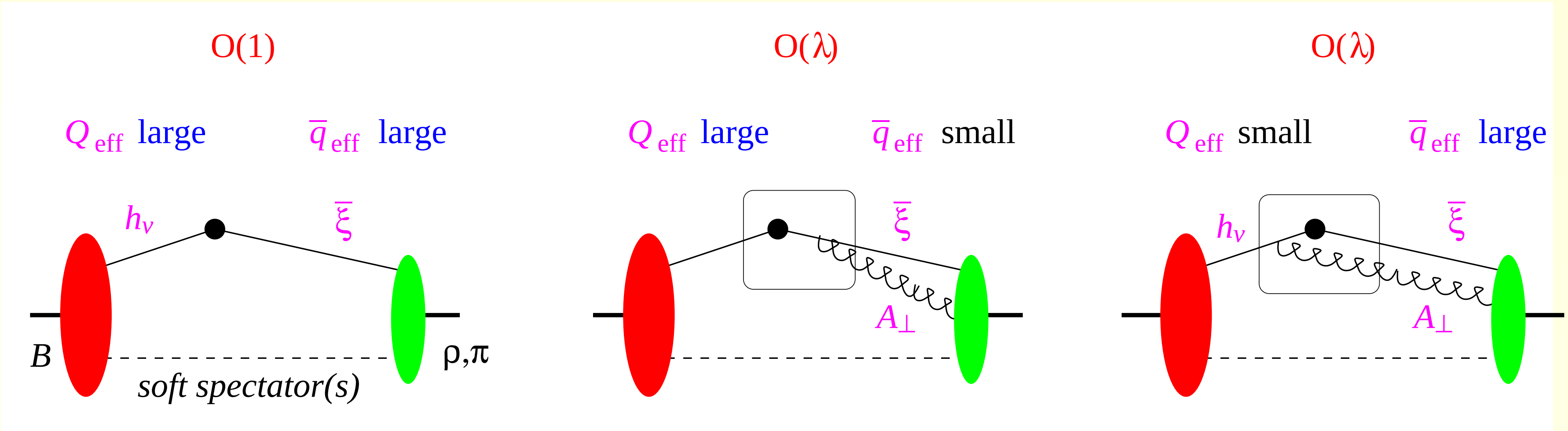
    $\Rightarrow$

L pseudoscalar:	2 independent f.f.s
L vector:	5 independent f.f.s

  - ⋆ at $O(\lambda^2)$ no form factor relations left

Form factor relations at order λ

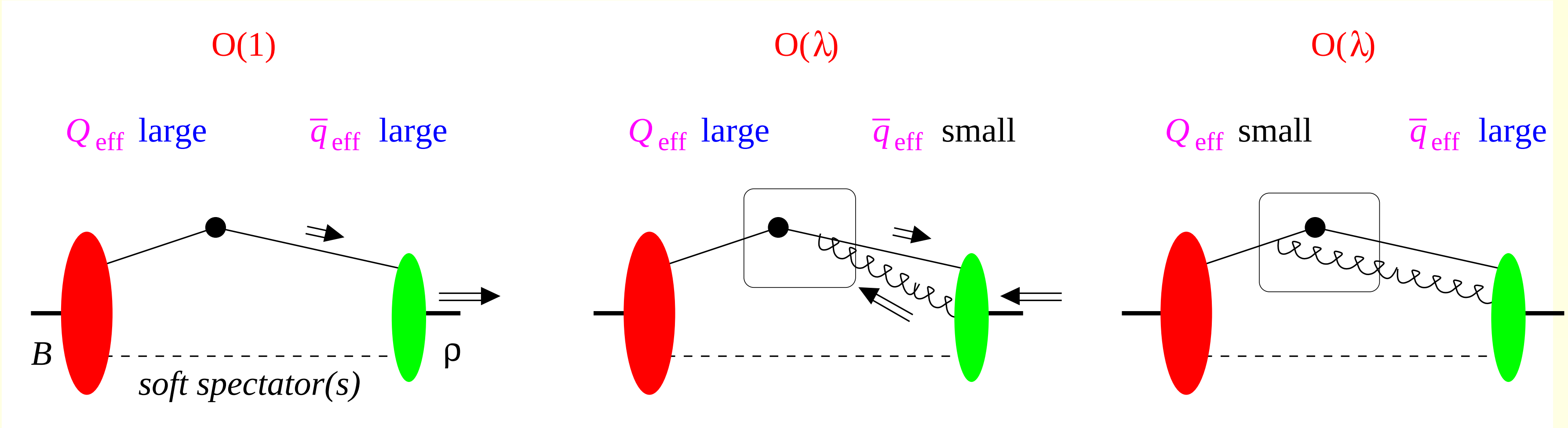
- caveat at order $\lambda = \sqrt{\Lambda_{\mathrm{QCD}}/m_b}$:



- $O(\lambda)$ operators involve different parton configurations of light meson than leading operators → further suppression possible from meson wave functions cannot decide within SCET

Form factor relations at order λ

- caveat at order $\lambda = \sqrt{\Lambda_{\text{QCD}}/m_b}$:



- $O(\lambda)$ operators involve different parton configurations of light meson than leading operators $\longrightarrow$ further suppression possible from meson wave functions
  cannot decide within SCET

- $O(\lambda)$ corrections violate helicity retention rule of [Burdman, Hiller, '00]
  size important for analysis of forw.-backw. asymmetry in $B \to K^* \ell^+ \ell^-$

5. Summary

- systematic treatment of power corrections in effective field theory formalism
- interactions between soft and collinear quarks and gluons: SCET
  - ⋆ general framework suitable for many different processes
  - ⋆ effective Lagrangian known including $O(\lambda^2)$ corrections
- heavy-to-light decays at large recoil energy
  - ⋆ effective Lagrangians from SCET and HQET
  - ⋆ effective heavy-to-light current known including $O(\lambda^2)$ corrections
- transitions $B \to$ light meson (leading $O(\alpha_s)$, no hard spectator interactions)
  - ⋆ form factor relations reduce hadronic uncertainties in analysis of decays
  - ⋆ at $O(\lambda)$: 1 relation for pseudoscalar mesons, 2 relations for vector mesons
    at $O(\lambda^2)$: no relations left
  - ⋆ further dynamical suppression at $O(\lambda)$ not ruled out