Heavy Meson Spectra and quark masses from Lattice QCD

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Outline

- Approaches to simulating b and c quarks on the lattice.
- Accuracy of lattice calculations.
- Results: mesons containing bs.
  - Υ, B, $b\bar{b}g$ hybrids and $\overline{m_b}(\overline{m_b})$.
- Results: mesons containing cs.
  - J/ψ, D, $c\bar{c}g$ hybrids and $\overline{m_c}(\overline{m_c})$.
- Summary.

Lattice simulations of b and c quarks

quarks on lattice: $S^{lattice}_{quark} = \int d^4x\overline{\psi}[D+m]\psi + O(a^2)$: a^2m^2, $a^2\Lambda^2$

present simulations: $1 < a^{-1} < 4$ GeV

problem: $4 < am_b < 1$ and $1 < am_c < .25$

Solutions:

- NRQCD: expand in v, in Υ: $v^2 \approx .1$

$$L = Q^\dagger(\Delta_t - \frac{\Delta^{(2)}}{2M_0} - c_B\frac{\sigma\cdot B}{2M_0} - c_3\frac{g}{8M_0^2}\sigma\cdot(\nabla\times E - E\times\nabla) + c_2\frac{ig}{8M_0^2}(\nabla\cdot E - E\cdot\nabla) - c_1\frac{(\Delta^{(2)})^2}{8M_0^3} + O(v^6))Q$$

  Truncate series, determine c_is.

  Continuum limit cannot be performed. [bottom]

- Fermilab: expansion in pa and g^2. Generalised set of operators, coefficients kept to all orders in am_q. Interpolates between rel. action and NRQCD. [bottom,charm]

- Relativistic action: live with significant discretisations errors, perform careful continuum extrapolation. [charm]

- Anisotropic lattices, rel. action: make $a_t \ll a_s$ such that $a_t m_q \ll 1$, computationally cheaper than isotropic $am_q \ll 1$. [charm,bottom]

Accuracy of lattice simulations

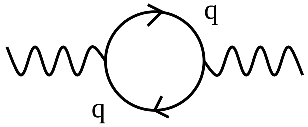
This depends on how well the sources of error have been controlled and/or estimated.

Sources of error:

- finite statistics, finite a, finite V, chiral extrapolation, renormalisation, ...

Most simulations have investigated these, have them under control.

- quenching: quark loops omitted



consequence: uncertainty in predictions due to differing a^{-1}.
Now first studies with sea quarks (note n_f, m_{sea}).

Note: first principles calculation: g^2 and m^0 input,
a and $m_{b,c}$ determined post simulation using expt.

Bottomonium

Most results with NRQCD: $m_b v^4$ action:

-spin-averaged spectrum to 1%

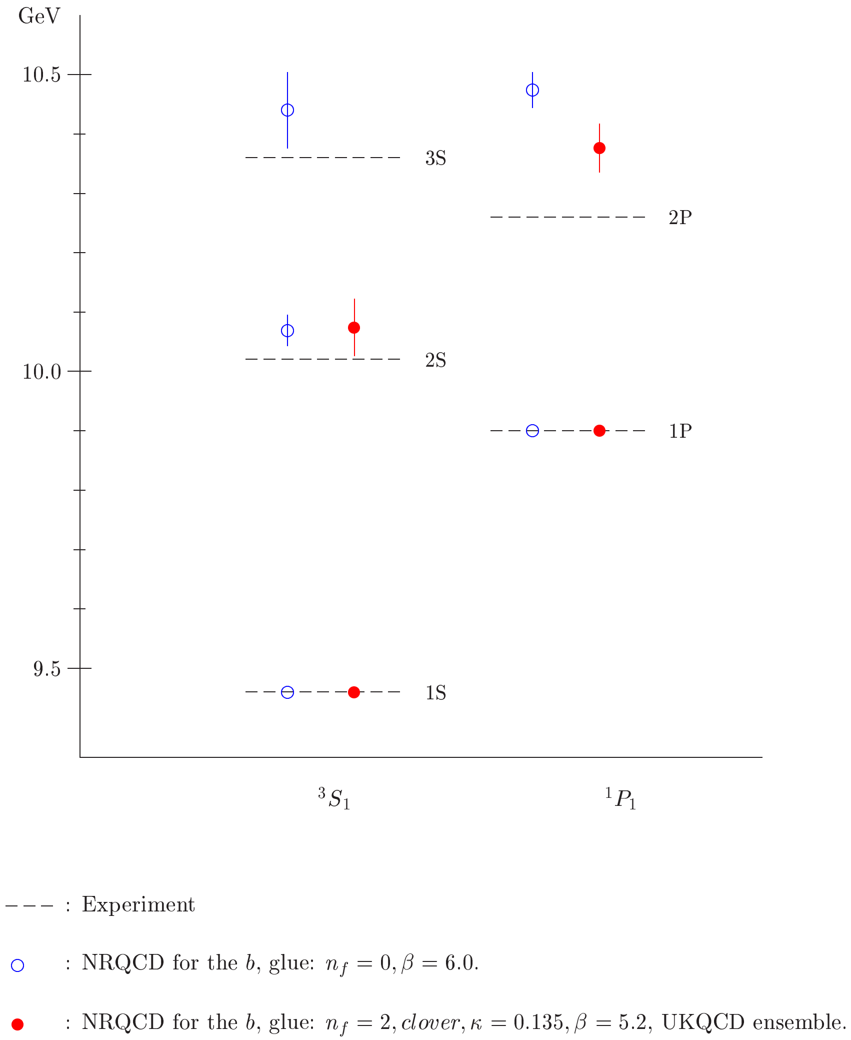
-fine structure to 10%

modulo only estimating c_is.

$b\bar{b}$ well studied by NRQCD collaboration, SESAM collaboration, CP-PACS collaboration, JLQCD collaboration ....

Υ radial and orbital splittings

L. Marcantonio et al '01

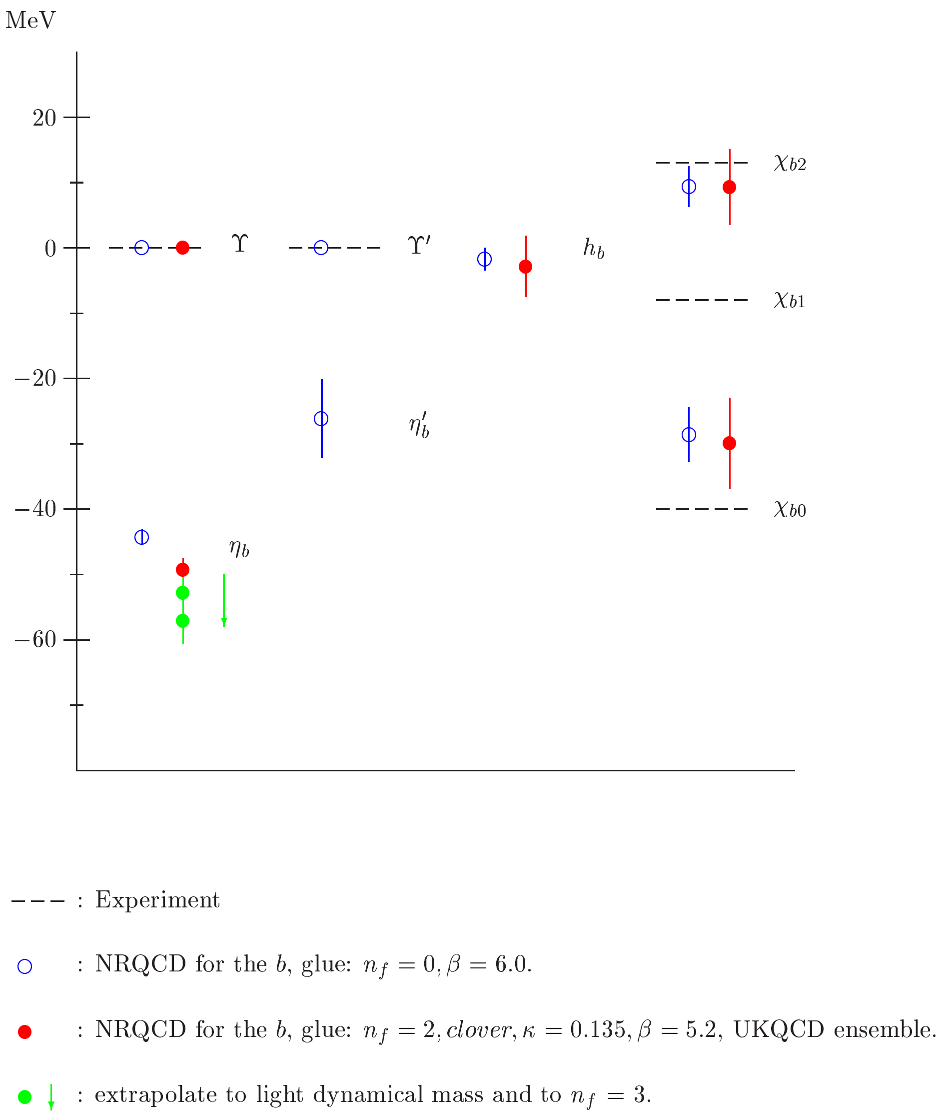


- a^{-1} set from $^1P_1 -^3 S_1$
- m_b from $^3S_1$

In quenched case scaling checked, finite V within statistical errors.

Υ fine structure

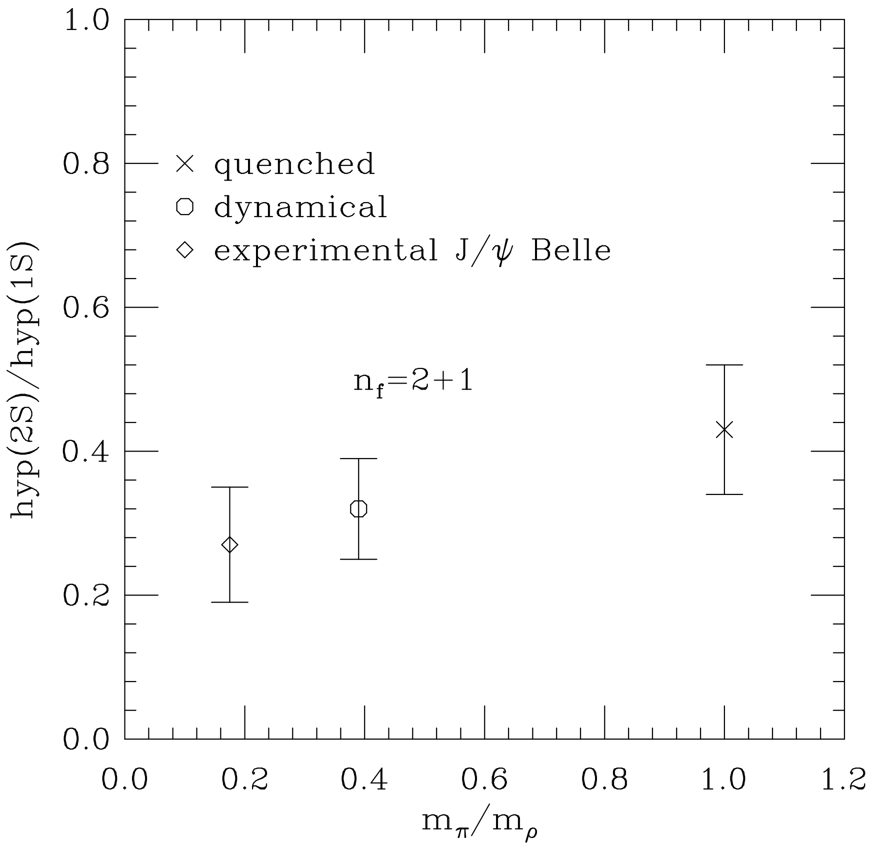
L. Marcantonio et al '01



- With a^{-1} from gross spectrum expect to see quenching effects.
- But: fine structure splittings depend on a.
  -need to calculate c_is more accurately
  -may be gluon action.

$\Upsilon \; 2(^3S_1 -^1 S_0)/1(^3S_1 -^1 S_0)$

A.Gray et al '02, preliminary



Ratio removes uncertainty from c_i.

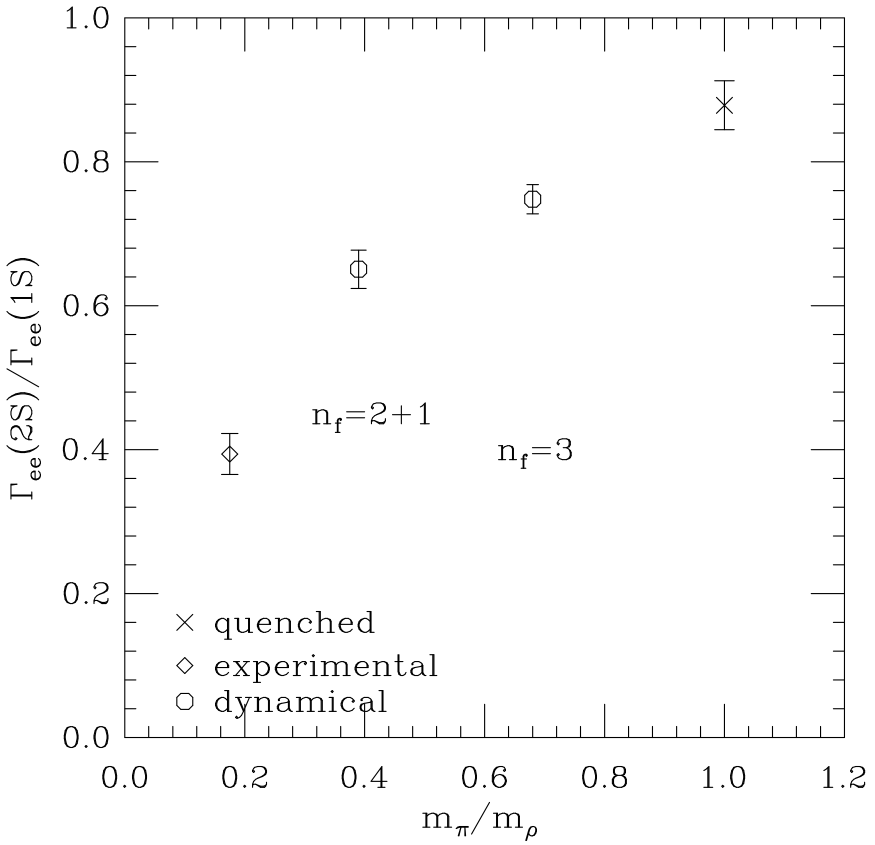
Reduced discretisation errors in gauge fields. (a coarser).

m_Q dependence also significantly reduced.

m_{sea} much lighter, n_f physically relevant.

Υ: $\Gamma_{ee}(2S)/\Gamma_{ee}(1S)$

A. Gray et al '02, preliminary



Renormalisation factor for current cancels in ratio.

Falling with m_{sea} but still some way from expt.

Only leading order calculation: need to go to next order.

(scaling must be checked etc)

$b\bar{b}g$ Hybrids

Mesons containing gluonic excitation $b\bar{b}g$

b quark more nonrelativistic in hybid meson.

- most simulations use NRQCD to Mv^2, omit spin-dependent terms

Thus, degenerate:

$$T_1^{+-} : \; 1^{--}, O^{-+}, \mathbf{1}^{-+}, 2^{-+}, \ldots$$

$$T_1^{-+} : \; 1^{++}, \mathbf{O}^{+-}, 1^{+-}, \mathbf{2}^{+-}, \ldots$$

Bold: exotic particles

Thresholds for stability:

- $B\bar{B}$ - not allowed
- $B^*\bar{B}$, relative P wave - allowed but suppressed (flux tube models)
- $B^{**}\bar{B}$ - allowed, most interesting.

Results

Difficult to extract hybrid signal on lattice:

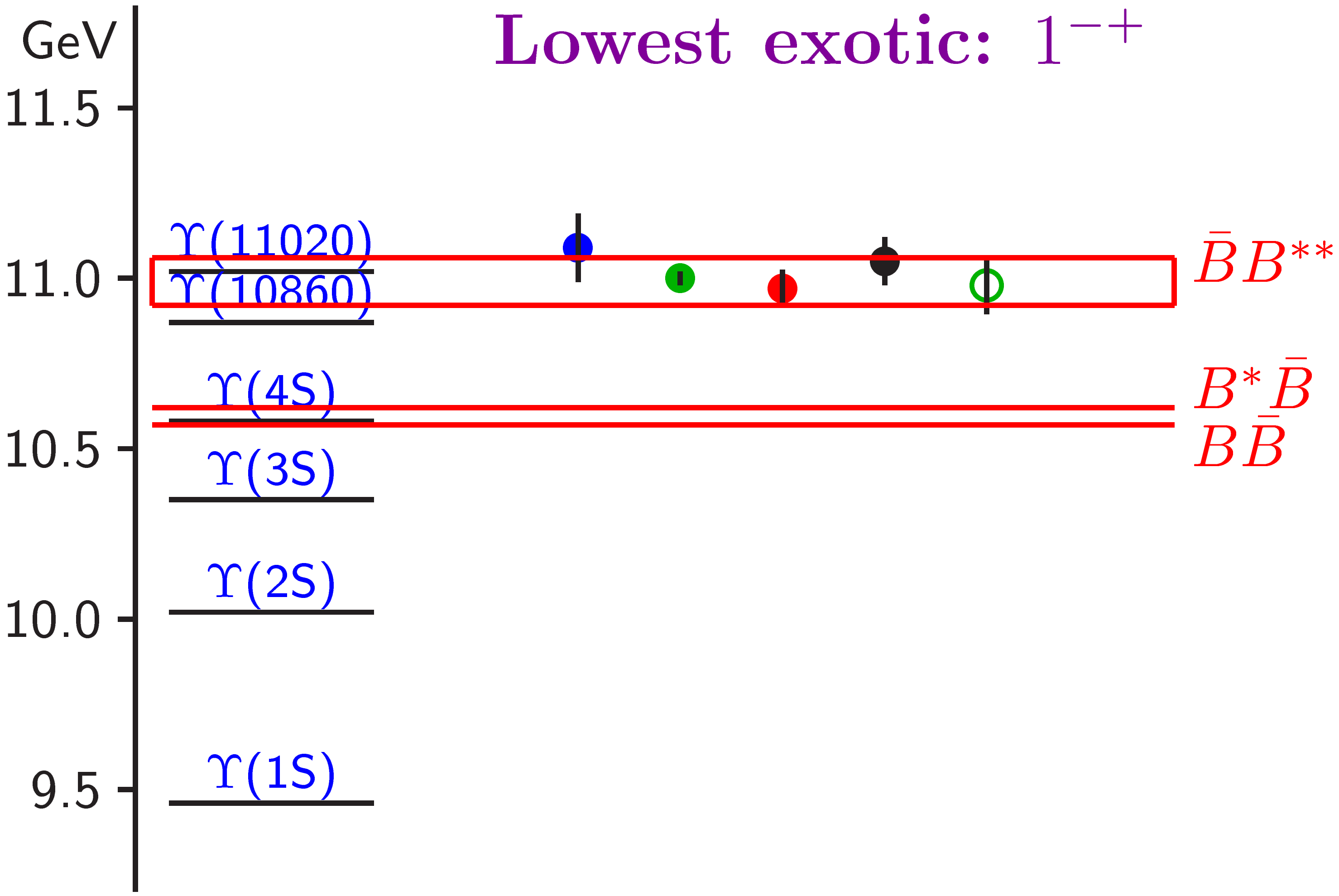
-several groups used anisotropic lattices with NRQCD

-$a_t \ll a_s$ allows the signal to be sampled more often before swamped by noise.

-NRQCD so $a_t m_b > 1$.

- lowest exotic, 1^{-+}, close to $B^{**}\bar{B}$ threshold
- aniostropic simulations, smaller statistical errors but more involved calculations: need checks.
- quenched results:
  - from potential models, unquenching decreases exotic mass
  - mixing!

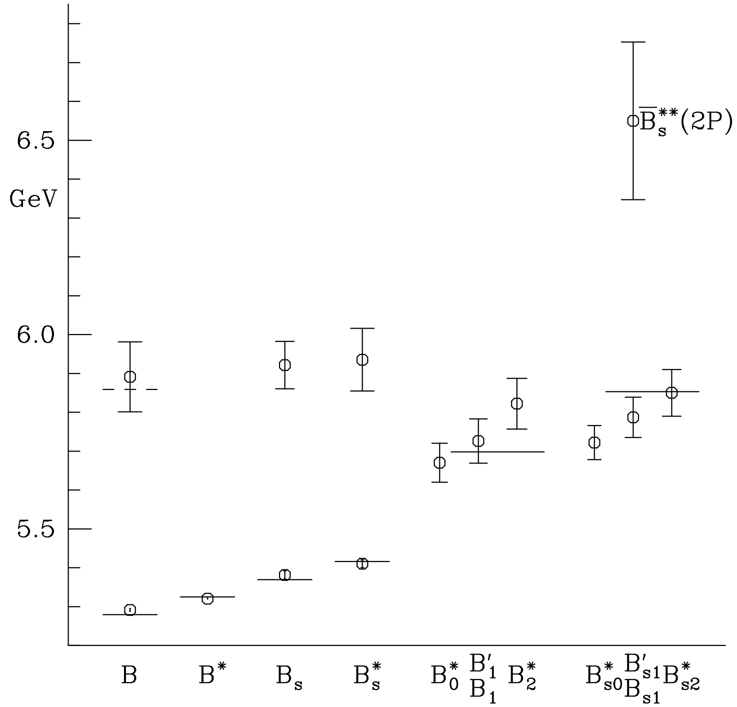
Lowest exotic: 1^{-+}



- ● UKQCD T. Manke et.al. '97 NRQCD sym
- ● CP-PACS T. Manke et.al. '98 NRQCD aniso
- ● Juge et.al. '99 NRQCD aniso
- ● X. Liao et.al. '01 rel. aniso
- ○ CP-PACS T. Manke et.al. '01 NRQCD iso $n_f = 2$

B and B_s spectrum

J. Hein et al '00



- most simulations of spectrum so far used NRQCD: (Lewis et al, Ishikawa et al), Hein: $Mv^4 \to \Lambda^2/M^2$.
- a^{-1} from m_ρ, am_b from $1\bar{S}$.
- reasonable agreement with experiment (except Δhyp).
- a dependence checked.
- no sea quark effects seen yet.

Once c_{hyp} etc are available at 1-loop, higher statistics are used, increased accuracy expected. Sea quark effects may then be visible.

b quark mass

Obtain $\overline{m_b}(\overline{m_b})$ via pole mass using results for the binding energy of the B meson.

$$\overline{m_b}(\mu) = \; Z_{cont}(\mu) M_{pole} \; = Z_{cont}(M_B^{expt} - \Lambda_{bind})$$

where $\Lambda_{bind} = E_{sim} - E_0$

aE_0 - at b known to $O(\alpha)$, in static limit $O(\alpha^3)$

$$aE_0 \;=\; 2.12\alpha_L + 11.15\alpha_L^2 + 86.6(5)\alpha_L^3 \quad \text{Trottier et al '01}$$
$$86.2(6)(10)\alpha_L^3 \quad \text{Di Renzo et al '01}$$

α_L not a good expansion parameter

$$aE_0 \;= 1.07\alpha_P + 0.12\alpha_P^2 + 3.56\alpha_P^3 \quad \text{Trottier et al : } \alpha_P(q^* = 0.84/a)$$
$$aE_0 \;= 2.12\alpha_{\overline{MS}} + 0.15\alpha_{\overline{MS}}^2 + 9.94\alpha_{\overline{MS}}^3 \quad \text{Di Renzo et al : } \alpha_{\overline{MS}}(\overline{m_b})$$

Similarly for Z_{cont}:

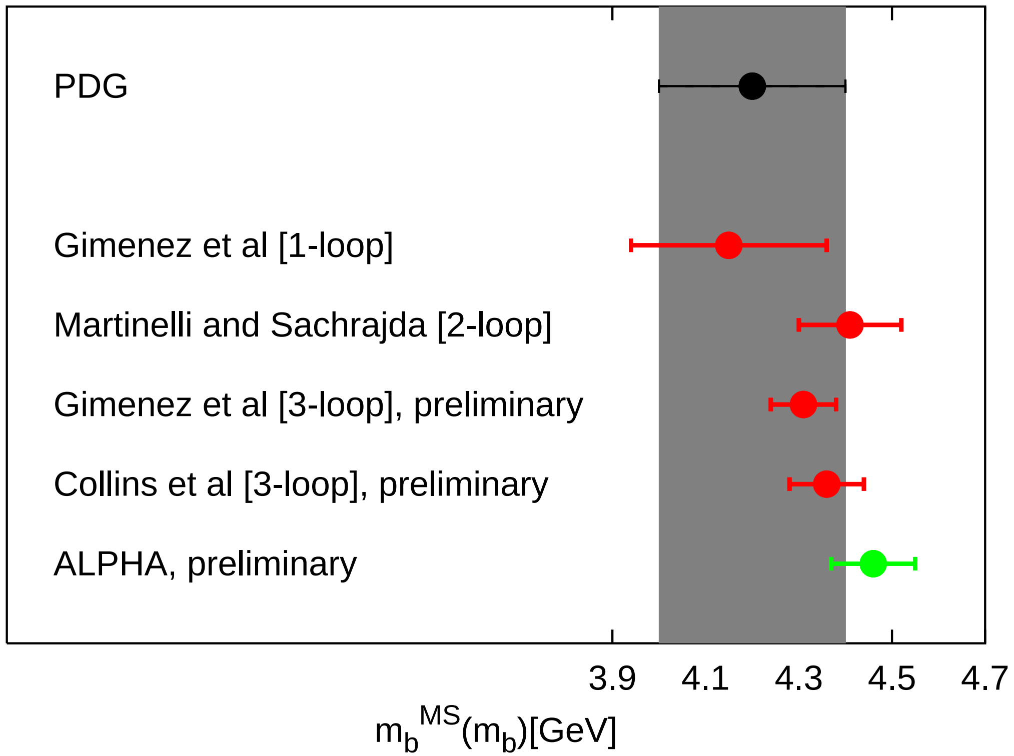
Trottier et al $\alpha_P(q^* = 0.62\overline{m_b})$, Gimenez et al $\alpha_{\overline{MS}}(\overline{m_b})$.

Nonperturbative approach: ALPHA collaboration

- also B in static limit.
- avoids the perturbative subtraction of power law divergences
- within Schrödinger functional approach

All groups simulated at several lattice spacings, performed continuum extrapolation (ALPHA). Convert to physical units with same or comparable a^{-1}.

Quenched $\overline{m_b}(\overline{m_b})$



Errors now small enough to consider:

- Quenching: first studies, $O(\alpha^2)$, suggest small decrease ($< 1\sigma$).
- Beyond the static limit: $O(1/M)$ corrections.

Charmonium

Simulations difficult as m_c lies between heavy and light regimes.

Previously:

-NRQCD, failed

-standard $O(a^2)$ relativistic action: Boyle '99.

-reinterpretation: Fermilab '00.

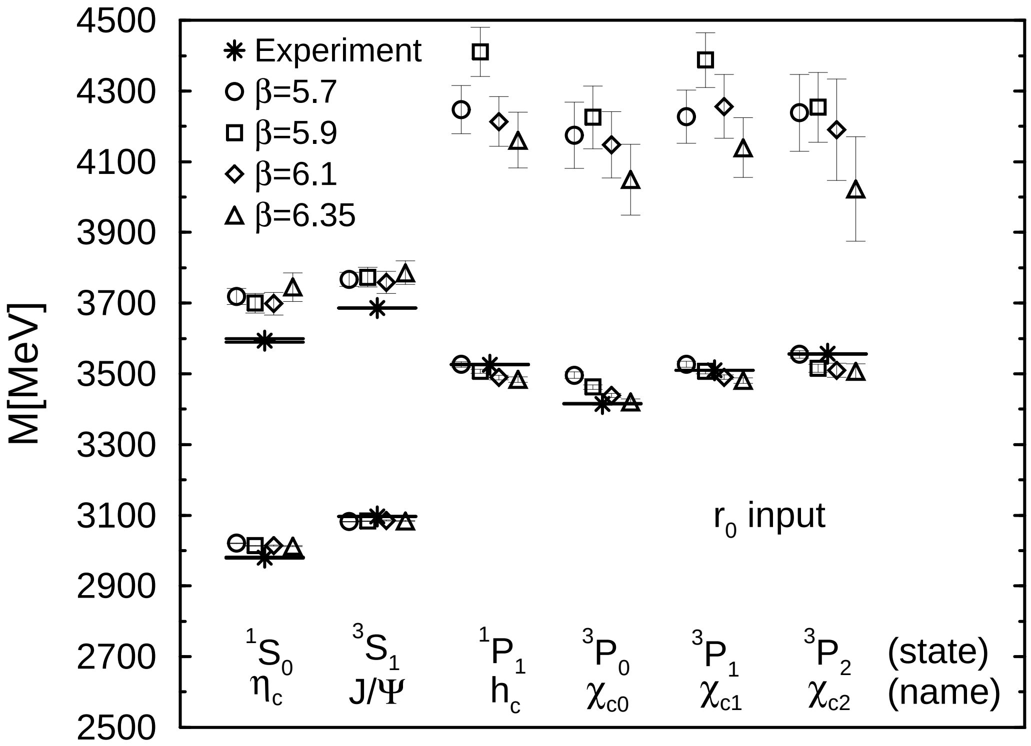
Most recent work: relativistic action $+$ anisotropic lattice

Chen et al '00

Okamoto et al '02

Quenched charmonium spectrum

CP-PACS: M. Okamoto et al. ’02



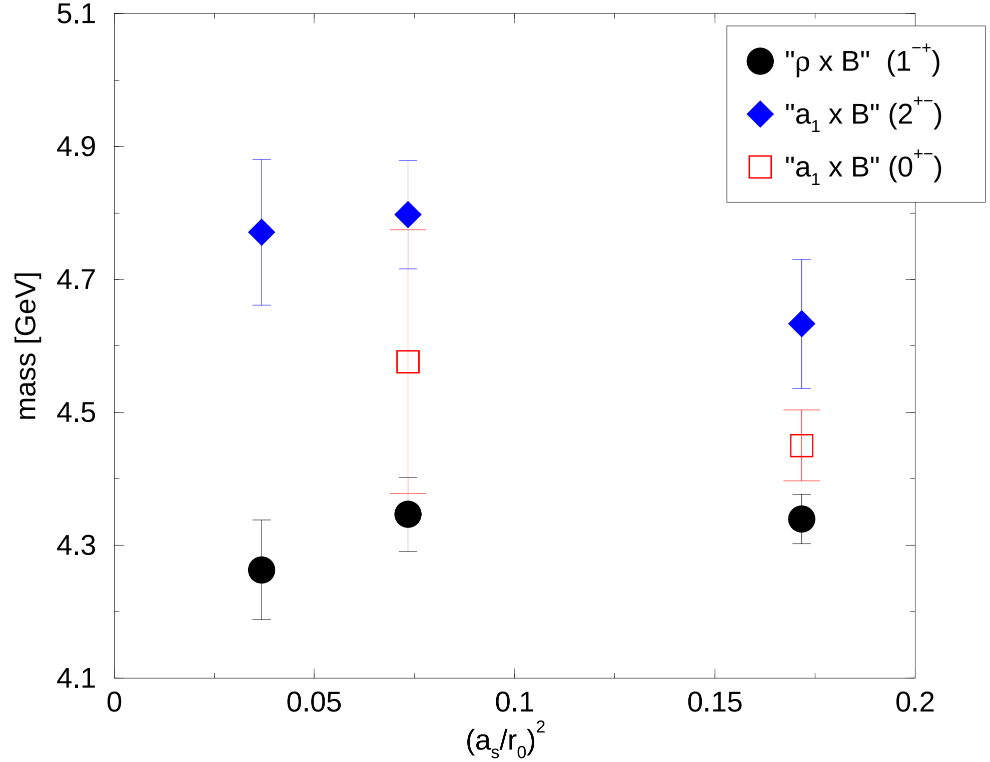
$$0.97 < (a_\sigma \text{ GeV})^{-1} < 2.81$$

$$2.90 < (a_\tau \text{ GeV})^{-1} < 8.44$$

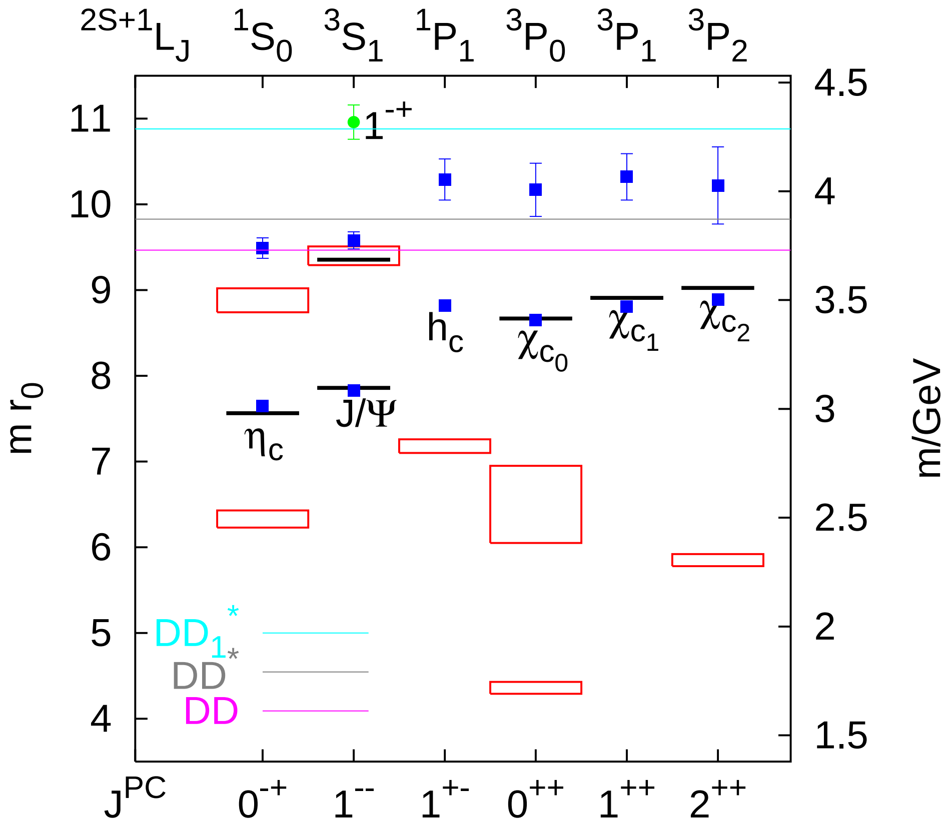
a^{-1} from $V_{q\bar{q}}$, am_c from $1\bar{S}$, $L_\sigma a_\sigma \approx 1.6$ fm.

Hybrid charmonia

Chen, Liao, Manke '00



Quenched flavour singlets



horizontal lines: experiment

squares: charmonia

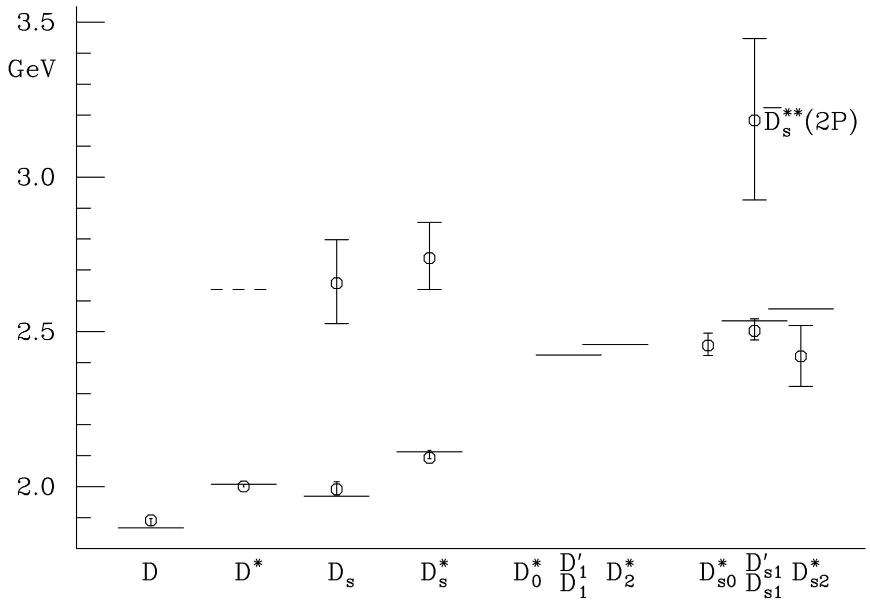
open boxes: glueballs

circle: exotic $c\bar{c}g$ hybrid

- continuum extrapolation of spectrum
- going beyond quenched approximation a challenge:
  -mixing
  -using anisotropic lattices.

D and D_s meson spectrum

J. Hein et al '00



- Not many extensive calculations:
  - Boyle '98, relativistic action
  - Lewis et al '98, NRQCD.
  - Hein et al: $O(\Lambda^2/M^2)$ action.
- on coarse lattices, NRQCD possible
  - c quark more non relativistic than in J/ψ
- hyperfine splitting too low

c quark mass in $\overline{MS}$ scheme

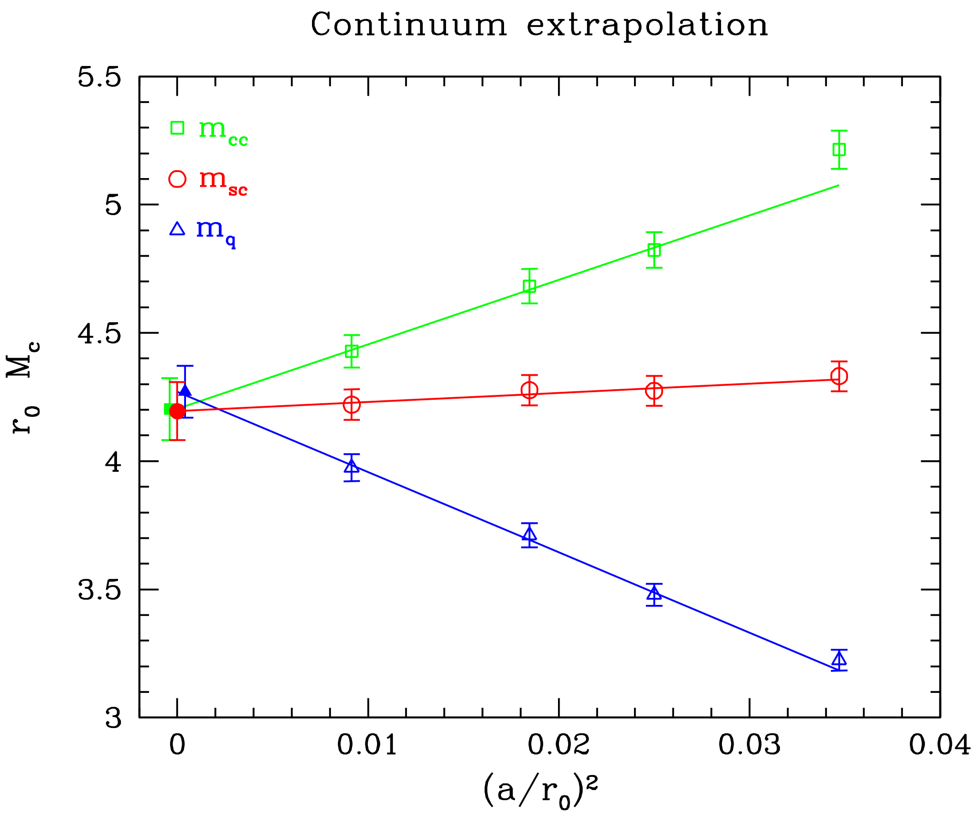
ALPHA Collaboration '01:

Input:

- mass D_s from expt.
- a^{-1} set from r_0, $r_0^2 F(r_0) = 1.65$
- previous calculation of m_s from m_K and $M_s/\hat{M} = 24.4(1.5)$.

Three definitions of bare m_c: $m_q = m_0 - m_c$, PCAC $m_{\bar{c}s}$, PCAC $m_{\bar{c}c}$

- standard rel. action $O(m^2a^2, \Lambda^2 a^2)$ errors.
- use light quark renormalisation constants + factors to convert to renormalisation group invariant mass, nonperturbatively determined within SF scheme.
- extrapolate to continuum.
- 4-loop running in $\overline{MS}$ scheme.



$\boxed{m_c(m_c) = 1.301(37)\ \text{GeV}}$ preliminary

Becirevic, Lubicz, Martinelli '01

Input: $m_{D_s}^{expt}$, a^{-1} set from m_{K^*}, previous calculation of m_s.

Two definitions of bare m_c: $m_q = m_0 - m_c$, PCAC $m_{\bar{c}s}$.

- standard rel. action $O(m^2a^2, \Lambda^2a^2)$ errors.
- light quark renormalisation factors + constants, factors nonperturbatively determined within RI/MOM scheme.
- one lattice spacing, fairly fine $a = 0.07$ fm.
- 4-loop conversion to RGI and $\overline{MS}$.

Average results from two definitions:

$$\boxed{m_c(m_c) = 1.26(4)(12)\ \text{GeV}}$$

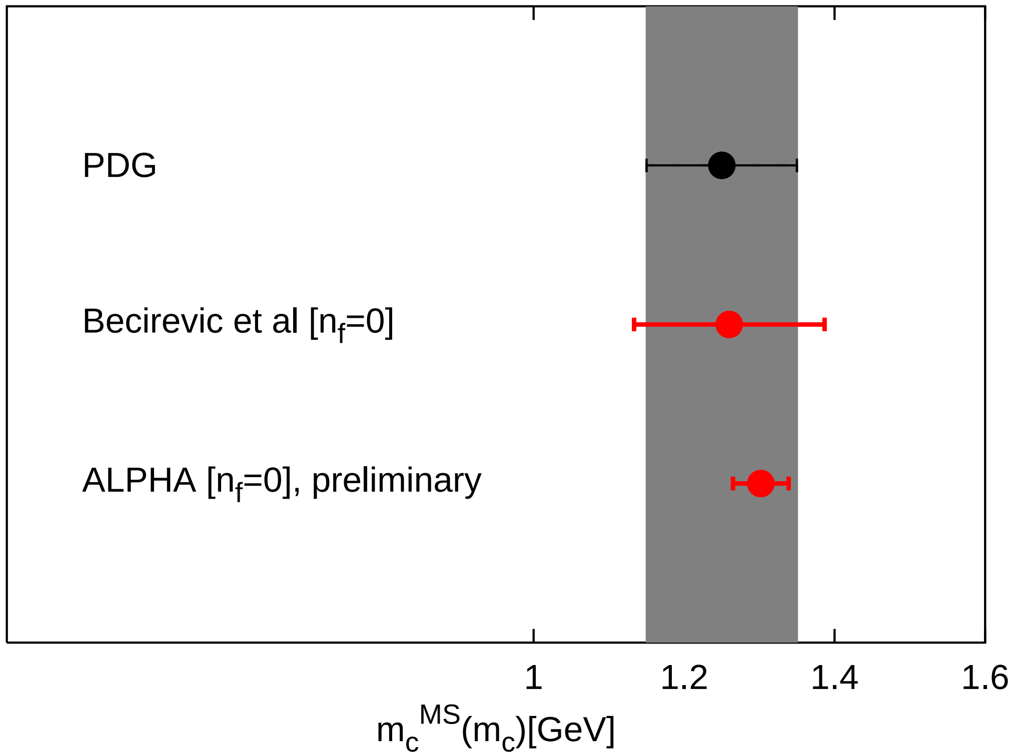
Juge et al '01:

Use: $1\overline{S}$ charmonium, a^{-1} from ${}^1P_1{-}1S$.

Method: via pole mass - $M_{pole} = M_{c\bar{c}}^{expt} - \Lambda_{bind}$

- Fermilab action: $O(\alpha a, a^2)$.
- E_0 to $O(\alpha^2)$ (numerical methods).
- results at several lattice spacings.
- results still very preliminary: $\overline{m_c}(\overline{m_c}) \sim 1.3$ GeV

$\overline{m_c}(\overline{m_c})$



remaining error: quenched approximation.

Summary

Heavy meson spectroscopy remains a very active area of research within the lattice community.

Extensive calculations of all mesons containing b and c quarks: Υ, $b\bar{b}g$, B, J/ψ, $c\bar{c}g$, D.

Recent progress in the quenched approximation

- extensive calculation of the charmonium spectrum $+$ exotics, bottomonium exotics.
- precise predictions of the b and c quark masses.

Next few years will be dominated by exploration of sea quark effects

- results for Υ hyperfine splittings and Γ_{ee} suggest long standing discrepancies will be resolved.
- mixing between flavour singlet states presents a challenge.