Next-to-leading Order Calculations of the Radiative and Semileptonic Rare B-Decays in the Standard Model and Comparison with Data

Ahmed Ali

DESY, Hamburg

(Work done in collaboration with C. Greub, G. Hiller, E. Lunghi, A. Parkhomenko, and A.S. Safir; hep-ph/0206242; hep-ph/0205254; hep-ph/0112300; hep-ph/0105302)

Contents

- Effective Hamiltonian in SM
- $B \to (K^*, \rho)\gamma$ Decay Rates in NLO, Comparison with Data & Implications for the CKM Phenomenology
- $B \to (X_s, K, K^*)\ell^+\ell^-$ Decay Rates & Differential Distributions in the NLO & Comparison with Data
- A Model-Independent Analysis of Data on $B \to X_s\gamma$ and $B \to (X_s, K, K^*)\ell^+\ell^-$
- Summary

Parallel Session 8, ICHEP 2002, Amsterdam

(ICHEP-2002 Abstract Numbers: ABS1067 – ABS1069)

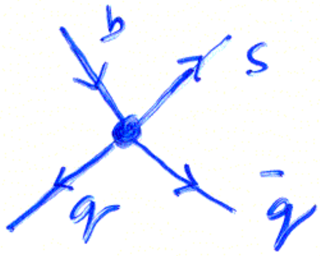
Effective Hamiltonian in SM

$$\mathcal{H}_{\text{eff}}(b \to s\gamma; b \to s\ell^+\ell^-) = -\frac{4G_F}{\sqrt{2}} V_{ts}^* V_{tb} \sum_{i=1}^{10} C_i(\mu)\, O_i(\mu)$$

- $O_i(\mu)$: Dimension-six operators at the scale μ
- $C_i(\mu)$: Corresponding Wilson coefficients
- G_F: Fermi coupling constant, V_{ij}: CKM matrix elements

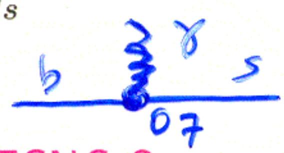
Four-Quark Operators O_i $(i = 1, \ldots, 6)$

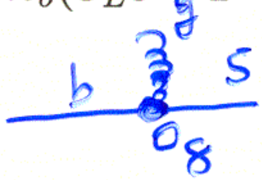
$$\begin{aligned}
O_1 &= (\bar{s}_L \gamma_\mu T^a c_L)(\bar{c}_L \gamma^\mu T^a b_L) \\
O_2 &= (\bar{s}_L \gamma_\mu c_L)(\bar{c}_L \gamma^\mu b_L) \\
O_3 &= (\bar{s}_L \gamma_\mu b_L) \textstyle\sum_q (\bar{q}\gamma^\mu q) \\
O_4 &= (\bar{s}_L \gamma_\mu T^a b_L) \textstyle\sum_q (\bar{q}\gamma^\mu T^a q) \\
O_5 &= (\bar{s}_L \gamma_{\mu_1}\gamma_{\mu_2}\gamma_{\mu_3} b_L) \textstyle\sum_q (\bar{q}\gamma^{\mu_1}\gamma^{\mu_2}\gamma^{\mu_3} q) \\
O_6 &= (\bar{s}_L \gamma_{\mu_1}\gamma_{\mu_2}\gamma_{\mu_3} T^a b_L) \textstyle\sum_q (\bar{q}\gamma^{\mu_1}\gamma^{\mu_2}\gamma^{\mu_3} T^a q)
\end{aligned}$$



Magnetic Moment Operators O_i $(i = 7, 8)$

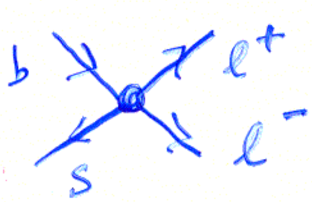
$$O_7 = \frac{e}{g_s^2} m_b (\bar{s}_L \sigma^{\mu\nu} b_R) F_{\mu\nu}\,, \quad O_8 = \frac{1}{g_s} m_b (\bar{s}_L \sigma^{\mu\nu} T^a b_R) G^a_{\mu\nu}$$

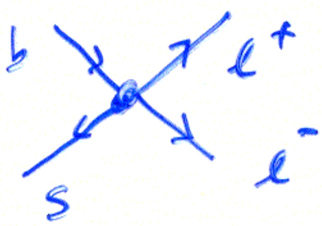




Semileptonic FCNC Operators O_i $(i = 9, 10)$

$$O_9 = \frac{e^2}{g_s^2} (\bar{s}_L \gamma_\mu b_L) \sum_\ell (\underbrace{\bar{\ell}\gamma^\mu \ell}_{\text{V}})\,, \quad O_{10} = \frac{e^2}{g_s^2} (\bar{s}_L \gamma_\mu b_L) \sum_\ell (\underbrace{\bar{\ell}\gamma^\mu \gamma_5 \ell}_{\text{A}})$$





$\mathcal{B}(B \to X_s\gamma)$ in LO & NLO

- A truly cooperative effort by several groups!

- LO Anomalous Dimension Matrix [Ciuchini et al.; Cella et al.; Misiak]
- NLO Anomalous Dimension Matrix [Chetyrkin,Misiak,Münz]
- NLO Virtual Corrections in ME [Greub,Hurth,Wyler; Buras et al.]
- Matching Conditions [Adil,Yao; Greub,Hurth; Buras,Kwiatkowski,Pott]
- Bremsstrahlung Corrections [Greub, A.A.; Pott]
- E_γ-spectrum [Greub;A.A.]
- Scale dependence, E_γ-spectrum [Neubert, Kagan]

$$\mathcal{B}(B \to X_s\gamma) = [\frac{\Gamma(B \to \gamma + X_s)}{\Gamma_{SL}}]^{th} \mathcal{B}(B \to X\ell\nu_\ell)$$

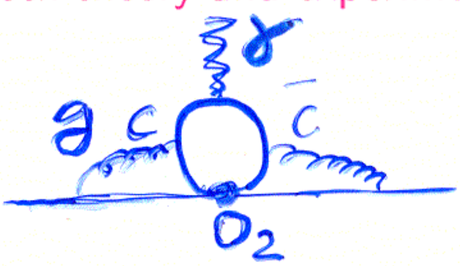
- SM (pole quark masses):
  $\mathcal{B}(B \to X_s\gamma) = [(3.35 \pm 0.30) \times 10^{-4}](|V_{ts}^* V_{tb}/V_{cb}|/0.976)^2$

- SM ($\overline{MS}$ quark masses) [Gambino, Misiak]:
  $\mathcal{B}(B \to X_s\gamma) = [(3.73 \pm 0.30) \times 10^{-4}](|V_{ts}^* V_{tb}/V_{cb}|/0.976)^2$

- Current theoretical uncertainty larger than usually assumed;
  Use: $\mathcal{B}(B \to X_s\gamma) = [(3.50 \pm 0.50) \times 10^{-4}](|V_{ts}^* V_{tb}/V_{cb}|/0.976)^2$

- Expt.(LP-01, Rome): $\mathcal{B}(B \to X_s\gamma) = (3.22 \pm 0.40) \times 10^{-4}$
  $\Longrightarrow \quad |\frac{V_{ts}^* V_{tb}}{V_{cb}}| = 0.96 \pm 0.10$ $\quad \left(3.88 \pm 0.36 \pm 0.37 {}^{+0.43}_{-0.23} (th)\right) \times 10^{-4}$ BABAR '02
  [cf. Unitarity fits: $|\frac{V_{ts}^* V_{tb}}{V_{cb}}| = 0.976 \pm 0.010$]
- $\mathcal{B}(B \to X_s\gamma)$ provides an indirect determination of V_{ts}, but currently limited in precision by both theory and experiment

- $\Delta\mathcal{B}(B \to X_s\gamma)_{th}$ dominated by $\Delta(m_c/m_b)$



NNLO (yet to be calculated)

B → Xsγ

(CLEO)

hep-ex/0108032

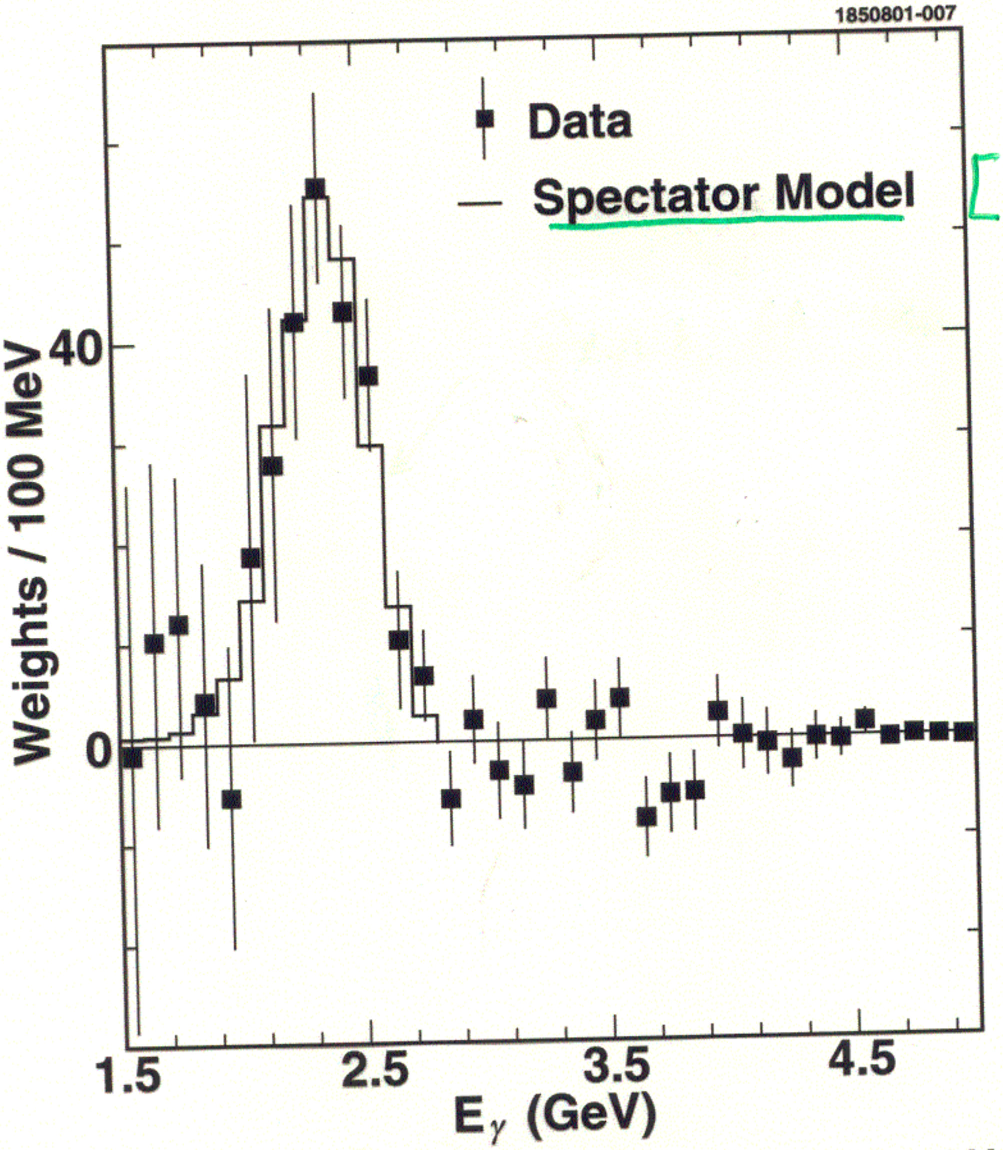


FIG. 2. Observed laboratory frame photon energy spectrum (weights per 100 MeV) for On minus scaled Off minus B backgrounds, the putative $b \to s\gamma$ plus $b \to d\gamma$ signal. No corrections have been applied for resolution or efficiency. Also shown is the spectrum from Monte Carlo simulation of the Ali-Greub spectator model with parameters $\langle m_b \rangle = 4.690$ GeV, $P_F = 410$ MeV/c, a good fit to the data.

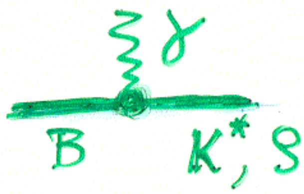
$B \to (K^*, \rho)\gamma$ Decay Rates in NLO

- Large Energy Effective Theory (LEET) ; Soft-Collinear Eff. Theory
  [Dugan, Grinstein '91; Charles et al. '99] [Bauer et al.; Beneke et al.]

$$E_V = \frac{m_B}{2}\left(1 - \frac{q^2}{m_B^2} + \frac{m_V^2}{m_B^2}\right)$$

For Large $E_V \sim m_B/2$, i.e., $q^2/m_B^2 \ll 1$; Symmetries in the Effective Theory $\Longrightarrow$ Relations among FFs:

$$f_k(q^2) = C_{\perp k}\xi_\perp(q^2) + C_{\parallel k}\xi_\parallel(q^2)$$



- (SCET). LEET symmetries broken by perturbation theory

Factorization Ansatz:

[Beneke, Buchalla, Neubert, Sachrajda; Beneke & Feldmann]

$$f_k(q^2) = C_{\perp k}\xi_\perp(q^2) + C_{\parallel k}\xi_\parallel(q^2) + \Phi_B \otimes T_k \otimes \Phi_V$$

Perturbative Corrections:

$$C_i = C_i^{(0)} + \frac{\alpha_s}{\pi} C_i^{(1)} + \ldots$$

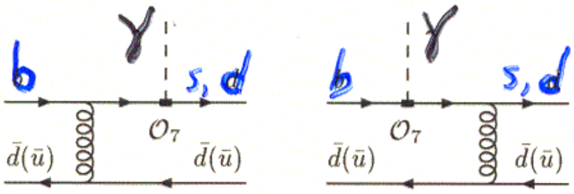
- T_k: Hard Spectator Corrections

$$\Delta\mathcal{M}^{(\mathrm{HSA})} = \frac{4\pi\alpha_s C_F}{N_c} \int_0^1 du \int_0^\infty dl_+ \, M_{jk}^{(B)} M_{li}^{(V)} T_{ijkl},$$

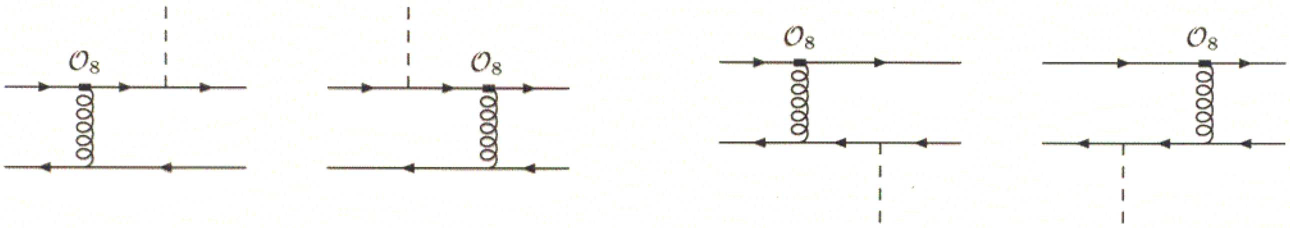
- $M_{jk}^{(B)}$ and $M_{li}^{(V)}$ B-Meson & V-Meson Projection Operators

Hard Spectator Contributions in $B \to (K^*, \rho)\gamma$

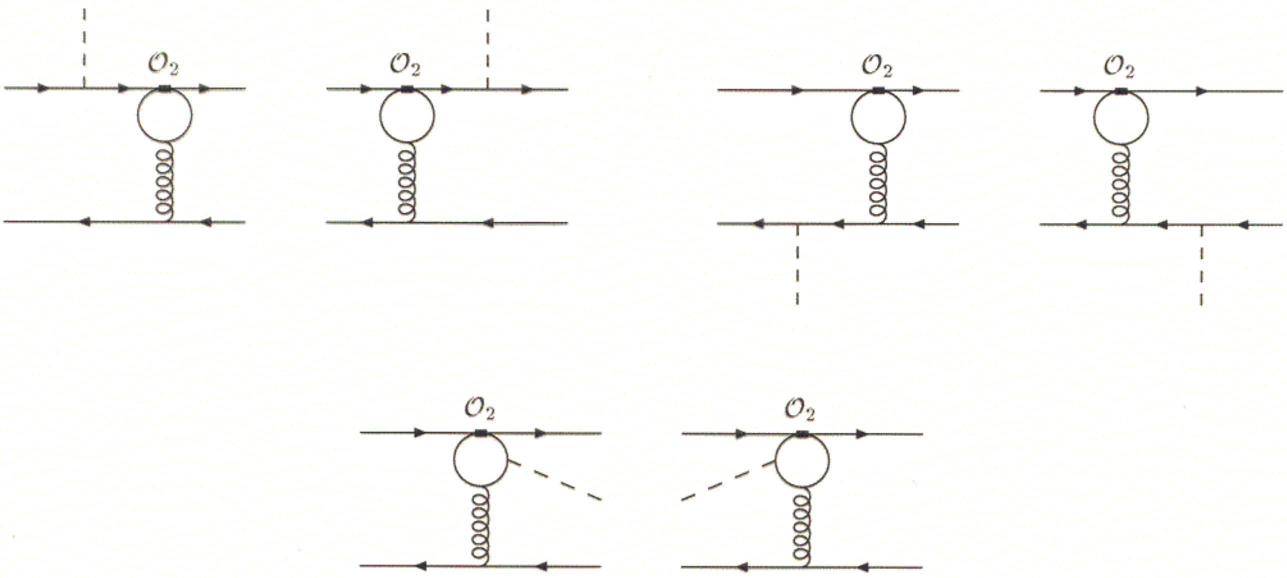
- Spectator corrections due to $\mathcal{O}_7$



- Spectator corrections due to $\mathcal{O}_8$



- Spectator corrections due to $\mathcal{O}_2$



Explicit $O(\alpha_s)$ Improvements

$$\mathcal{B}(B \to K^* \gamma) = \tau_B \frac{G_F^2 \alpha |V_{tb} V_{ts}^*|^2}{32\pi^4} m_{b,\text{pole}}^2 M^3 \times \left[\xi_\perp^{(K^*)}\right]^2 \left(1 - \frac{m_{K^*}^2}{M^2}\right)^3 \left|C_7^{(0)\text{eff}} + A^{(1)}(\mu)\right|^2$$

$$A^{(1)}(\mu) = A_{C_7}^{(1)}(\mu) + A_{\text{ver}}^{(1)}(\mu) + A_{\text{sp}}^{(1)K^*}(\mu_{\text{sp}})$$

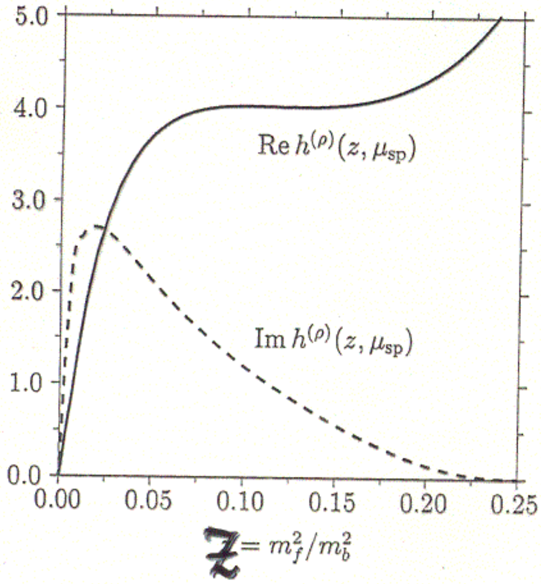
↑ RGE ↑ Vertex Corrections ↑ Hard spectator Corrections

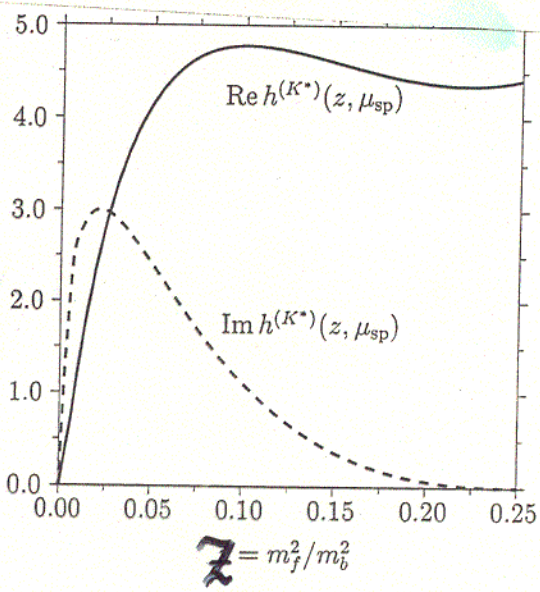
- $$A_{C_7}^{(1)}(\mu) = \frac{\alpha_s(\mu)}{4\pi} C_7^{(1)\text{eff}}(\mu),$$
- $$A_{\text{ver}}^{(1)}(\mu) = \frac{\alpha_s(\mu)}{4\pi} \left\{ \frac{32}{81}\left[13C_2^{(0)}(\mu) + 27C_7^{(0)\text{eff}}(\mu) - 9\,C_8^{(0)\text{eff}}(\mu)\right] \ln\frac{m_b}{\mu} - \frac{20}{3} C_7^{(0)\text{eff}}(\mu) + \frac{4}{27}\left(33 - 2\pi^2 + 6\pi i\right) C_8^{(0)\text{eff}}(\mu) + r_2(z)\, C_2^{(0)}\right.$$
- $$A_{\text{sp}}^{(1)K^*}(\mu_{\text{sp}}) = \frac{\alpha_s(\mu_{\text{sp}})}{4\pi} \frac{2\Delta F_\perp^{(K^*)}(\mu_{\text{sp}})}{9\xi_\perp^{(K^*)}} \left\{ 3C_7^{(0)\text{eff}}(\mu_{\text{sp}}) + C_8^{(0)\text{eff}}(\mu_{\text{sp}}) \left[1 - \frac{6a_{\perp 1}^{(K^*)}(\mu_{\text{sp}})}{\langle \bar{u}^{-1}\rangle_\perp^{(K^*)}(\mu_{\text{sp}})}\right] + C_2^{(0)}(\mu_{\text{sp}}) \left[1 - \frac{h^{(K^*)}(z,\mu_{\text{sp}})}{\langle \bar{u}^{-1}\rangle_\perp^{(K^*)}(\mu_{\text{sp}})}\right]\right\}$$

- Two Scales: $\mu \simeq \mathcal{O}(m_b) \sim 5$ GeV
  $\mu_{sp} \simeq \mathcal{O}(\sqrt{m_b \Lambda}) \sim 1.5$ GeV

[Parkhomenko, A.A.]

$h^{(\rho)}(z, \mu_{sp})$ $h^{K^*}(z, \mu_{sp})$





$$\Delta_{\rm sp} T_1^{(\rho)}(0) = \Delta_{\rm sp} T_2^{(\rho)}(0) \simeq \frac{\alpha_s C_F}{4\pi} \frac{\Delta F_\perp^{(\rho)}(\mu)}{2} \times \left[1 + \frac{C_8^{(0)\rm eff}(\mu)}{3 C_7^{(0)\rm eff}(\mu)} + \frac{C_2^{(0)}(\mu)}{3 C_7^{(0)\rm eff}(\mu)} \left(1 + \frac{V_{cd}^* V_{cb}}{V_{td}^* V_{tb}} \frac{h^{(\rho)}(z,\mu)}{\langle \bar{u}^{-1} \rangle_\perp^{(\rho)}(\mu)} \right) \right],$$ [redacted]

for the ρ-meson, and

$$\Delta_{\rm sp} T_1^{(K^*)}(0) = \Delta_{\rm sp} T_2^{(K^*)}(0) \simeq \frac{\alpha_s C_F}{4\pi} \frac{\Delta F_\perp^{(K^*)}(\mu)}{2} \times \left[1 + \frac{C_8^{(0)\rm eff}(\mu)}{3 C_7^{(0)\rm eff}(\mu)} \frac{\langle u^{-1} \rangle_\perp^{(K^*)}(\mu)}{\langle \bar{u}^{-1} \rangle_\perp^{(K^*)}(\mu)} + \frac{C_2^{(0)}(\mu)}{3 C_7^{(0)\rm eff}(\mu)} \left(1 - \frac{h^{(K^*)}(z,\mu)}{\langle \bar{u}^{-1} \rangle_\perp^{(K^*)}(\mu)} \right) \right],$$ [redacted]

$$\mathcal{B}_{\rm th}(B \to K^*\gamma) = \tau_B \frac{G_F^2 \alpha |V_{tb}V_{ts}^*|^2}{32\pi^4} m_{b,\rm pole}^2 M^3$$

$$\times \left[\xi_\perp^{(K^*)}\right]^2 \left(1 - \frac{m_{K^*}^2}{M^2}\right)^3 \left|C_7^{(0)\rm eff} + A^{(1)}(\mu)\right|^2$$

LEET FF

$$K = \frac{\left|C_7^{(0)\rm eff} + A^{(1)}(\mu)\right|^2}{\left|C_7^{(0)\rm eff}\right|^2} \quad \text{with} \quad 1.5 \le K \le 1.7$$

Large!

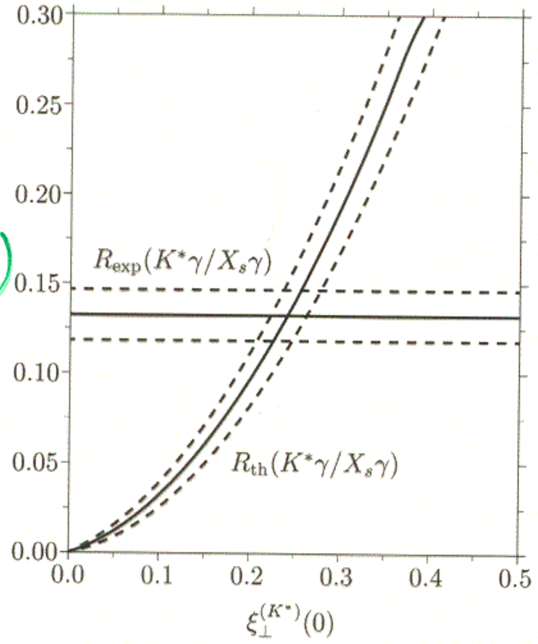
[Beneke, Feldmann, Seidel; Bosch, Buchalla; Parkhomenko, A.A.]

$$\langle \mathcal{B}_{\rm th}(B \to K^*\gamma)\rangle \simeq (7.2 \pm 1.1) \times 10^{-5} \left(\frac{\tau_B}{1.6\ \rm ps}\right) \left(\frac{m_{b,\rm pole}}{4.65\ \rm GeV}\right)^2 \left(\frac{\xi_\perp^{(K^*)}}{0.35}\right)^2$$

$$= (7.2 \pm 2.7) \times 10^{-5} \quad [Expt.: (4.22 \pm 0.28) \times 10^{-5}]$$

[Parkhomenko, A.A.]

R(K*γ/X_sγ)



} Experiment

$$R(K^*\gamma/X_s\gamma) \equiv \frac{\mathcal{B}(B \to K^*\gamma)}{\mathcal{B}(B \to X_s\gamma)} = 0.13 \pm 0.02 \implies \bar{\xi}_\perp^{(K^*)}(0) = 0.25 \pm 0.04$$

Relation between HQET FF $\xi_\perp$ and QCD FF $T_1^{K^*}$

[Benke, Feldmann; hep-ph/0008255]

$$T_1^{K^*}(s) = \xi_\perp^{K^*}(s)\left(1 + \frac{\alpha_s C_F}{4\pi}[\ln\frac{m_b^2}{\mu^2} - L]\right) + \frac{\alpha_s C_F}{4\pi}\Delta T_1$$

$$L = -\frac{2E}{M-2E}\ln\frac{2E}{M}; \quad \Delta T_1 = \frac{M}{4E}\Delta F_\perp$$

Limiting case: $L \to 1$ for $E \to M/2$; $\Delta F_\perp$ a Non-pert. parameter

$T_1^{(K^*)}(0, \bar{m}_b) \simeq 1.08\xi_\perp(0) \implies \boxed{T_1^{(K^*)}(0, \bar{m}_b) = 0.27 \pm 0.04}$

$= 0.38 \pm 0.05$ [LC-QCD Sum Rules; Ball & Braun; AA, Ball, Handoko, Hiller]

$= 0.32^{+0.04}_{-0.02}$ [Lattice-QCD; Del Debbio et al.]

- QCD Factorization & Current Data $\implies$ smaller value for the FF $T_1^{K^*}$ than the LC-QCD Sum Rules or the Lattice QCD
- New Lattice-QCD Cal. for $T_1^{K^*}$ under way (*)
- The consistency of the QCD Factorization theory has to be checked by independent measurements, such as $\mathcal{B}(B \to \rho\gamma)$ and $d\mathcal{B}(B \to K^*\ell^+\ell^-/ds)$

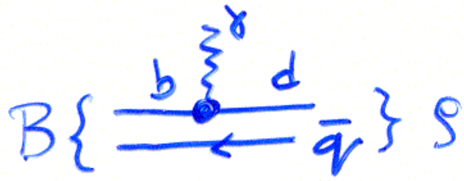
(*) $T_1^{K^*}(0) = 0.23(6)^{+1}_{-2}$

Becirevic (Rome-Orsay)

(*) (also @ BNL (Soni, Private Communication))

$B \to \rho\gamma$ Decay

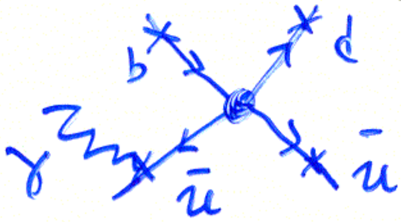
Penguin Amplitude $\mathcal{M}_{\rm P}(B \to \rho\gamma)$



$$-\frac{G_F}{\sqrt{2}} V_{tb} V_{td}^* C_7 \frac{e m_b}{4\pi^2} \epsilon^{(\gamma)\mu} \epsilon^{(\rho)\nu} \left(\epsilon_{\mu\nu\alpha\beta} p^\alpha q^\beta - i[g^{\mu\nu}(q.p) - p^\mu q^\nu] \right) T_1^{(\rho)}(0)$$

Annihilation Amplitude $\mathcal{M}_{\rm A}(B^\pm \to \rho^\pm\gamma)$

$$e \frac{G_F}{\sqrt{2}} V_{ub} V_{ud}^* a_1 m_\rho \epsilon^{(\gamma)\mu} \epsilon^{(\rho)\nu} \left(\epsilon_{\mu\nu\alpha\beta} p^\alpha q^\beta F_A^{(\rho);\rm p.v.}(0) - i[g^{\mu\nu}(q.p) - p^\mu q^\nu] F_A^{(\rho);\rm p.c.} \right)$$

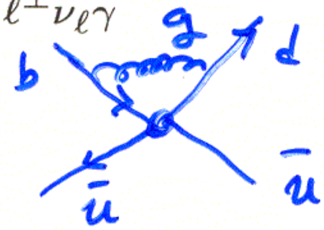


- $F_A^{(\rho);\rm p.v.}(0) \simeq F_A^{(\rho);\rm p.c.}(0) = F_A^{(\rho)}(0)$
  [more recently Byer, Melikhov, Stech]

$\frac{A}{P}$:

$$\epsilon_{\rm A}(\rho^\pm\gamma) = \frac{4\pi^2 m_\rho a_1}{m_b C_7^{eff}} \frac{F_A^{(\rho)}(0)}{T_1^{(\rho)}} = 0.30 \pm 0.07$$

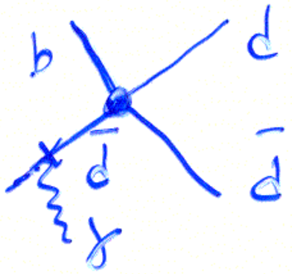
- Holds in factorization approximation
- $O(\alpha_s)$ corrections to annihilation amplitude $\mathcal{M}_{\rm A}(B^\pm \to \rho^\pm\gamma)$: Leading-twist contribution vanishes in the chiral limit [Grinstein, Pirjol]; non-factorizing annihilation contribution likely small; testable in $B^\pm \to \ell^\pm \nu_\ell \gamma$



Annihilation Amplitude $\mathcal{M}_{\rm A}(B^0 \to \rho^0\gamma)$

- Suppressed due to the electric charges ($Q_d/Q_u = -1/2$) and colour factors (BSW Parameters: $a_2/a_1 \simeq 0.25$)

$$\Longrightarrow \epsilon_{\rm A}(\rho^0\gamma) \simeq 0.05$$



$\frac{A}{P}(B^0 \to \rho^0\gamma) << 1$

$B \to \rho\gamma$ Decay Rates

[Parkhomenko,A.A.; Bosch, Buchalla]

$$R(\rho\gamma/K^*\gamma) = \frac{\overline{\mathcal{B}}_{\rm th}(B \to \rho\gamma)}{\overline{\mathcal{B}}_{\rm th}(B \to K^*\gamma)} = S_\rho \left|\frac{V_{td}}{V_{ts}}\right|^2 \frac{(1-m_\rho^2/M^2)^3}{(1-m_{K^*}^2/M^2)^3} \zeta^2 \left[1 + \Delta R(\rho/K^*)\right]$$

Annihilation + $O(\alpha_s)$ Corrections

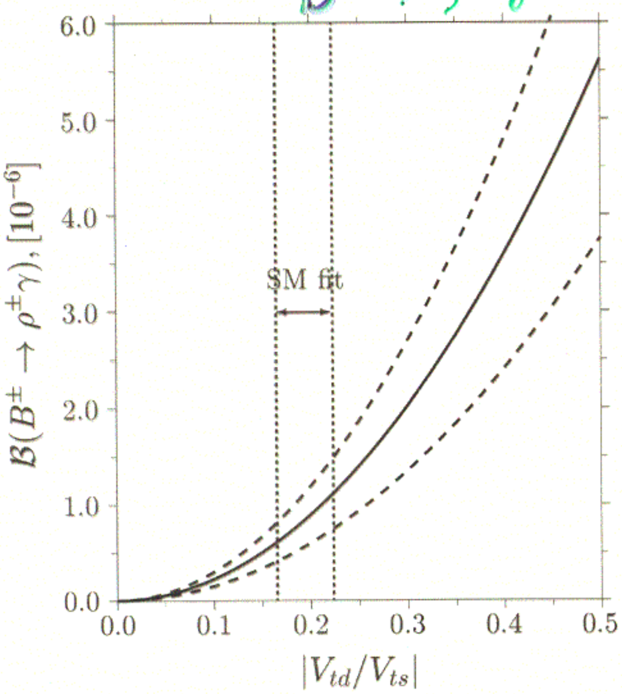
$$S_\rho = 1 \text{ for } B^\pm \to \rho^\pm\gamma; \quad = 1/2 \text{ for } B^0 \to \rho^0\gamma$$

$$\zeta = \frac{T_1^{(\rho)}(0)}{T_1^{(K^*)}(0)} \simeq 0.76 \pm 0.06 \quad \text{[Braun, Simma, A.A.'94]}$$

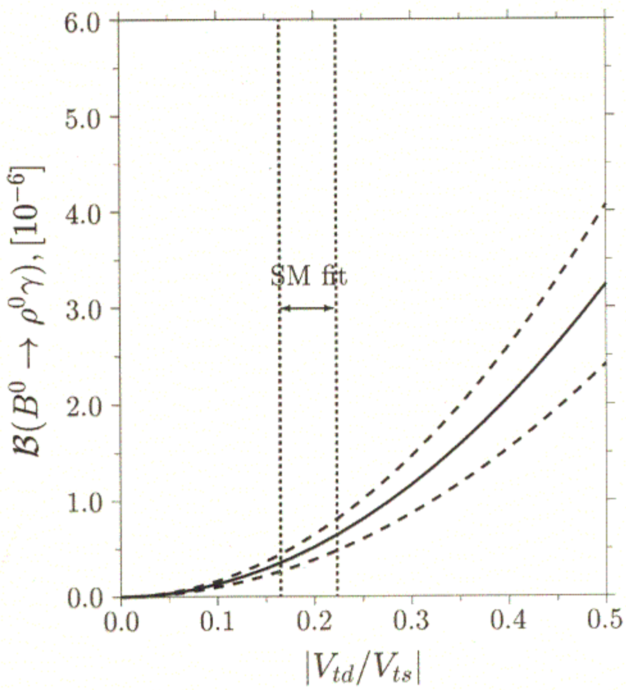
$$\Delta R(\rho^\pm/K^{*\pm}) = 0.07 \pm 0.13; \quad \Delta R(\rho^0/K^{*0}) = 0.02 \pm 0.11$$

[Parkhomenko,A.A. '01; Lunghi,A.A. '02]

$B^\pm \to \rho^\pm\gamma$



$B^0 \to \rho^0\gamma$



$\mathcal{B}_{\rm SM}(B^\pm \to \rho^\pm\gamma) = (0.90 \pm 0.33) \times 10^{-6}$ [Expt. : $< 2.3 \times 10^{-6}$ (BABAR)]

$\mathcal{B}_{\rm SM}(B^0 \to \rho^0\gamma) = (0.49 \pm 0.18) \times 10^{-6}$ [Expt. : $< 1.4 \times 10^{-6}$ (BABAR)]

$\mathcal{B}_{\rm SM}(B^0 \to \omega\gamma) \simeq \mathcal{B}_{\rm SM}(B^0 \to \rho^0\gamma)$ [Expt. $< 1.2 \times 10^{-6}$ (~~CLEO~~ BABAR)]

$R(\rho\gamma/K^*\gamma) \equiv \frac{\mathcal{B}(B \to \rho\gamma)}{\mathcal{B}(B \to K^*\gamma)} = 0.01 - 0.04$ [Expt. < 0.06 (BABAR)]

- also Upper limits from BELLE [S. Nishida]

ICHEP-2002 Update
$R(\rho\gamma/K^*\gamma) < 0.047$
[C. Jessop]

Lunghi, AA
hep-ph/0206242

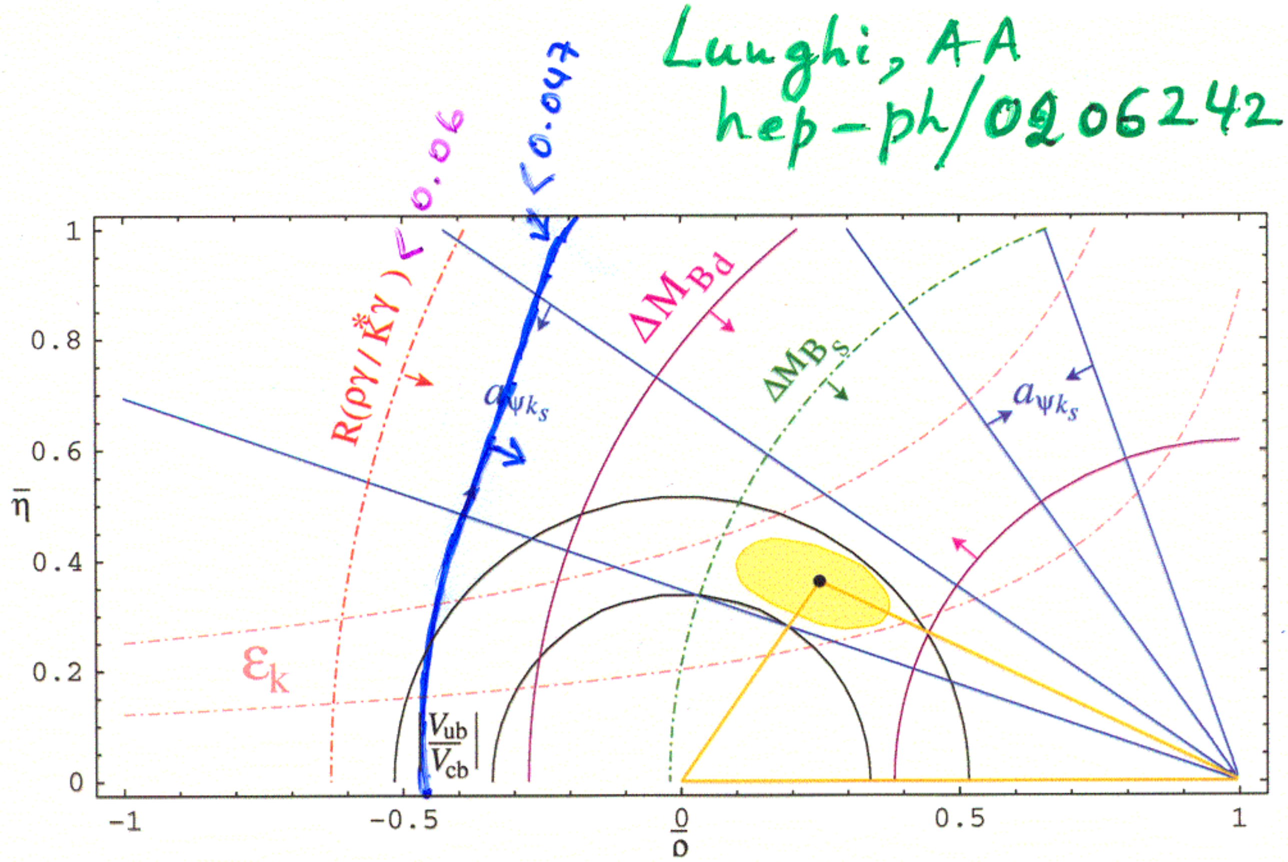


Figure 1: *Unitary triangle fit in the SM and the resulting 95% C.L. contour in the* $\bar{\rho}$ - $\bar{\eta}$ *plane. The impact of the* $R(\rho\gamma/K^*\gamma) < 0.06$ *constraint is also shown.*

Theoretical Uncertainties

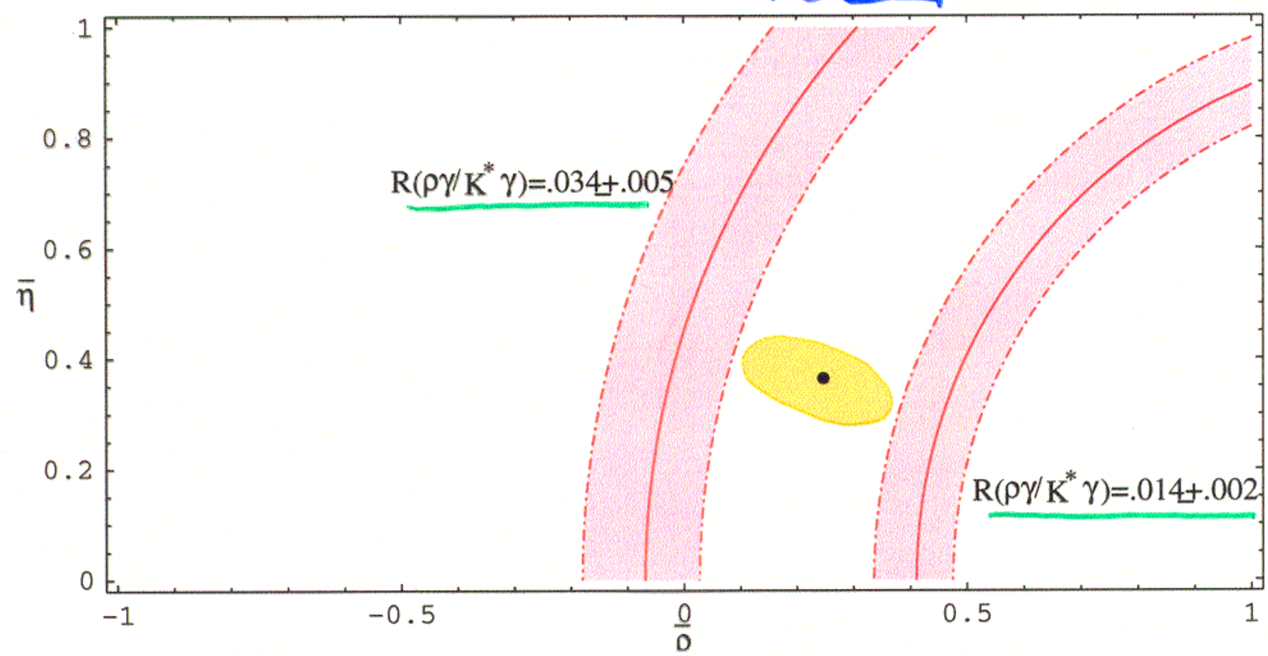


Figure 2: *Extremal values of* $R(\rho\gamma/K^*\gamma)$ *that are compatible with the SM unitarity triangle analysis.*

Need to reduce theoretical Uncertainty

@ present $\delta\left(\frac{V_{td}}{V_{ts}}\right) \approx O(15\%)$

Asymmerties in $B \to \rho\gamma$ Decays

[Parkhomenko,A.A.; Bosch, Buchalla; Handoko, London, A.A.]

- Isospin-Violating Ratios $\Delta^{\pm 0}$

$$\Delta = \frac{1}{2}\left[\Delta^{+0} + \Delta^{-0}\right], \qquad \Delta^{\pm 0} = \frac{\Gamma(B^\pm \to \rho^\pm \gamma)}{2\Gamma(B^0(\bar{B}^0) \to \rho^0 \gamma)} - 1$$

$$\Delta_{\rm LO} \simeq 2\epsilon_A \left[F_1 + \frac{\epsilon_A}{2}(F_1^2 + F_2^2)\right]$$

$$\Delta_{\rm NLO} \simeq \Delta_{\rm LO} - \frac{2\epsilon_A}{C_7^{(0)\rm eff}}\left[F_1 A_R^{(1)t}\right.$$

$$\left. + (F_1^2 - F_2^2) A_R^u + \epsilon_A (F_1^2 + F_2^2)(A_R^{(1)t} + F_1 A_R^u)\right]$$

$$\frac{V_{ub}V_{ud}^*}{V_{tb}V_{td}^*} = -\left|\frac{V_{ub}V_{ud}^*}{V_{tb}V_{td}^*}\right| e^{i\alpha} = F_1 + iF_2$$

$$B^\pm \to \rho^\pm \gamma: \quad \epsilon_A = +0.3 \pm 0.03; \quad B^0 \to \rho^0 \gamma: \quad \epsilon_A \simeq 0.05$$

[Braun,AA; Khodjamirian, Stoll, Wyler; Pirjol, Grinstein; Byer, Melikhov, Stech]

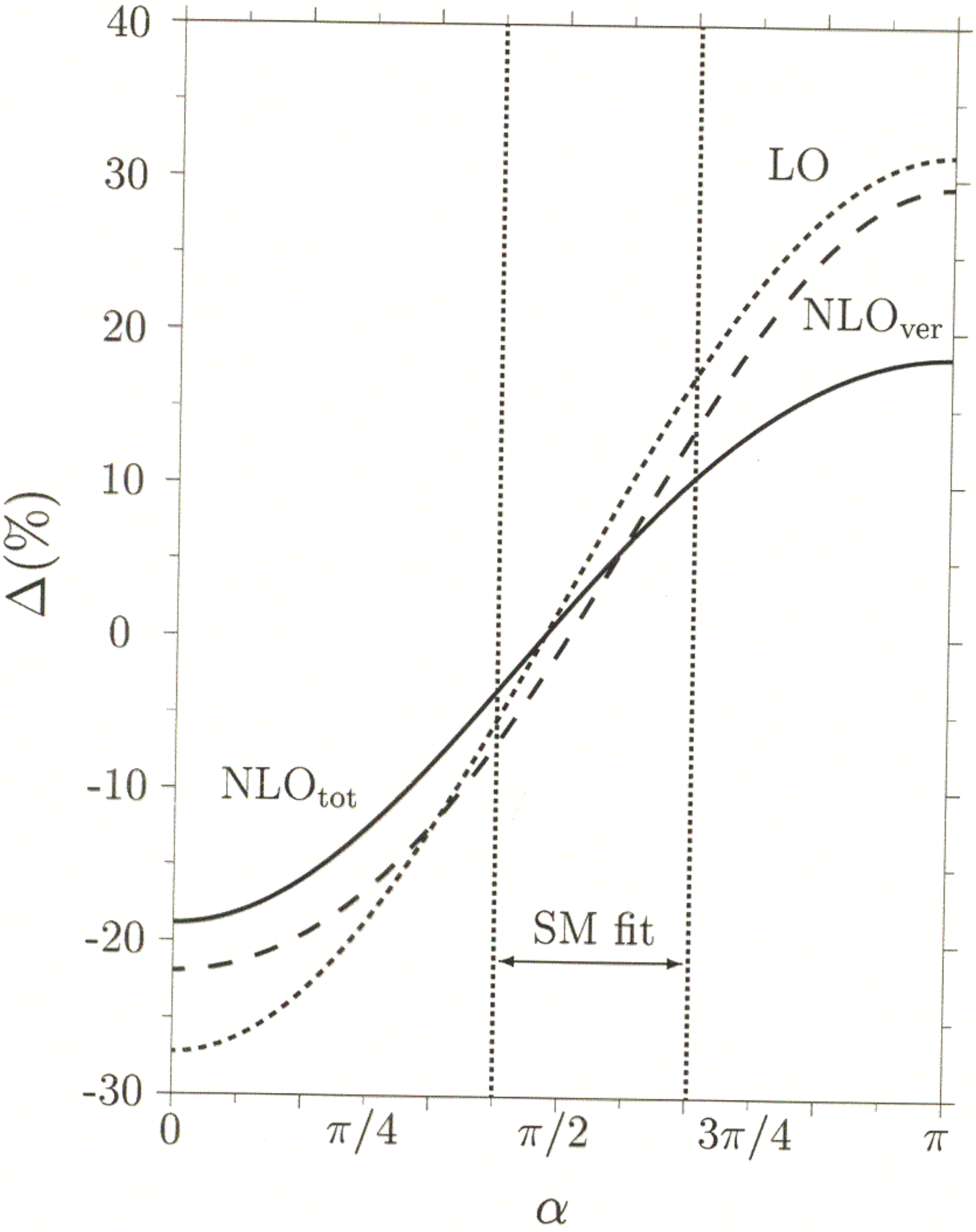
- Sign of ϵ_A alternates in literature!

  Recent calculations in the QCD Factorization framework now agree on the sign [Bosch, Buchalla; Kagan, Neubert; Parkhomenko, AA (revised version)]

$\Delta(\rho\gamma)$

$\Delta(\rho\gamma)_{SM} = 0.035^{+0.14}_{-0.07}$

[Parkhomenko; A.A.; in agreement with Bosch & Buchalla]



Direct CP-Asymmetries $\mathcal{A}_{CP}(\rho^{\pm}\gamma)$ and $\mathcal{A}_{CP}(\rho^{0}\gamma)$

- **Annihilation Contribution important in $\mathcal{A}_{CP}(\rho^{\pm}\gamma)$**

$$\mathcal{A}_{CP}(\rho^{\pm}\gamma) = \frac{\mathcal{B}(B^- \to \rho^-\gamma) - \mathcal{B}(B^+ \to \rho^+\gamma)}{\mathcal{B}(B^- \to \rho^-\gamma) + \mathcal{B}(B^+ \to \rho^+\gamma)}$$

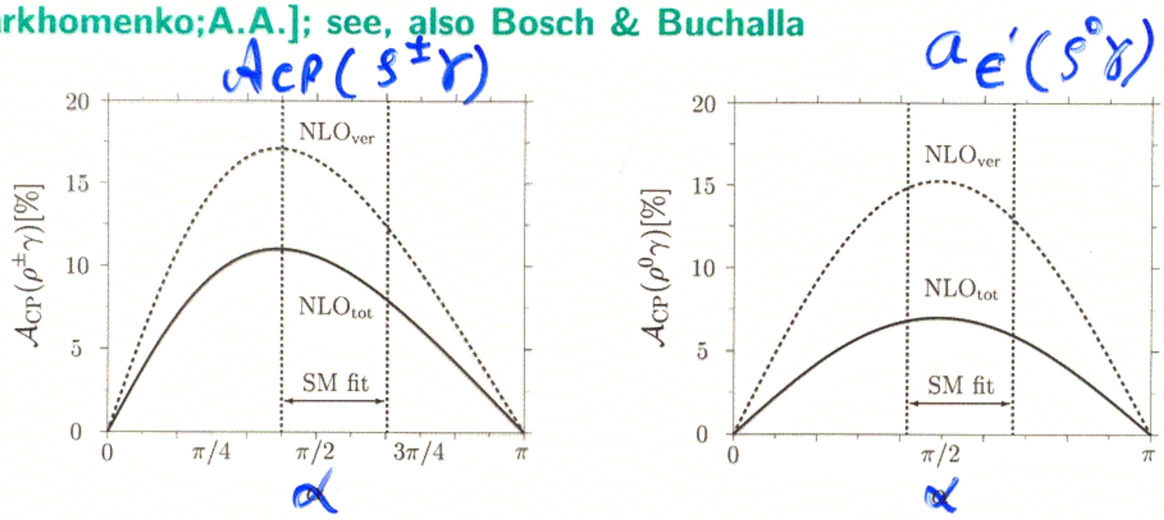
$$\mathcal{A}_{CP}(\rho^{\pm}\gamma) = \frac{2F_2(A_I^u - \epsilon_A\, A_I^{(1)t})}{C_7^{(0)\mathrm{eff}}\,(1+\Delta_{LO})}$$

- **Annihilation Contribution small in $\mathcal{A}_{CP}(\rho^{0}\gamma)$**

$$\mathcal{A}_{CP}(\rho^{0}\gamma)(t) = a_{\epsilon'}\cos(\Delta M_d t) + a_{\epsilon+\epsilon'}\sin(\Delta M_d t)$$

$$a_{\epsilon}'(\rho^{0}\gamma) = \frac{2F_2 A_I^u}{C_7^{(0)\mathrm{eff}}\,(1+\Delta_{LO})}$$

[Parkhomenko;A.A.]; see, also Bosch & Buchalla



- **Hard Spectator Corrections reduce $\mathcal{A}_{CP}(\rho^{\pm}\gamma)$ and $\mathcal{A}_{CP}(\rho^{0}\gamma)$**
- $\mathcal{A}_{CP}(\rho\gamma)$ **sensitive to μ, m_c/m_b, and ϵ_A**
- **SM:** $\mathcal{A}_{CP}(\rho^{\pm}\gamma) = 0.10 \pm 0.03$; $\mathcal{A}_{CP}(\rho^{0}\gamma) = 0.06 \pm 0.02$

Possible Supersymmetric Effects

Lunghi, AA
hep-ph/0206242

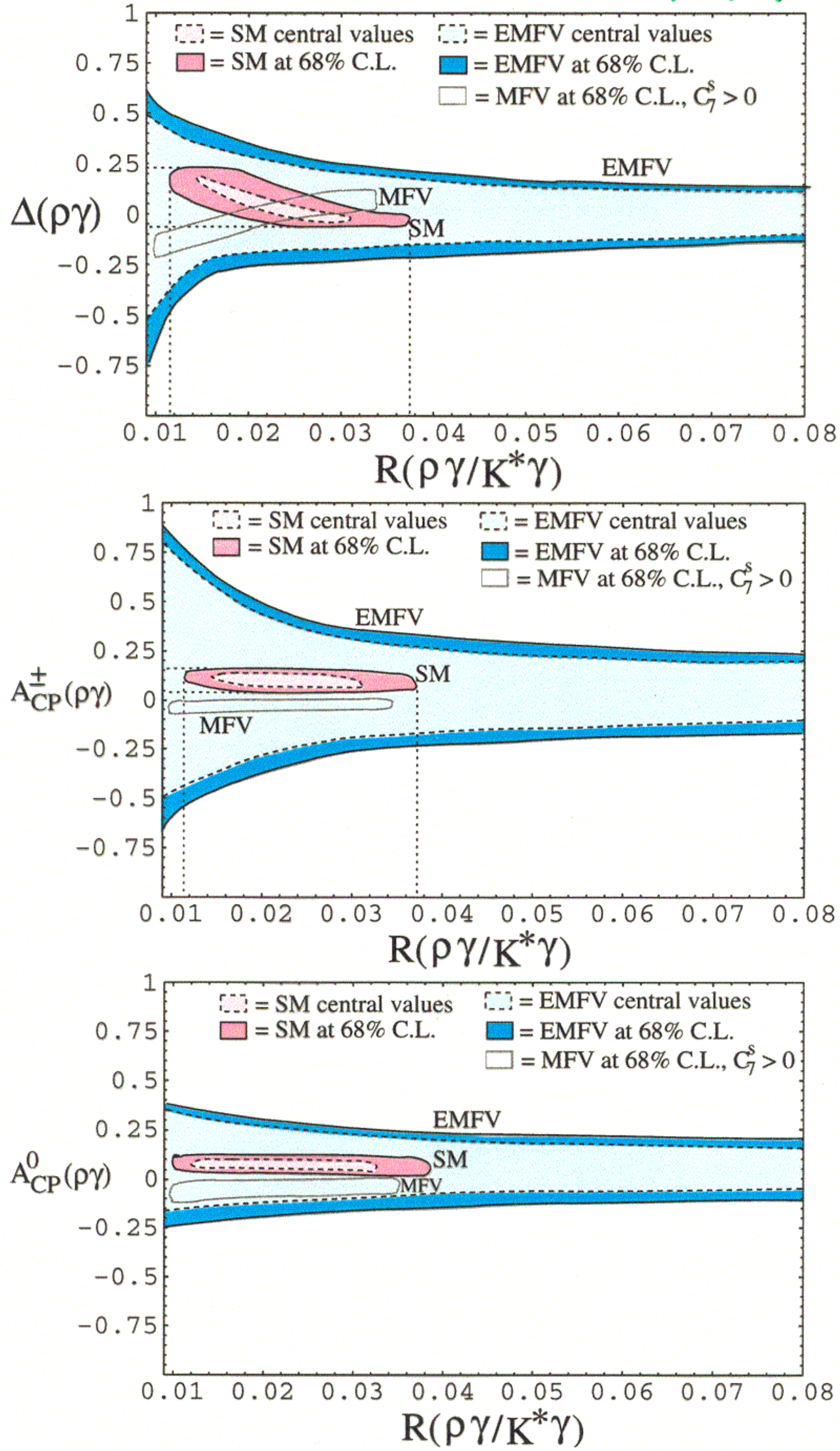


Figure 3: *Correlation between $R(\rho\gamma/K^*\gamma)$, $\Delta(\rho\gamma)$, $A^{\pm}_{CP}(\rho\gamma)$ and $A^0_{CP}(\rho\gamma)$ in the SM and in MFV and EMFV models. The light-shaded regions are obtained varying $\bar\rho$, $\bar\eta$, the supersymmetric parameters (for the MFV and EMFV models) and using the central values of all the hadronic quantities. The darker regions show the effect of $\pm 1\sigma$ variation of the hadronic parameters.*

$B \to X_s \ell^+ \ell^-$ in the SM

- Dilepton invariant mass shows resonant structure
  ($s = (p_{\ell^+} + p_{\ell^-})^2 = m^2_{J/\psi}, m^2_{\psi'}, ..$; (Long-distance physics)

- Principal interest: Measurements of dilepton invariant mass and Forward-Backward asymmetry $\mathcal{A}_{\rm FB}(s)$ in the non-resonant part (short-distance physics)

<u>Main Theoretical Developments</u>

- Lowest order Estimate of dilepton mass spectrum [Grinstein, Savage, Wise '89]

- Next-to-Leading Log (NLL) QCD Corrections; NLL matching conditions result in a substantial ($\pm 16\%$) dependence on the decay rate due to the scale (μ_W) [Misiak '93; Buras & Münz '95]

- μ_W-dependence reduced by Next-Next-Leading-Log (NNLL) matching [Bobeth, Misiak, Urban '99]; but the decay rate still uncertain by $\pm 13\%$ due to the lower scale ($= \mu_b$)-dependence

- Explicit $\mathcal{O}(\alpha_s)$ two-loop virtual corrections to the matrix elements and $d\mathcal{B}/d\hat{s}$ for $\hat{s} = s/m_b^2 < 0.25$ [Asatryan, Asatryan, Greub, Walker '01] $\Longrightarrow$ Reduction in the uncertainty in $d\mathcal{B}/d\hat{s}$ to $\pm 6\%$

- Leading power corrections in $1/m_b$ [Falk, Luke, savage '94; AA, Hiller, Handoko, Morozumi '97; Buchalla, isidori '98] and in $1/m_c$ [Buchalla, Isidori, Rey '98; Chen, Rupak, Savage '97] known

- Power $(1/m_b)$ corrected hadron energy and hadron invariant mass spectra in NLO in $B \to X_s \ell^+ \ell^-$ in HQET and Fermi Motion Model [Hiller, AA, '98, '99]

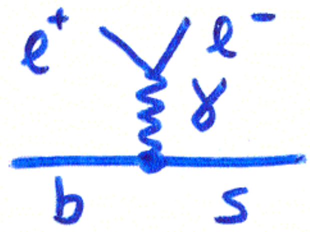
- Forward-Backward Asymmetry in $B \to X_s \ell^+ \ell^-$ in NNLO [Ghinculov, Hurth, Isidori, Yao (CERN-TH/2002-161); Asatrian, Bieri, Greub, Hovhannisyan (in preparation)]

Effective Low Energy Hamiltonian

$$\mathcal{H}_{\rm eff}(b \to s\ell^+\ell^-) = -\frac{4G_F}{\sqrt{2}} V_{ts}^* V_{tb} \sum_{i=1}^{10} C_i(\mu) O_i(\mu)$$

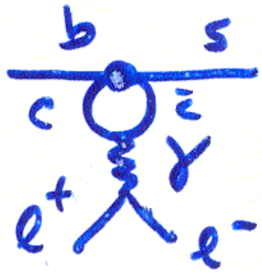
- Obtained using the CKM unitarity & $V_{us}^* V_{ub} \ll V_{ts}^* V_{tb}$
- $O_{1,\ldots,6}$: 4-quark operators; O_8: bsg-Vertex; enter due to operator mixing and explicit $O(\alpha_s)$ corrections

Dominant Operators



- $O_7 = \frac{e}{16\pi^2} \bar{s}_\alpha \sigma_{\mu\nu} (m_b R + m_s L) b_\alpha F^{\mu\nu}$

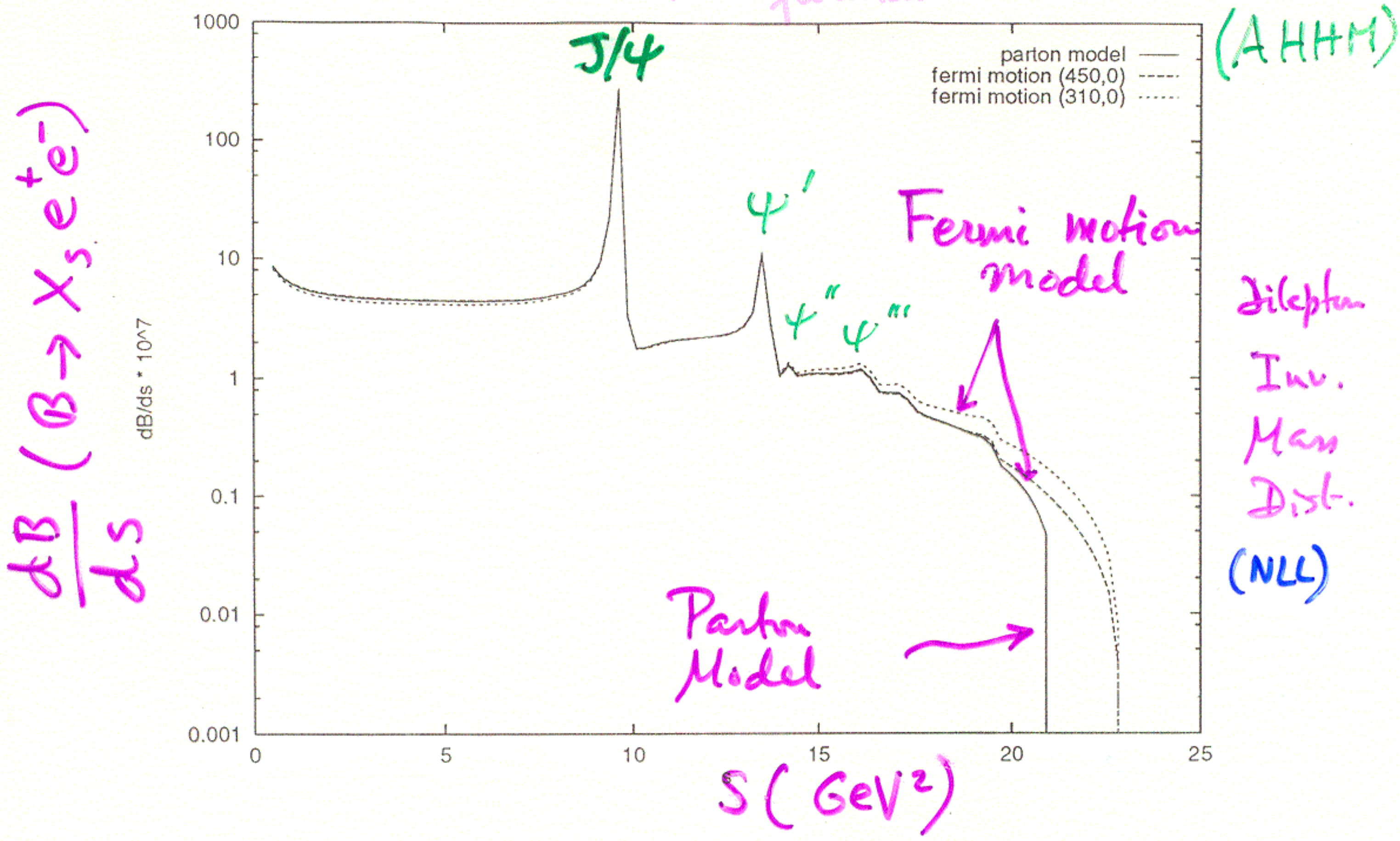
- $O_9 = \frac{e^2}{16\pi^2} \bar{s}_\alpha \gamma^\mu L b_\alpha \bar{\ell} \gamma_\mu \ell \quad (V)$
- $O_{10} = \frac{e^2}{16\pi^2} \bar{s}_\alpha \gamma^\mu L b_\alpha \bar{\ell} \gamma_\mu \gamma_5 \ell \quad (A)$
- Additional Non-local contribution to C_9



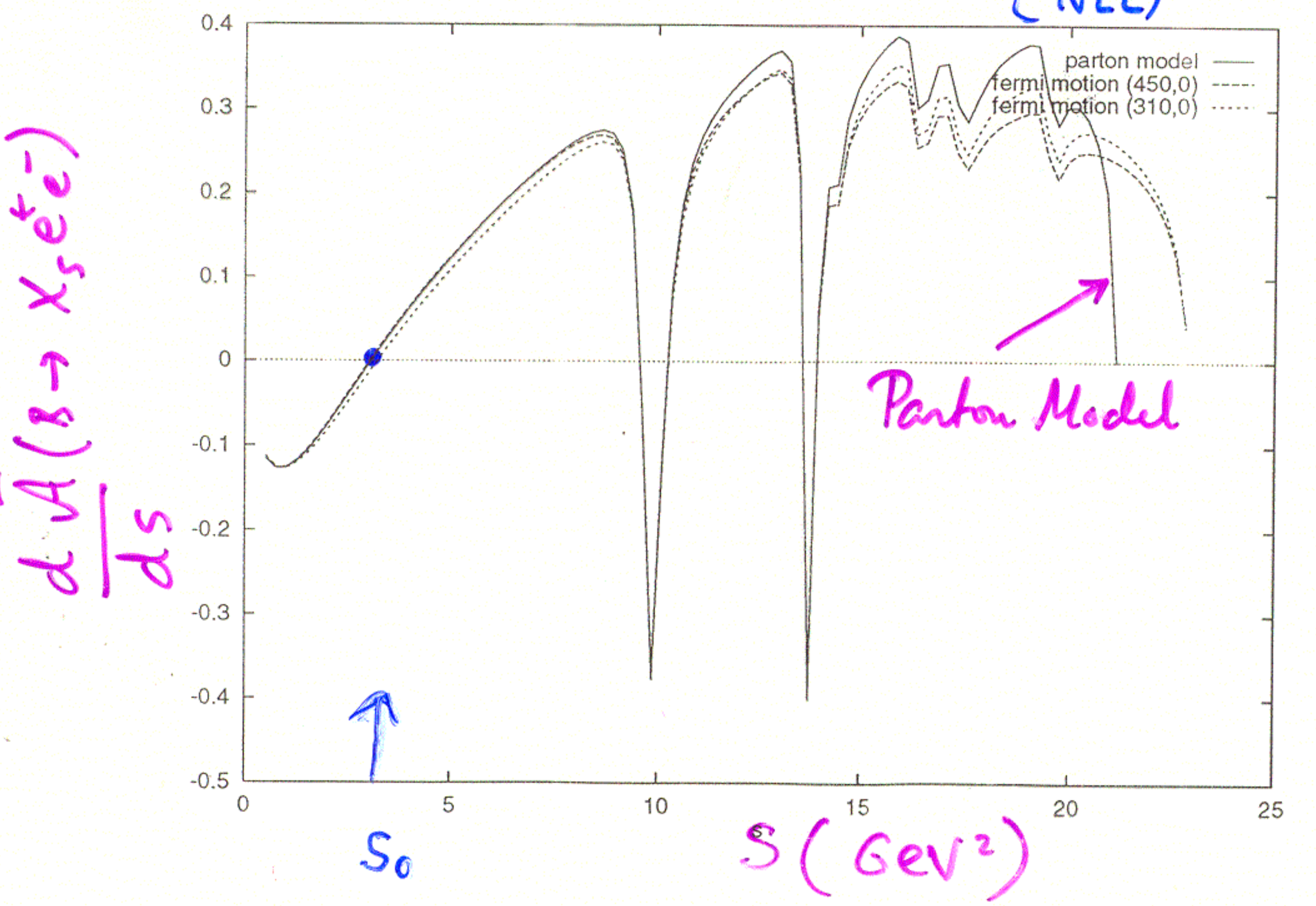
$$C_9^{\rm eff}(\hat{s}) = C_9 \eta(\hat{s}) + Y(\hat{s})$$

- $\eta(\hat{s}) = (1 + O(\alpha_s))$ [Jezabek, Kühn]
- $Y(\hat{s})$ contains perturbative charm loops and Charmonim resonances $(J/\psi, \psi', \ldots)$
- Several presciptions to combine SD- and Resonant parts [AA, Mannel, Morozumi '91; Krüger, Sehgal '96; ...]
- Residual uncertainty can be reduced by experimental cuts and using HQET $(1/m_c)$ power corrections

(including LD effects of Wave function) A.A., Hiller, Handoko, Morozumi, (AHHM)



Diff. Forward - Backward Asymmetry (NLL)



Inclusive $B \to X_s \ell^+ \ell^-$ in NNLO in SM

Dilepton Invariant Mass

$$\frac{d\Gamma(b \to X_s \ell^+ \ell^-)}{d\hat{s}} = \left(\frac{\alpha_{em}}{4\pi}\right)^2 \frac{G_F^2 m_{b,pole}^5 \left|V_{ts}^* V_{tb}\right|^2}{48\pi^3} (1-\hat{s})^2 \times$$

$$\left((1+2\hat{s}) \left(\left|\tilde{C}_9^{\text{eff}}\right|^2 + \left|\tilde{C}_{10}^{\text{eff}}\right|^2 \right) + 4\,(1+2/\hat{s}) \left|\tilde{C}_7^{\text{eff}}\right|^2 + 12\text{Re}\left(\tilde{C}_7^{\text{eff}} \tilde{C}_9^{\text{eff}*}\right) \right)$$

$$\begin{aligned}
\tilde{C}_7^{\text{eff}} &= \left(1 + \frac{\alpha_s(\mu)}{\pi}\omega_7(\hat{s})\right) A_7 \\
&\quad - \frac{\alpha_s(\mu)}{4\pi}\left(C_1^{(0)} F_1^{(7)}(\hat{s}) + C_2^{(0)} F_2^{(7)}(\hat{s}) + A_8^{(0)} F_8^{(7)}(\hat{s})\right), \\
\tilde{C}_9^{\text{eff}} &= \left(1 + \frac{\alpha_s(\mu)}{\pi}\omega_9(\hat{s})\right) \left(A_9 + T_9\, h(\hat{m}_c^2, \hat{s}) + U_9\, h(1, \hat{s}) + W_9\, h(0, \hat{s})\right) \\
&\quad - \frac{\alpha_s(\mu)}{4\pi}\left(C_1^{(0)} F_1^{(9)}(\hat{s}) + C_2^{(0)} F_2^{(9)}(\hat{s}) + A_8^{(0)} F_8^{(9)}(\hat{s})\right), \\
\tilde{C}_{10}^{\text{eff}} &= \left(1 + \frac{\alpha_s(\mu)}{\pi}\omega_9(\hat{s})\right) A_{10}\,,
\end{aligned}$$

- $h(\hat{m}_c^2, \hat{s})$ **and** $\omega_9(\hat{s})$
  [Bobeth, Misiak; Urban NP B574 (2000) 291]
- $\omega_7(\hat{s})$, **and** $F_{1,2,8}^{(7,9)}(\hat{s})$
  [Asatrian, Asatrian, Greub, Walker; Phys. Lett. B507 (2001) 162; hep-ph/0109140]

- A_7, A_8, A_9, A_{10}, T_9, U_9, W_9 **are linear combinations of the Wilson coefficients**

[Asatrian, Asatrian, Greub, Walker; Phys. Lett. B507 (2001) 162; hep-ph/0109140]

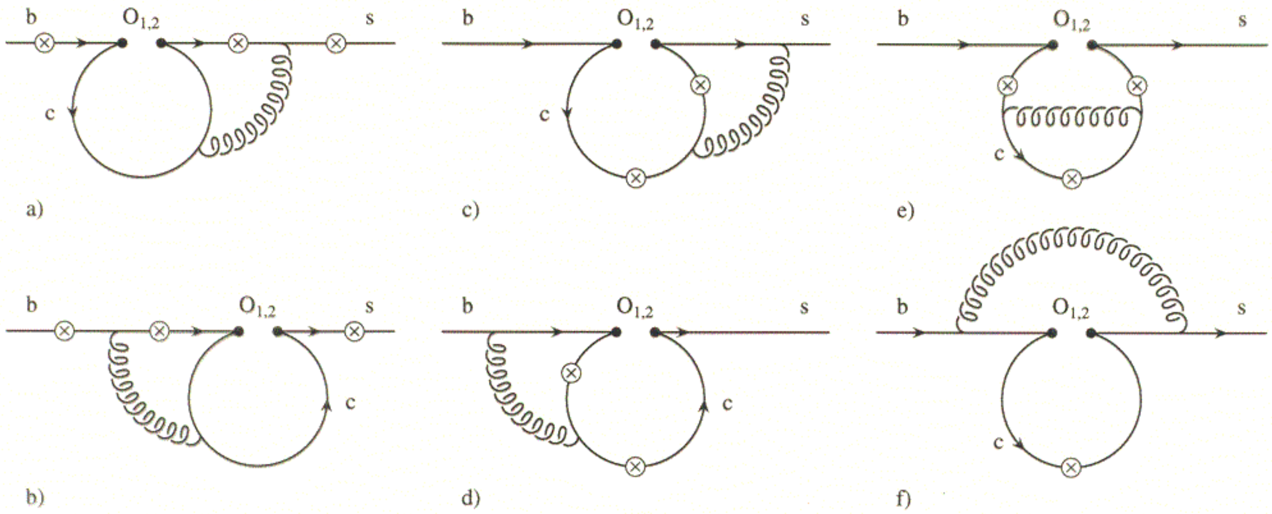


Figure 1: Matrix Elements from the operators $\mathcal{O}_{1,2}$

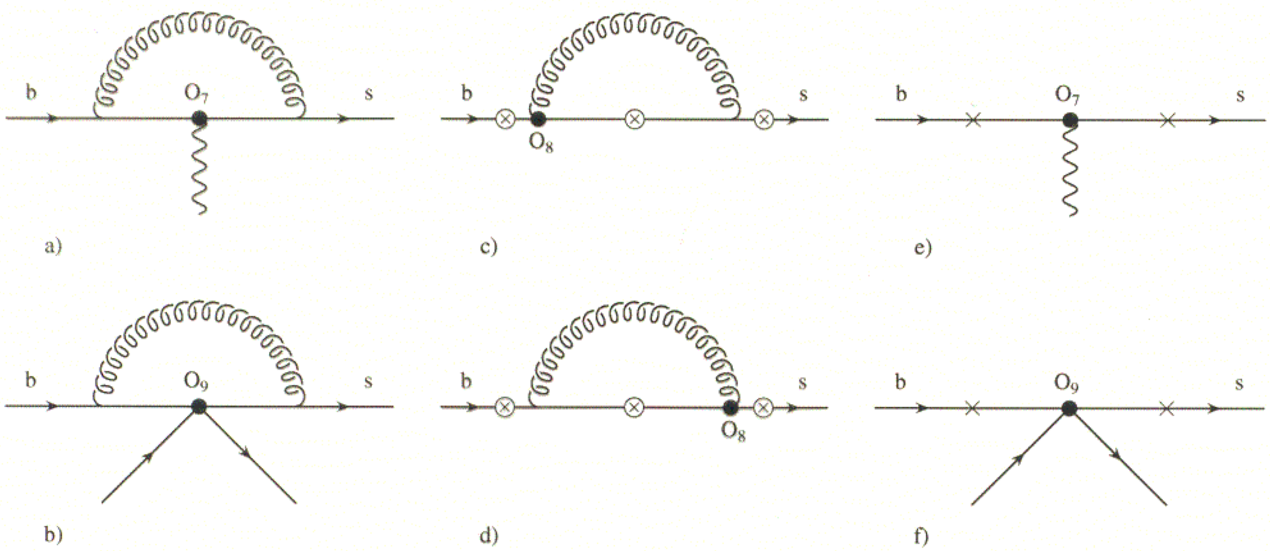


Figure 2: Matrix Elements from the operators $\mathcal{O}_7, \mathcal{O}_8, \mathcal{O}_9$

Power corrections in $B \to X_s \ell^+ \ell^-$ decays

- $1/m_b$ corrections [A. Falk et al., Phys. Rev. D49 (1994) 4553; AA, Handoko, Morozumi,Hiller, Phys. Rev. D55 (1997) 4105; Buchalla, Isidori, Rey; Nucl. Phys. B511 (1998) 594]

$$
\begin{aligned}
\frac{d\Gamma(b \to s\ell^+\ell^-)}{d\hat{s}} &= \left(\frac{\alpha em}{4\pi}\right)^2 \frac{G_F^2 m_{b,pole}^5 |\lambda_{ts}|^2}{48\pi^3}(1-\hat{s})^2 \left[(1+2\hat{s})\left(\left|\widetilde{C}_9^{\text{eff}}\right|^2 + \left|\widetilde{C}_{10}^{\text{eff}}\right|^2\right) G_1\right. \\
&+ \left. 4\,(1+2/\hat{s})\left|\widetilde{C}_7^{\text{eff}}\right|^2 G_2(\hat{s}) + 12\text{Re}\left(\widetilde{C}_7^{\text{eff}}\widetilde{C}_9^{\text{eff}*}\right) G_3(\hat{s}) + G_c(\hat{s})\right]
\end{aligned}
$$

where

$$
\begin{aligned}
G_1(\hat{s}) &= 1 + \frac{\lambda_1}{2m_b^2} + 3\frac{1-15\hat{s}^2+10\hat{s}^3}{(1-\hat{s})^2(1+2\hat{s})}\frac{\lambda_2}{2m_b^2} \\
G_2(\hat{s}) &= 1 + \frac{\lambda_1}{2m_b^2} - 3\frac{6+3\hat{s}-5\hat{s}^3}{(1-\hat{s})^2(2+\hat{s})}\frac{\lambda_2}{2m_b^2} \\
G_3(\hat{s}) &= 1 + \frac{\lambda_1}{2m_b^2} - \frac{5+6\hat{s}-7\hat{s}^2}{(1-\hat{s})^2}\frac{\lambda_2}{2m_b^2}
\end{aligned}
$$

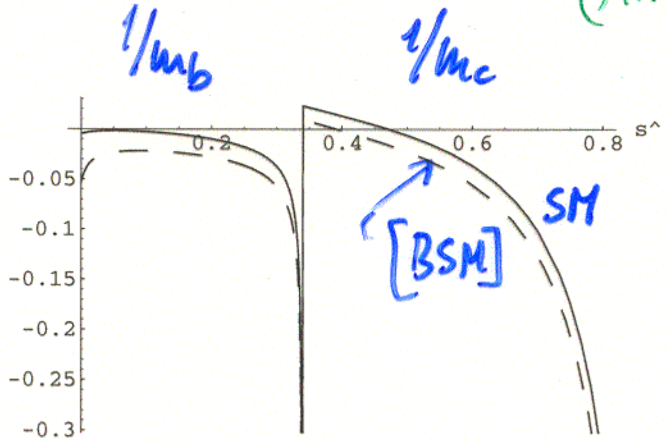
- $1/m_c$ corrections [Buchalla, Isidori, Rey; Nucl. Phys. B511 (1998) 594]

$$
G_c(\hat{s}) = -\frac{8}{9}(C_2 - \frac{C_1}{6})\frac{\lambda_2}{m_c^2}\text{Re}\left(F(r)\left[\widetilde{C}_9^{\text{eff}*}(2+\hat{s}) + \widetilde{C}_7^{\text{eff}*}\frac{1+6\hat{s}-\hat{s}^2}{\hat{s}}\right]\right)
$$

where $F(r)$ ($r = \hat{s}/(4\hat{m}_c^2)$) is:

$$
F(r) = \frac{3}{2r}\begin{cases} \dfrac{1}{\sqrt{r(1-r)}}\arctan\sqrt{\dfrac{r}{1-r}} - 1 & 0 < r < 1 \\[2ex] \dfrac{1}{2\sqrt{r(r-1)}}\left(\ln\dfrac{1-\sqrt{1-1/r}}{1+\sqrt{1-1/r}} + i\pi\right) - 1 & r > 1 \end{cases}
$$

(AA, Lunghi, Greub, Hiller)



Power Corrections

Figure 4: Power correction $R(\hat{s})$ in decay rate for $B \to X_s \ell^+ \ell^-$: SM (solid), $C_7 = -C_7^{SM}$ (dashed)

Scale-dependence

$B \to X_s e^+ e^-$ $\qquad$ $B \to X_s \ell^+ \ell^-$, $\ell = e, \mu$

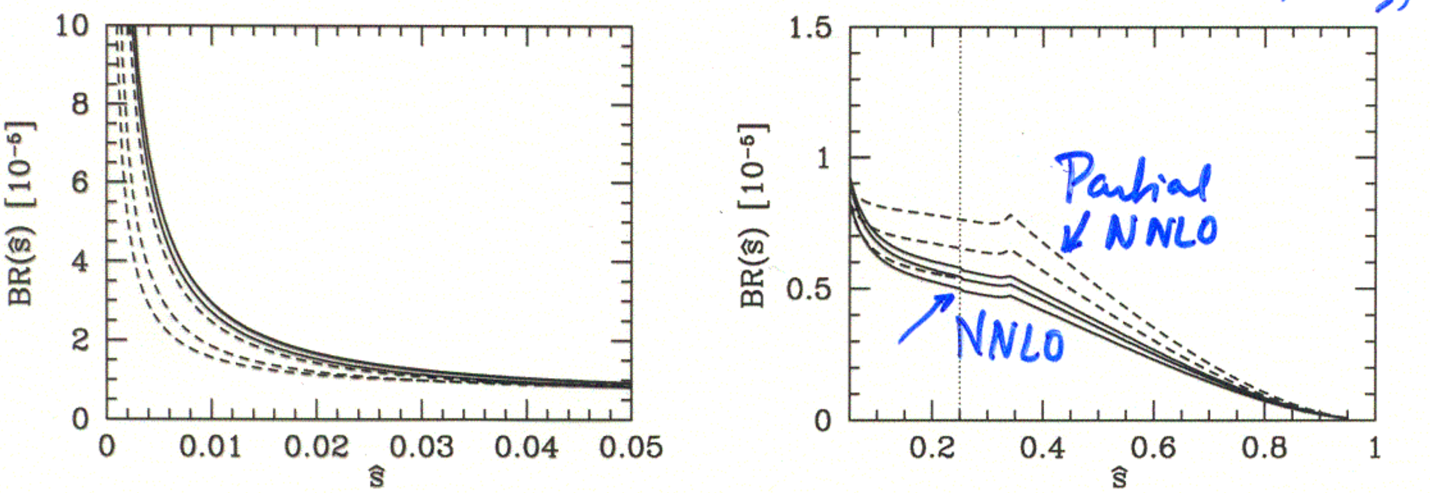
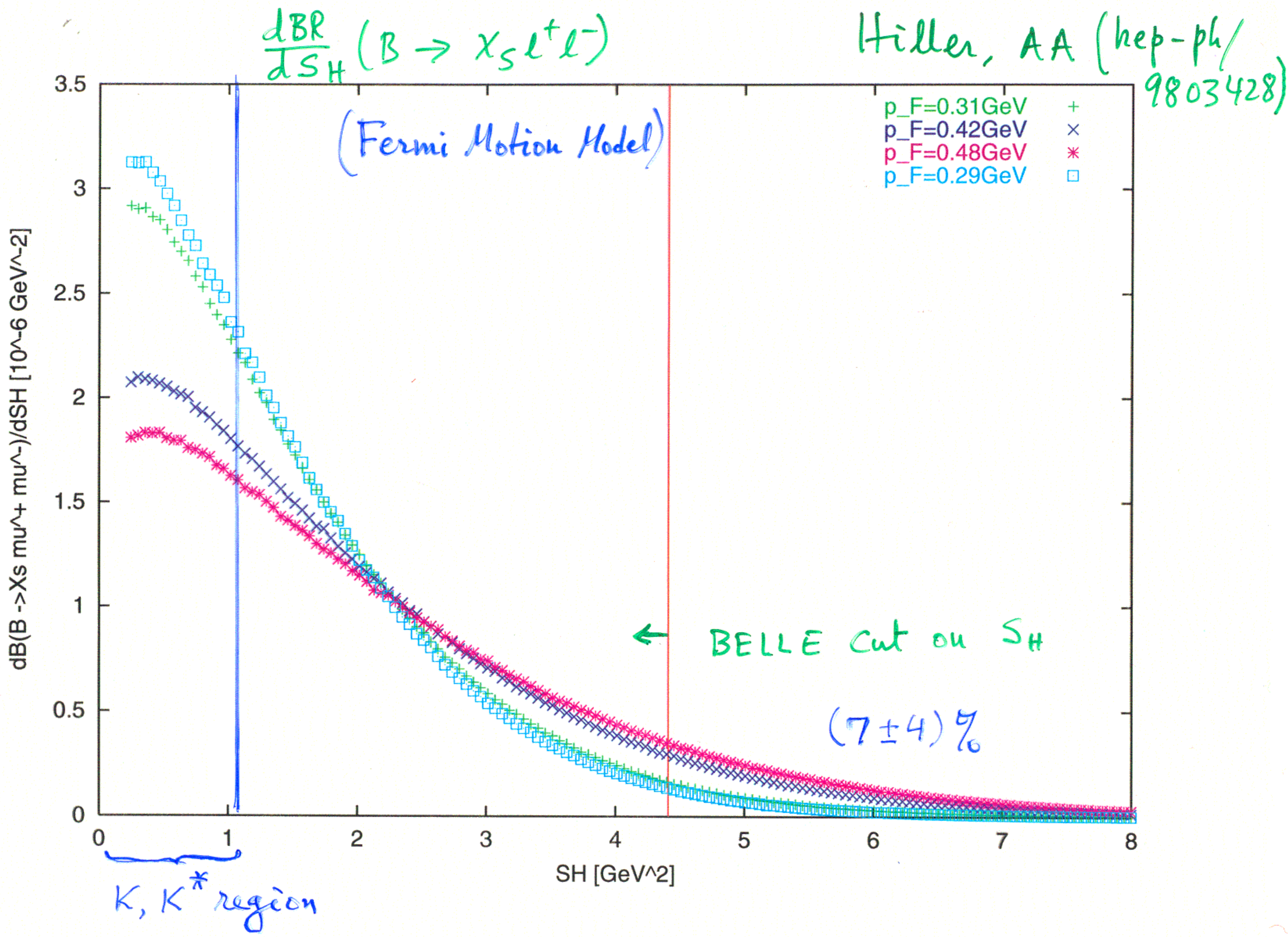


Figure 5: Dilepton inv. mass distributions in $B \to X_s \ell^+ \ell^-$; Partial NNLO (dashed lines) vs. full NNLO (solid lines). Left plot ($\hat{s} \in [0, 0.05]$): lower most curves are for $\mu = 10$ GeV, uppermost ones for $\mu = 2.5$ GeV. Right plot: μ dependence reversed

- **Scale-dependence in NNLO reduced**
- $\mathcal{B}(B \to X_s \ell^+ \ell^-)_{\mathrm{NNLO}} < \mathcal{B}(B \to X_s \ell^+ \ell^-)_{\mathrm{NLO}}$
- $\mathcal{B}(B \to X_s e^+ e^-) = (6.9 \pm 0.7) \times 10^{-6}$

  $\mathcal{B}(B \to X_s \mu^+ \mu^-) = (4.15 \pm 1.0) \times 10^{-6}$


dBR/dS_H (B → X_s l⁺l⁻)
Hiller, AA (hep-ph/9803428)
(Fermi Motion Model)
p_F=0.31GeV
p_F=0.42GeV
p_F=0.48GeV
p_F=0.29GeV
dB(B ->Xs mu^+ mu^-)/dSH [10^-6 GeV^-2]
SH [GeV^2]
← BELLE Cut on S_H
(7±4)%
K, K* region
0
0.5
1
1.5
2
2.5
3
3.5
0
1
2
3
4
5
6
7
8

$B \to X_s \ell^+ \ell^-$

BELLE [hep-ex/0207005]

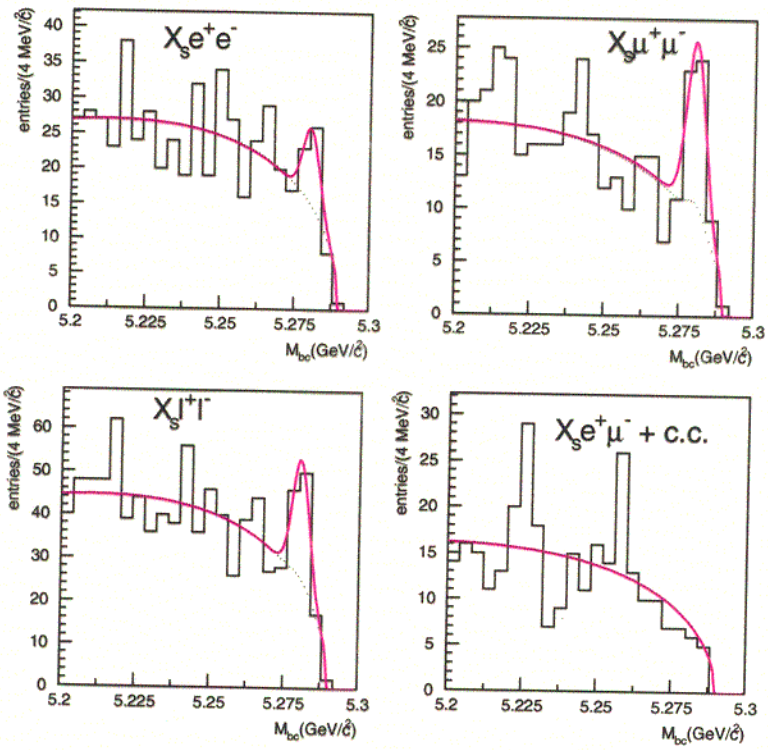
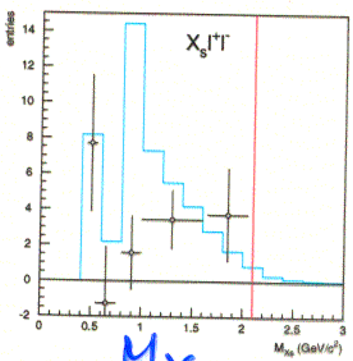
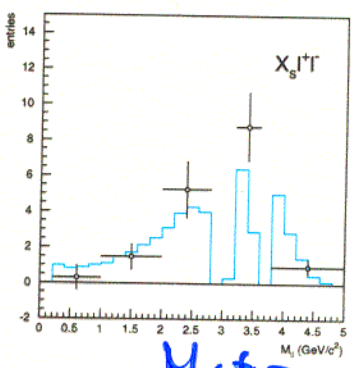


Figure 1: M_{bc} distributions and fit results. Top-left: $X_s e^+e^-$ candidates, top-right: $X_s\mu^+\mu^-$ candidates, bottom-left: $X_s\ell^+\ell^- = (X_s e^+e^-) + (X_s\mu^+\mu^-)$ candidates, and bottom-right: $X_s e^\pm \mu^\mp$ to estimate combinatorial background. The significance is determined from the statistical error only.



M_{X_s}

Figure 2: Comparison of the M_{X_s} distribution for data and MC.



$M_{\ell\ell}$

Figure 3: Comparison of the $M_{\ell\ell}$ distribution for data and MC.

$$\mathcal{B}(B \to X_s \ell^+ \ell^-) = (6.1 \pm 1.4 \,^{+1.3}_{-1.1}) \times 10^{-6}$$

$$\text{SM} = (5.6 \pm 0.9) \times 10^{-6}$$

Exclusive Decays $B \to (K, K^*)\ell^+\ell^-$

- $B \to K$ (pseudoscalar P); $B \to K^*$ (Vector V) Transitions involve the currents:

$$\Gamma^1_\mu = \bar{s}\gamma_\mu(1-\gamma_5)b, \quad \Gamma^2_\mu = \bar{s}\sigma_{\mu\nu}q^\nu(1+\gamma_5)b$$

$$\langle P|\gamma^1_\mu|B\rangle \supset f_+(q^2), f_-(q^2)$$

$$\langle P|\gamma^2_\mu|B\rangle \supset f_T(q^2)$$

$$\langle V|\gamma^1_\mu|B\rangle \supset V(q^2), A_1(q^2), A_2(q^2), A_3(q^2)$$

$$\langle V|\gamma^2_\mu|B\rangle \supset T_1(q^2), T_2(q^2), T_3(q^2)$$

- 10 non-perturbative q^2-dependent functions (Form factors) $\Longrightarrow$ model-dependence
- Data on $B \to K^*\gamma$ provides normalization of $T_1(0) = T_2(0) \simeq 0.28$
- HQET/LEET-Approach allows to reduce the number of independent form factors from 10 to 3; perturbative symmetry-breaking corrections [Beneke, Feldmann, Seidel; Beneke, feldmann]
- HQET & SU(3) relate $B \to (\pi,\rho)\ell\nu_\ell$ and $B \to (K, K^*)\ell^+\ell^-$ to determine the remaining FF's
- Need good measurements of the decays $B \to (\pi,\rho)\ell\nu_\ell$ and V_{ub} to make model-independent predictions for the decays $B \to (K, K^*)\ell^+\ell^-$

Dilepton Invariant Mass Distribution for $B \to K\ell^+\ell^-$

$$\frac{d\Gamma}{d\hat{s}} = |V_{ts}^* V_{tb}|^2 (|C_9^{eff} f_+ + \frac{2\hat{m}_b}{1+\hat{m}_K} C_7^{eff} f_T|^2 + |C_{10} f_+|^2)$$

- For $m_\ell = 0$, no contribution from the FF f_-
- In SM, $|C_7^{eff}| \ll |C_9^{eff}, C_{10}$, and no kinematical enhancement at low $\hat{s}$ (as opposed to $B \to K^*\ell^+\ell^-$); To a good approximation ($O(10\%)$)

$$\frac{d\Gamma}{d\hat{s}} \sim |f_+(\hat{s})|^2$$

- $f_+(\hat{s})$ determined from $B \to \pi\ell\nu_\ell$ and SU(3)-breaking

Constraints on the CKM Matrix Elements

- $\mathcal{B}(B \to K\ell^+\ell^-) \implies$ a determination of $|V_{ub}/V_{ts}^* V_{tb}|$ [Ligeti, Stewart, Wise]
- $\mathcal{B}(B \to \pi\ell^+\ell^-) \implies$ a precise determination of $|V_{ub}/V_{td}^* V_{tb}|$
- SM estimates (in NNLO) [AA, Lunghi, Greub, Hiller '01]:

$$\mathcal{B}(B \to K\ell^+\ell^-) = (0.35 \pm 0.12) \times 10^{-6}$$
$$\mathcal{B}(B \to \pi\ell^+\ell^-) = (0.24 \pm 0.10) \times |\frac{V_{td}}{V_{ts}}|^2 \times 10^{-6} \simeq 10^{-8}$$

Sensitivity to New Physics

- $B \to X_s\gamma$ Data implies $|C_7^{eff}| \simeq |C_7^{eff}(\mathrm{SM})| \implies$ Two possible solutions
- $C_7^{eff} \simeq C_7^{eff}(\mathrm{SM})$; (SUGRA-type solutions for low $\tan\beta$); hard to distinguish from SM
- $C_7^{eff} \simeq -C_7^{eff}(\mathrm{SM})$; (allowed solution in SUGRA-type models with large $\tan\beta$); distinguishable through a precise measurement of the dilepton mass spectrum [Okada et al.; AA, Ball, Handoko, Hiller]

Dilepton Invariant Mass Distribution for $B \to K^* \ell^+ \ell^-$

- For $m_\ell = 0$, no contribution from the FF $A_0(\hat{s})$

- Enhancement in dilepton mass spectrum in low $\hat{s}$-region due to the dependence $\frac{d\Gamma}{d\hat{s}} \sim C_7^{eff\,2}/\hat{s}$; photon pole cotribution dominant for $q^2 < 1$ GeV^2
$\Longrightarrow \mathcal{B}(B \to K^* e^+ e^-) > \mathcal{B}(B \to K^* \mu^+ \mu^-)$

- Like $B \to K \ell^+ \ell^-$, following combinations of WC's are involved:
$|C_{10}|^2, \quad |C_9|^2, \quad |C_7^{eff}|^2, \quad Re(C_7^{eff} C_9^{eff})$

- HQET/LEET can be used advantageously to reduce the number of independent form factors to 2; $O(\alpha_s)$-corrections to the HQET/LEET symmetry calculated [Beneke, Feldmann; Beneke, Feldmann, Seidel]

- Residual FF-related uncertainties can be reduced by relating $B \to K^* \ell^+ \ell^-$ with $B \to \rho \ell \nu_\ell$ and SU(3)-breaking; Data on $B \to \rho \ell \nu_\ell$ not yet precise enough to warrant this analysis

- Helicity analysis of $B \to K^* \ell^+ \ell^-$ in terms of the components $H_+(\hat{s}), H_-(\hat{s})$ and $H_0(\hat{s})$ and using data on $B \to K^* \gamma \Longrightarrow$ rather precise dilepton mass spectrum for the $H_-(\hat{s})$ component [Safir, AA '02]

- SM estimates (in NNLO) [AA, Lunghi, Greub, Hiller '01]:

$$\mathcal{B}(B \to K^* e^+ e^-) = (1.58 \pm 0.52) \times 10^{-6}$$
$$\mathcal{B}(B \to K^* \mu^+ \mu^-) = (1.2 \pm 0.4) \times 10^{-6}$$

- $\mathcal{B}(B \to K^* \ell^+ \ell^-)$ and $\mathcal{B}(B \to \rho \ell^+ \ell^-)$ can be used (like $B \to (K, \pi) \ell^+ \ell^-$) to determine the CKM matrix elements $|V_{ts}|$ and $|V_{td}|$

- Sizable distrotion of the dilepton spectrum allowed in New Physics scenarios, such as supersymmetry

Dilepton inv. mass distributions   Ball, Handoko, Hiller, A-A. hep-ph/9910221

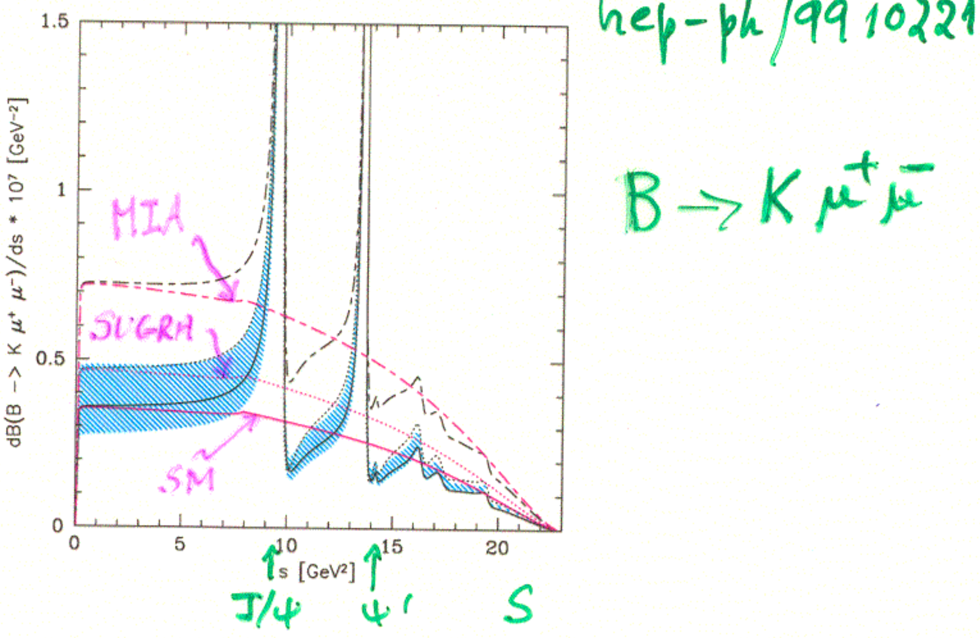


Figure 6: *The dilepton invariant mass distribution in $B \to K\mu^+\mu^-$ decays, using the form factors from LCSR as a function of s. All resonant $c\bar{c}$ states are parametrized as in Ref. [29]. The solid line represents the SM and the shaded area depicts the form factor-related uncertainties. The dotted line corresponds to the SUGRA model with $R_7 = -1.2$, $R_9 = 1.03$ and $R_{10} = 1$. The long-short dashed lines correspond to an allowed point in the parameter space of the MIA-SUSY model, given by $R_7 = -0.83$, $R_9 = 0.92$ and $R_{10} = 1.61$. The corresponding pure SD spectra are shown in the lower part of the plot.*

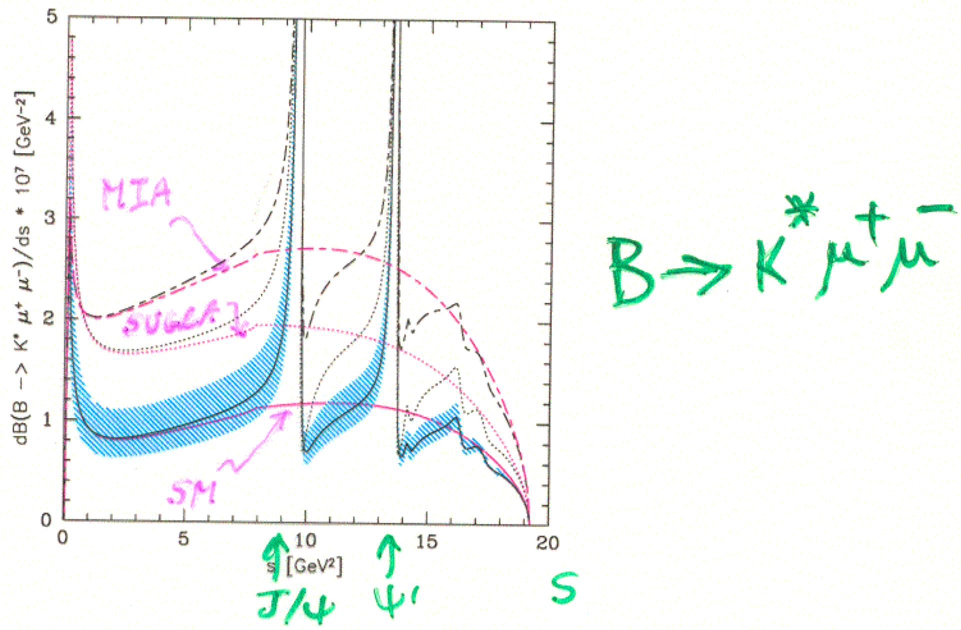


Figure 7: *The dilepton invariant mass distribution in $B \to K^*\mu^+\mu^-$ decays, using the form factors from LCSR as a function of s. All resonant $c\bar{c}$ states are parametrized as in Ref. [29]. The legends are the same as in Fig. 6.*

SM predictions at NNLO accuracy & Comparison with Data

(all in units of 10^{-6})

[A.A., Lunghi, Greub, Hiller, DESY 01-217; hep-ph/0112300]

Decay Mode	Theory (SM)	BELLE	BABAR
$B \to K \ell^+ \ell^-$	0.35 ± 0.12	$0.75^{+0.25}_{-0.21} \pm 0.09$ (*) $0.58^{+0.17}_{-0.15} \pm 0.06$	$0.84^{+0.30+0.10}_{-0.24-0.18}$ (*) $0.78^{+0.24+0.11}_{-0.20-0.18}$
$B \to K^* e^+ e^-$	1.58 ± 0.52	< 5.1	$1.68^{+0.68}_{-0.58} \pm .28$
$B \to K^* \mu^+ \mu^-$	1.2 ± 0.4	< 3.0	< 3.6; weighted $e^+e^-, \mu^+\mu^-$ $B \to K^* \ell^+ \ell^-$
$B \to X_s \mu^+ \mu^-$	4.15 ± 1.0	$8.9^{+2.3+1.6}_{-2.1-1.7}$ (*) $7.9 \pm 2.1^{+2.0}_{-1.5}$	–
$B \to X_s e^+ e^-$	6.9 ± 0.70	< 11.0 (*) $5.0 \pm 2.3^{+1.2}_{-1.1}$	–
$B \to X_s \ell^+ \ell^-$	5.6 ± 0.9	$7.1 \pm 1.6^{+1.6}_{-1.2}$ (*) $6.1 \pm 1.4^{+1.3}_{-1.1}$	–

(*) S. Nishida (ICHEP 2002)

(*) J. Richman (ICHEP 2002)

- Experiments & SM in good accord;
- Improved precision crucial to disentangle NP effects

Forward-Backward Asymmetry in $B \to K^* \ell^+ \ell^-$

$$(\hat{u} \sim \cos\theta; \quad \theta = \sphericalangle(p_B, p_{\ell^+}) \ \text{ in dilepton CMS}$$

$$\frac{dA_{FB}}{d\hat{s}} = -\int_0^{\hat{u}(\hat{s})} d\hat{u} \frac{d\Gamma}{d\hat{u}d\hat{s}} + \int_{-\hat{u}(\hat{s})}^{0} d\hat{u} \frac{d\Gamma}{d\hat{u}d\hat{s}}$$

$$\sim C_{10}[\mathrm{Re}(C_9^{eff})VA_1 + \frac{\hat{m}_b}{\hat{s}} C_7^{eff}(VT_2(1-\hat{m}_V) + A_1T_1(1+\hat{m}_V))]$$

- Probes different combinations of WC's than dilepton mass spectrum; has a characteristic zero in the SM $(\hat{s}_0)$ below $m^2_{J/\psi}$

Position of the $A_{FB}(\hat{s})$ zero $(\hat{s}_0)$ in $B \to K^* \ell^+ \ell^-$

$$\mathrm{Re}(C_9^{eff}(\hat{s}_0)) = -\frac{\hat{m}_b}{\hat{s}_0} C_7^{eff}\left(\frac{T_2(\hat{s}_0)}{A_1(\hat{s}_0)}(1-\hat{m}_V) + \frac{T_1(\hat{s}_0)}{V(\hat{s}_0)}(1+\hat{m}_V)\right)$$

- Model-dependent studies $\Longrightarrow$ small FF-related uncertainties in $\hat{s}_0$ [Burdman '98]
- HQET/LEET provide a symmetry argument why the uncertainty in $\hat{s}_0$ is small. In leading order in $1/m_B$, $1/E$ ($E = \frac{m_B^2 + m_{K^*}^2 - q^2}{2m_B}$) and $O(\alpha_s)$:

$$\frac{T_2}{A_1} = \frac{1+\hat{m}_V}{1+\hat{m}_V^2 - \hat{s}}\left(1 - \frac{\hat{s}}{1-\hat{m}_V^2}\right)$$

$$\frac{T_1}{V} = \frac{1}{1+\hat{m}_V}$$

- No hadronic uncertainty in $\hat{s}_0$ [AA, Ball, Handoko, Hiller '99]

$$C_9^{eff}(\hat{s}_0) = -\frac{2m_b M_B}{s_0} C_7^{eff}$$

- $O(\alpha_s)$ corrections to the LEET-symmetry relations lead to substantial perturbative shift in $\hat{s}_0$ [Beneke, Feldmann, Seidel '01]

$$C_9^{eff}(\hat{s}_0) = -\frac{2m_b M_B}{s_0} C_7^{eff}(1 + \frac{\alpha_s C_F}{4\pi}[\ln \frac{m_b^2}{\mu^2} - L] + \frac{\alpha_s C_F}{4\pi} \frac{\Delta F_\perp}{\xi_\perp(s_0)})$$

AA, A.S. Safir, DESY Report '02-005 [hep-ph/02054]

H

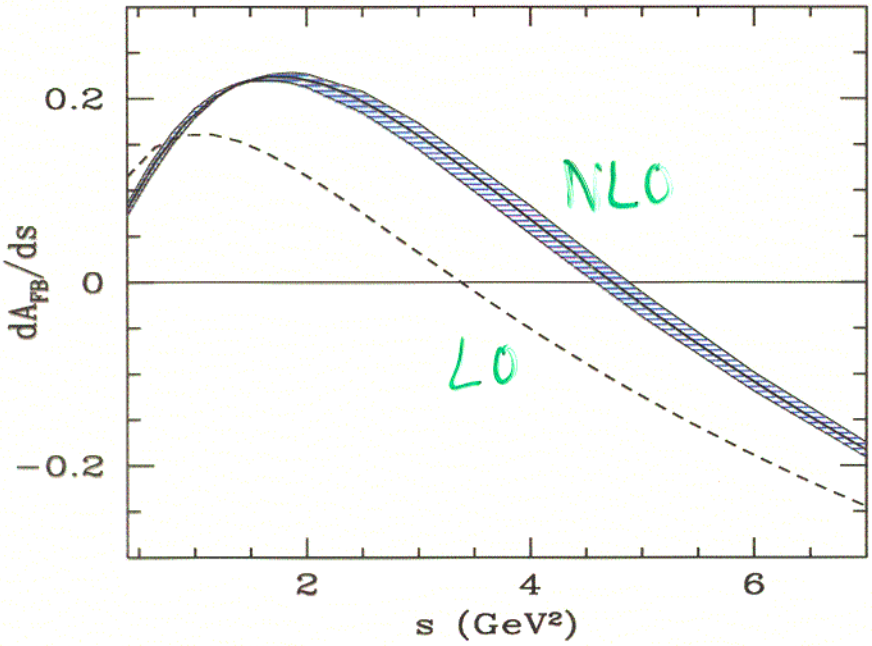
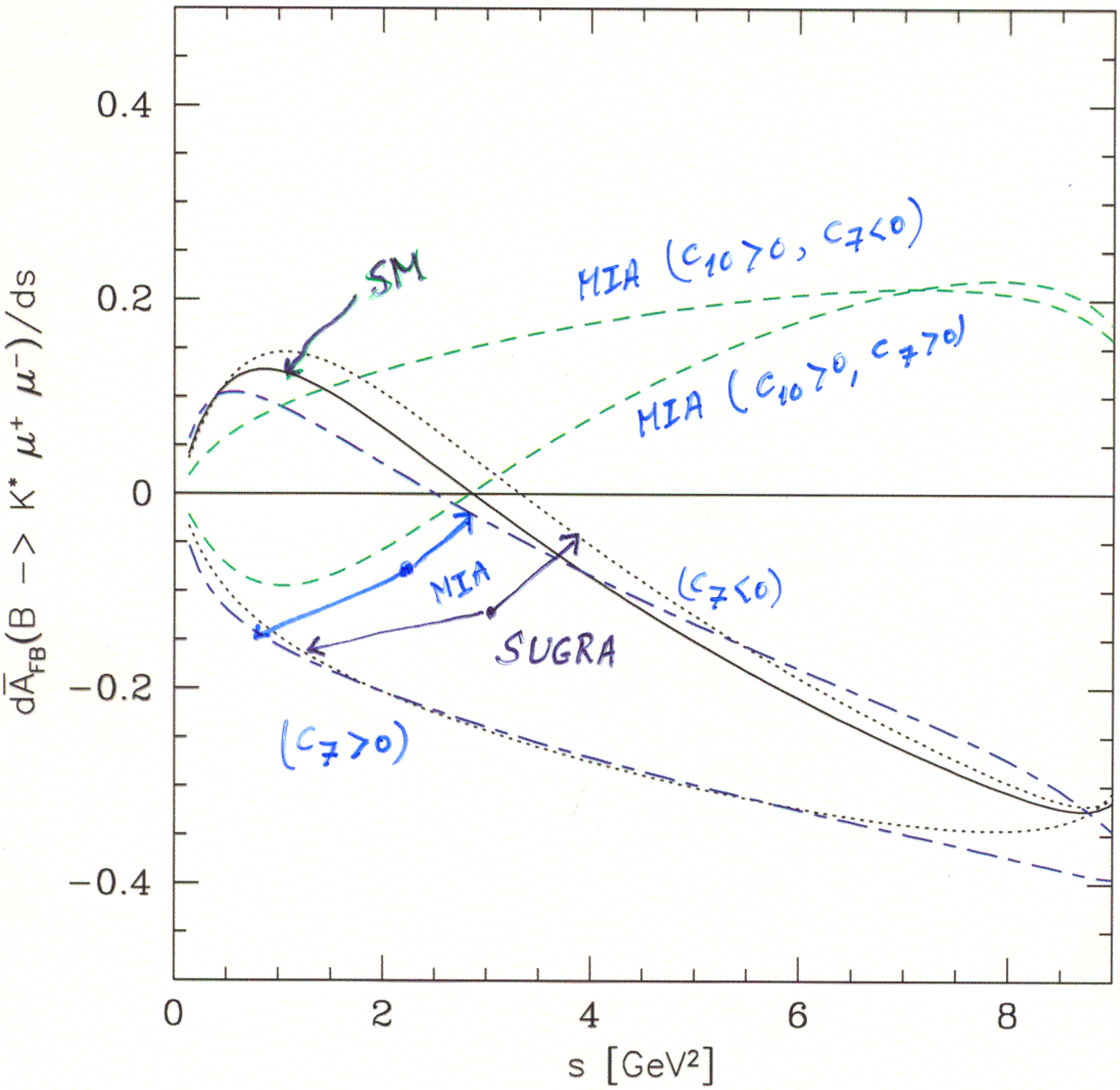


Figure 5: Forward-backward asymmetry $dA_{FB}(B \to K^* l^+ l^-)/ds$ at next-to-leading order (solid center line) and leading order (dashed). The band reflects the theoretical uncertainties from the input parameters.

Ball, Hiller, Handoko, AA

FB Asymmetry ($B \to K^* \ell^+ \ell^-$)



A Model-independent Analysis of $B \to X_s \gamma$ & $B \to X_s \ell^+ \ell^-$

- Assume $\mathcal{H}_{eff}^{\rm SM}$ a sufficient operator basis also for Beyond-the-SM physics
- Shifts due to Beyond-the-SM [BSM] physics only in $C_7(\mu_W)$,$C_8(\mu_W)$, $C_9(\mu_W)$, and $C_{10}(\mu_W)$
- BSM Coefficients: $R_7 - 1$, $R_8 - 1$, $C_9^{\rm NP}$, & $C_{10}^{\rm NP}$
- Define:

$$R_{7,8}(\mu_W) \equiv \frac{C_{7,8}^{\rm tot}(\mu_W)}{C_{7,8}^{\rm SM}(\mu_W)}$$

  with $C_{7,8}^{\rm tot}(\mu_W) = C_{7,8}^{\rm SM}(\mu_W) + C_{7,8}^{\rm NP}(\mu_W)$

- Set the scale $\mu_W = M_W$, and use RGE to evolve
  $R_{7,8}(\mu_W) \to R_{7,8}(\mu_b) = \frac{A_{7,8}^{\rm tot}(\mu_b)}{A_{7,8}^{\rm SM}(\mu_b)}$
- RGE $\Longrightarrow$ modificications in $\tilde{C}_7^{eff}$, $\tilde{C}_9^{eff}$, $\tilde{C}_{10}^{eff}$
- Impose constraints from $R_7(\mu_b)$ and $R_8(\mu_b)$ from $B \to X_s \gamma$ Data
- Use Data on $B \to (X_s, K^*, K)\ell^+\ell^-$ BRs to constrain $C_9^{\rm NP}$ and $C_{10}^{\rm NP}$
- Two-fold ambiguity due to the sign of C_7^{eff}
  $\Longrightarrow$ Two-fold ambiguity for $C_9^{\rm NP}$ and $C_{10}^{\rm NP}$

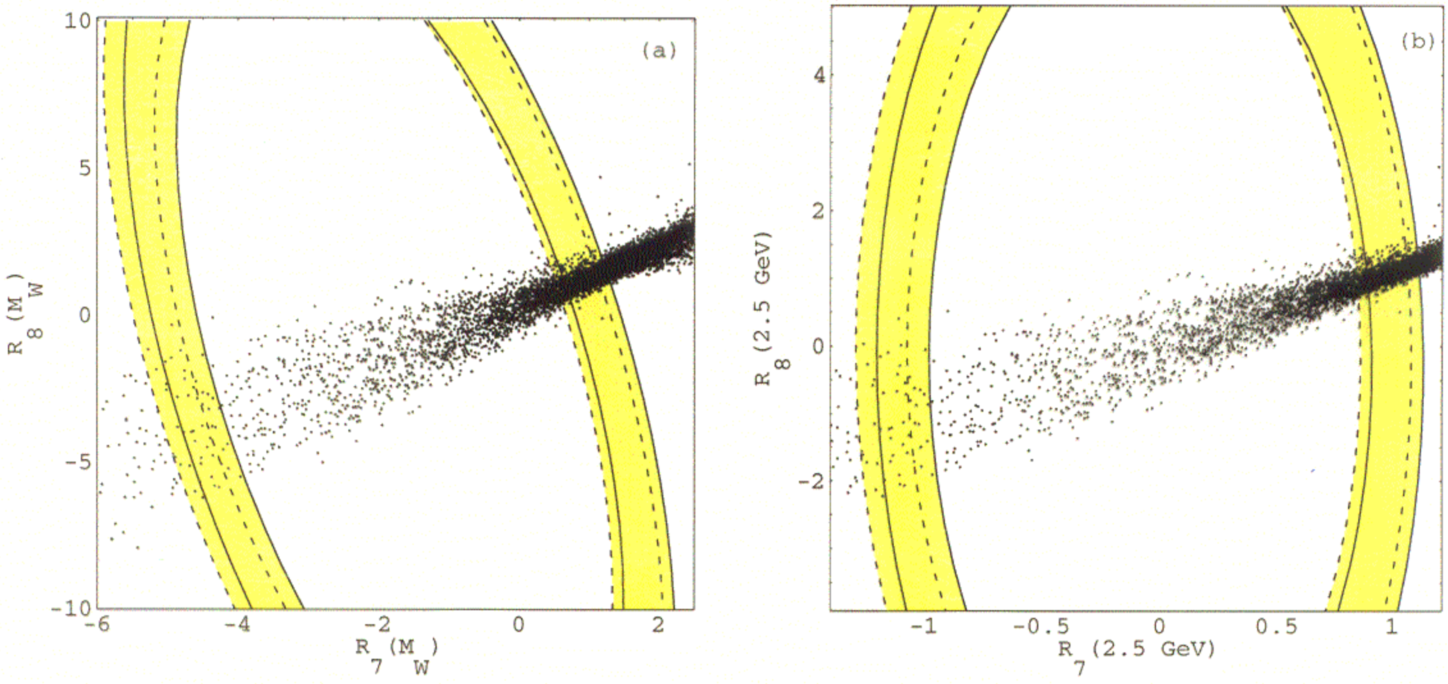


Figure 6: 90% C.L. **bounds in the** $[R_7(\mu), R_8(\mu)]$ **plane from the** $\mathcal{B}(B \to X_s\gamma)$ **for two choices of** m_c/m_b**.** $\mu = m_W$ **(left-hand plot) and** $\mu = 2.5$ **GeV (right-hand plot). The scattered points are generated in the SUSY-MFV model.**

$$\begin{cases} m_c/m_b = 0.29: & A_7^{tot}(2.5\ \mathbf{GeV}) \in [-0.37, -0.18]\ \&\ [0.21, 0.40]\ , \\ m_c/m_b = 0.22: & A_7^{tot}(2.5\ \mathbf{GeV}) \in [-0.35, -0.17]\ \&\ [0.25, 0.43]\ . \end{cases}$$

$\Longrightarrow$

$$A_7^{tot} - \text{negative}: -0.37 \leq A_7^{tot,<0}(2.5\ \mathbf{GeV}) \leq -0.17$$
$$A_7^{tot} - \text{positive}: 0.21 \leq A_7^{tot,>0}(2.5\ \mathbf{GeV}) \leq 0.43$$

- **Data allows a larger range for** $R_8(2.5\ \text{GeV})$

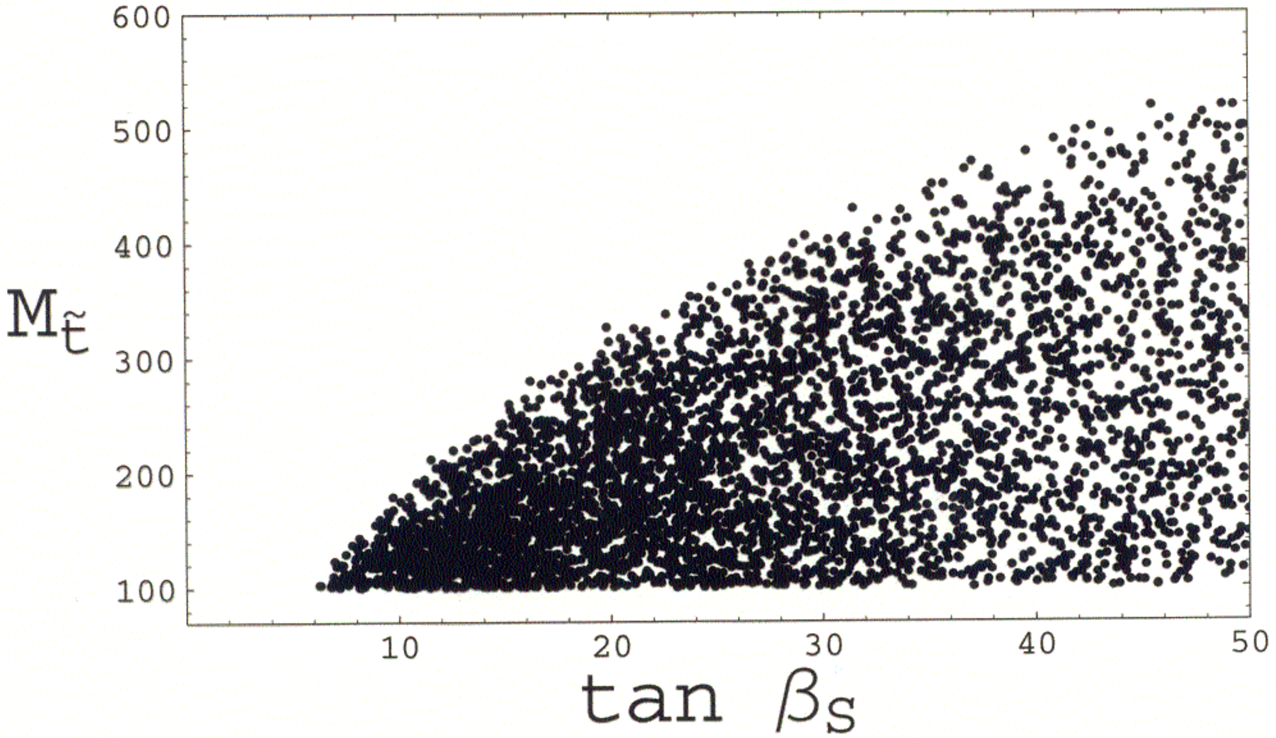


Figure 4: *Correlation between* $\tan\beta_S$ *and* $M_{\tilde{t}}$ *in the MFV-SUSY model for the* $C_7^s > 0$ *scenario.*

[A.A., Lunghi, Greub, Hiller; DESY 01-217; hep-ph/0112300]

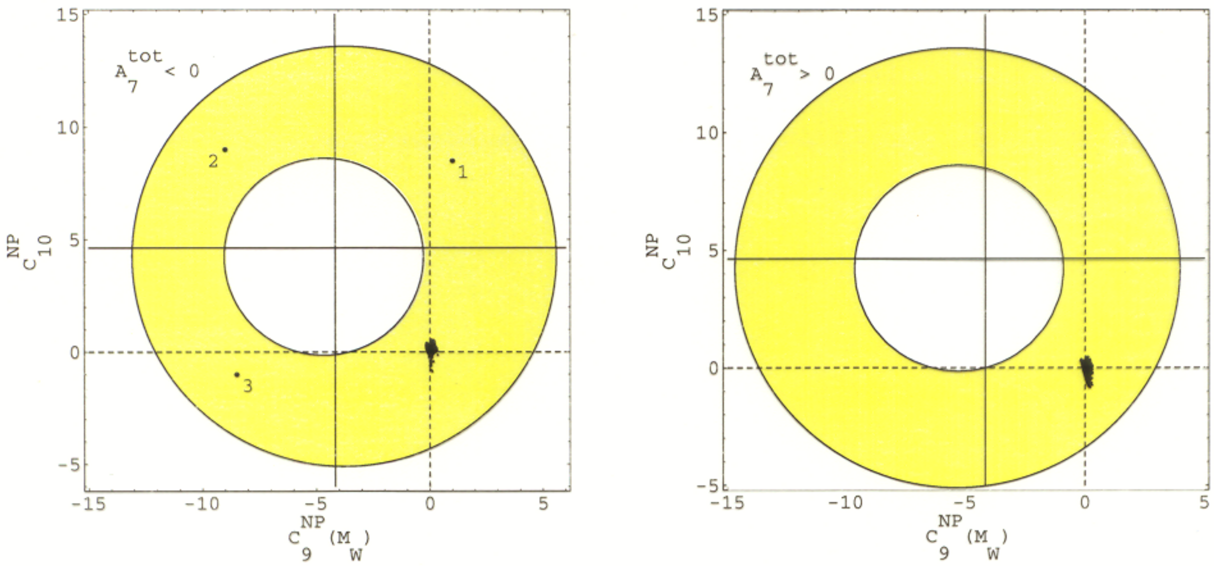


Figure 7: NNLO Case. Superposition of all the constraints from radiative and semileptonic rare decays (Points refer to the SUSY-MFV Model)

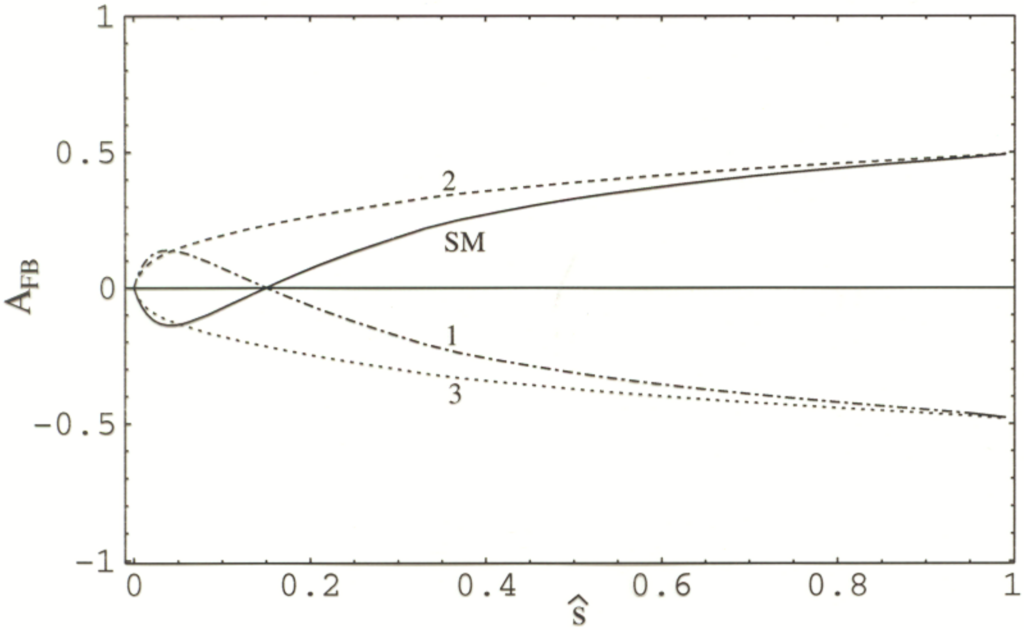


Figure 8: Differential Forward–Backward asymmetry for $B \to X_s \ell^+ \ell^-$. The four curves correspond to the points indicated above

Summary

- SM is in comfortable agreement with data on $B \to X_s\gamma$; Supersymmetric theories in agreement as well!; current theoretical uncertainty dominated by the quark masses; theoretical precision requires $O(\alpha_s)^2$ corrections

- Radiative and Semileptonic decays $B \to (K^*, \rho)\gamma$ and $B \to (K^*, \rho)\ell^+\ell^-$ provide excellent testing grounds for ideas on QCD factorization

- $\mathcal{B}(B \to \rho\gamma)$, Isopspin violating asymmetry $\Delta(\rho\gamma)$ and Direct CP-Asymmetries $\mathcal{A}_{\rm CP}(\rho\gamma)$ will lead to complementary constriants on the CKM parameters

- $\mathcal{B}(B \to \rho\gamma)$, $\Delta(\rho\gamma)$ and $\mathcal{A}_{\rm CP}(\rho\gamma)$ are also promising observables to search for physics beyond the SM

- $B \to X_s\ell^+\ell^-$ under theoretical control; the next frontier in Rare B-Decays!

- SM is in agreement with the present measurements in semileptonic rare B-decays $B \to (X_s, K^*, K)\ell^+\ell^-$; Form Factors an issue!

- Dilepton invariant mass distribution and Forward-Backward asymmetry crucial measurements in semileptonic rare B-decays
  $\Longrightarrow$ precise determination of Wilson coefficients
  $\Longrightarrow$ Precision tests of SM in flavour physics, or discovery of BSM-Physics; Supersymmetry is a case in point