Charm and Bottom Baryon Masses in the 1/N Expansion

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Large-N_c Baryons

- There is a *bona fide* spin-flavor symmetry for baryons in large-N_c limit of QCD. Large-N_c baryons form irreducible representations of spin-flavor algebra.
- Spin-flavor symmetry broken for baryons with finite N_c by $1/N_c$ corrections. Spin-flavor structure of baryons can be computed in a systematic expansion in $1/N_c$.
- Symmetry relations are obtained at various orders in $1/N_c$ by neglecting subleading $1/N_c$ corrections.

QCD Baryons: $N_c = 3$

- $1/N_c$ expansion useful for QCD baryons with $N_c = 3$. Suppression factors $1/N_c = 1/3$ evident in experimental data
- $SU(3)$ flavor symmetry breaking $\epsilon \sim m_s/\Lambda_\chi$ comparable to $1/N_c = 1/3$ and cannot be neglected.
- Heavy quark baryons in HQET: heavy quark spin-flavor symmetry breaking $\Lambda_{\rm QCD}/m_Q$, as well as $1/N_c$ and $SU(3)$ flavor breaking ϵ

(Qqq) Baryon Spin-Flavor Reps

$$J = \frac{1}{2} : J_\ell = 0, J_Q = \frac{1}{2}, \text{flavor } \bar{3} \quad \Lambda_c(cud), \Xi_c(csq)$$

$$J = \frac{1}{2} \oplus \frac{3}{2} : J_\ell = 1, J_Q = \frac{1}{2}, \text{flavor } 6 \quad \Sigma_c^{(*)}, \Xi_c'^{(*)}, \Omega_c^{(*)}(c$$

Experimental Status vs. Theory

- All singly charmed baryon masses measured except spin-$\frac{3}{2}$ Ω_c^*. $1/N_c$ analysis successfully predicted $\Xi_c' = 2580.8 \pm 2.1$ MeV. Expt. $\Xi_c' = 2576.5 \pm 2.3$ MeV.
- Compute charm baryon mass hierarchy using $\Omega_c^* = 2770.7 \pm 5.9$ MeV extracted from $\frac{1}{4}\left[(\Sigma_c^* - \Sigma_c) - 2\left(\Xi_c^* - \Xi_c'\right) + (\Omega_c^* - \Omega_c)\right] = 0$
- Only Λ_b^0 measured. Can predict all bottom baryon masses to ± 8 MeV from charm baryon masses. Many bottom splittings predicted to sub-MeV accuracy.

Spin-flavor Symmetry of HQ Baryons

- $SU(6)_\ell$ symmetry in $N_c \to \infty$ limit
  generators: J_ℓ^i, T^a, G^{ia} suppressed by $1/N_c$
- $SU(2)_Q$ symmetry in $m_Q \to \infty$ and $N_c \to \infty$
  generator: J_Q^i suppressed by $1/N_c m_Q$.
- For two heavy quark flavors $Q = c$ and $Q = b$, heavy quark spin-flavor symmetry generalizes to spin-flavor $SU(4)_Q$. Can relate spin-flavor properties of charm and bottom baryons.

$1/N_c$ Operator Expansion for Baryons

$$(\mathcal{O}_m)_{\rm QCD} = (N_c)^m \sum_{n=0}^{N_c} \left(\frac{1}{N_c}\right)^n c_n\, \mathcal{O}_n$$

- $\mathcal{O}_n$ all independent n-body operators in same spin $\otimes$ flavor representation as QCD operator. Matrix elements of $\mathcal{O}_n$ known.
- Coefficients c_n are unknown, but $O(1)$ at leading order in $1/N_c$
- Order in $1/N_c$ of each operator $\mathcal{O}_n$ known.

HQET Mass Expansion

$$M(H_Q) = m_Q + \bar{\Lambda} - \frac{\lambda_1}{2m_Q} - d_H \frac{\lambda_2}{2m_Q} + \mathcal{O}\left(\frac{1}{m_Q^2}\right)$$

$$\lambda_1 = \langle H_Q(v)|\bar{Q}_v (iD)^2 Q_v|H_Q(v)\rangle$$

$$d_H \lambda_2 = \frac{1}{2} Z_Q \langle H_Q(v)|\bar{Q}_v g G_{\mu\nu}\sigma^{\mu\nu} Q_v|H_Q(v)\rangle$$

$$d_H = -4\left(J_\ell \cdot J_Q\right)$$

HQET Baryon Mass in $1/N_c$

$$T_Q = m_Q + \bar{\Lambda}_T - \frac{\lambda_{1T}}{2m_Q} + \cdots$$

$$S_Q = m_Q + \bar{\Lambda}_S - \frac{\lambda_{1S}}{2m_Q} - \frac{4\lambda_{2S}}{2m_Q} + \cdots$$

$$S_Q^* = m_Q + \bar{\Lambda}_S - \frac{\lambda_{1S}}{2m_Q} + \frac{2\lambda_{2S}}{2m_Q} + \cdots$$

$$\bar{\Lambda} = N_c \mathbb{1} + \frac{J_\ell^2}{N_c}, \qquad \frac{\lambda_1}{2m_Q} = \frac{1}{m_Q} N_Q + \frac{1}{N_c^2}\frac{1}{m_Q} N_Q J_\ell^2$$

$$-d_H \frac{\lambda_2}{2m_Q} = \frac{1}{N_c}\frac{1}{m_Q}\left(J_\ell \cdot J_Q\right)$$

(Qqq) Mass Splittings

Operator	Mass Splitting	$1/m_Q$	$1/N_c$	Expt. $Q=c$
1	Λ_Q	*	*	2284.9 ± 0.6 MeV
J_ℓ^2	$\frac{1}{3}\left(\Sigma_Q + 2\Sigma_Q^*\right) - \Lambda_Q$	1	$1/N_c$	210.8 ± 0.9 MeV
$J_\ell \cdot J_Q$	$\Sigma_Q^* - \Sigma_Q$	$1/m_Q$	$1/N_c$	65.6 ± 1.1 MeV

$$* = m_Q + N_c \Lambda$$

(Qqq) vs. (qqq)

$$M_{(qqq)} = N_c \mathbf{1} + \frac{1}{N_c} J_\ell^2$$

$$M_{(Qqq)} = N_c \mathbf{1} + N_Q m_Q + \frac{1}{N_c} J_\ell^2 + \frac{1}{N_c^2 m_Q} N_Q J_\ell^2 + \frac{1}{N_c m_Q} (J_\ell \cdot J_Q)$$

Operator	Mass Splitting	$1/m_Q$	$1/N_c$	Expt. $Q = c$
N_Q	$*$	m_Q	1	1420.2 ± 0.7 MeV
$N_Q J_\ell^2$	$*$	$1/m_Q$	$1/N_c^2$	16 ± 1 MeV

$* = \Lambda_Q - \frac{1}{4} (5N - \Delta)$

$* = \left[\frac{1}{3} \left(\Sigma_Q + 2\Sigma_Q^*\right) - \Lambda_Q\right] - \frac{2}{3} (\Delta - N)$

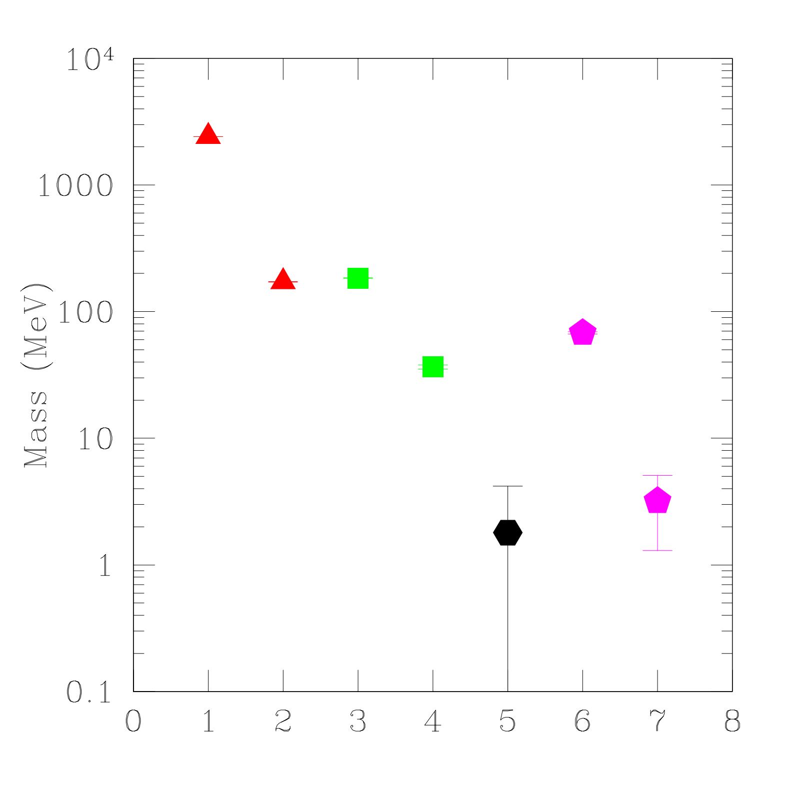
Charm Baryon Mass Hierarchy

Operator	$1/m_Q$	$1/N_c$	Flavor	Expt. $Q = c$
1	$*$	$*$	1	2407.7 ± 0.7 MeV
J_ℓ^2	1	$1/N_c$	1	172.1 ± 1.1 MeV
T^8	1	$1/N_c$	ϵ	184.2 ± 1.2 MeV
$J_\ell^i G^{i8}$	1	$1/N_c$	ϵ	36.6 ± 1.3 MeV
$\{T^8, T^8\}$	1	$1/N_c$	ϵ^2	1.8 ± 2.4 MeV
$(J_\ell \cdot J_Q)$	$1/m_Q$	$1/N_c$	1	68.1 ± 1.4 MeV
$(J_\ell \cdot J_Q)\, T^8$	$1/m_Q$	$1/N_c^2$	ϵ	3.2 ± 1.9 MeV
$J_Q^i \{T^8, G^{i8}\}$	$1/m_Q$	$1/N_c^2$	ϵ^2	0.0 ± 1.9 MeV

Charm Baryon Mass Hierarchy

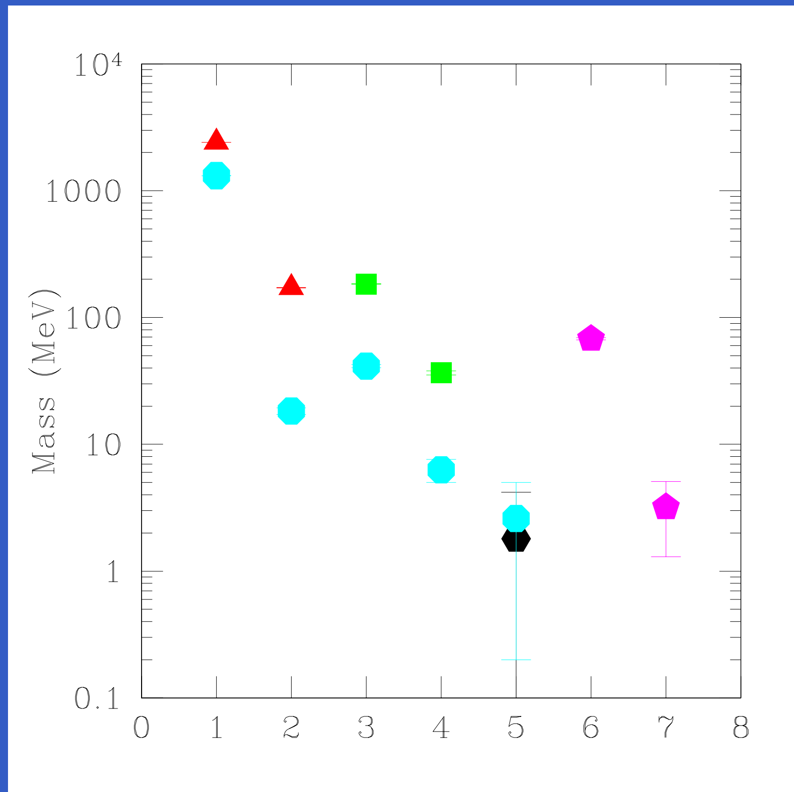
Operator	Mass Splitting
1	$\frac{1}{3}\left(\Lambda_Q + 2\Xi_Q\right)$
J_ℓ^2	$-\frac{1}{3}\left(\Lambda_Q + 2\Xi_Q\right) + \frac{1}{6}\Big[\Big(\Sigma_Q + 2\Sigma_Q^*\Big) + \frac{2}{3}\Big(\Xi'_Q + 2\Xi_Q^*\Big) + \frac{1}{3}\Big(\Omega_Q + 2\Omega_Q^*$
T^8	$\Xi_Q - \Lambda_Q$
$J_\ell^i G^{i8}$	$-\frac{5}{8}\left(\Lambda_Q - \Xi_Q\right) + \frac{1}{8}\Big[\Big(\Sigma_Q + 2\Sigma_Q^*\Big) - \frac{1}{3}\Big(\Xi'_Q + 2\Xi_Q^*\Big) - \frac{2}{3}\Big(\Omega_Q + 2\Omega_Q^*\Big)$
$\{T^8, T^8\}$	$-\frac{1}{2}\left[\frac{1}{3}\left(\Sigma_Q + 2\Sigma_Q^*\right) - \frac{2}{3}\left(\Xi'_Q + 2\Xi_Q^*\right) + \frac{1}{3}\left(\Omega_Q + 2\Omega_Q^*\right)\right]$
$\left(J_\ell \cdot J_Q\right)$	$\frac{1}{6}\left[3\left(\Sigma_Q^* - \Sigma_Q\right) + 2\left(\Xi_Q^* - \Xi'_Q\right) + \frac{1}{3}\left(\Omega_Q^* - \Omega_Q\right)\right]$
$\left(J_\ell \cdot J_Q\right) T^8$	$-\frac{1}{6}\left[3\left(\Sigma_Q^* - \Sigma_Q\right) - \left(\Xi_Q^* - \Xi'_Q\right) - 2\left(\Sigma_Q^* - \Sigma_Q\right)\right]$
$J_Q^i\left\{T^8, G^{i8}\right\}$	$\frac{1}{4}\left[\left(\Sigma_Q^* - \Sigma_Q\right) - 2\left(\Xi_Q^* - \Xi'_Q\right) + \left(\Omega_Q^* - \Omega_Q\right)\right]$

Charm Baryon Mass Splittings



$$N_c \Lambda + m_Q : \frac{\Lambda}{N_c} : \epsilon : \frac{\epsilon}{N_c} : \frac{\epsilon^2}{N_c} : \frac{\Lambda^2}{N_c m_Q} : \frac{\epsilon \Lambda}{N_c^2 m_Q}$$

Charm Baryon Mass Splittings



$$m_Q : \frac{\Lambda^2}{N_c^2 m_Q} : \frac{\epsilon\Lambda}{N_c m_Q} : \frac{\epsilon\Lambda}{N_c^2 m_Q} : \frac{\epsilon^2\Lambda}{N_c^2 m_Q} : \frac{\Lambda^2}{N_c m_Q} : \frac{\epsilon\Lambda}{N_c^2 m_Q}$$

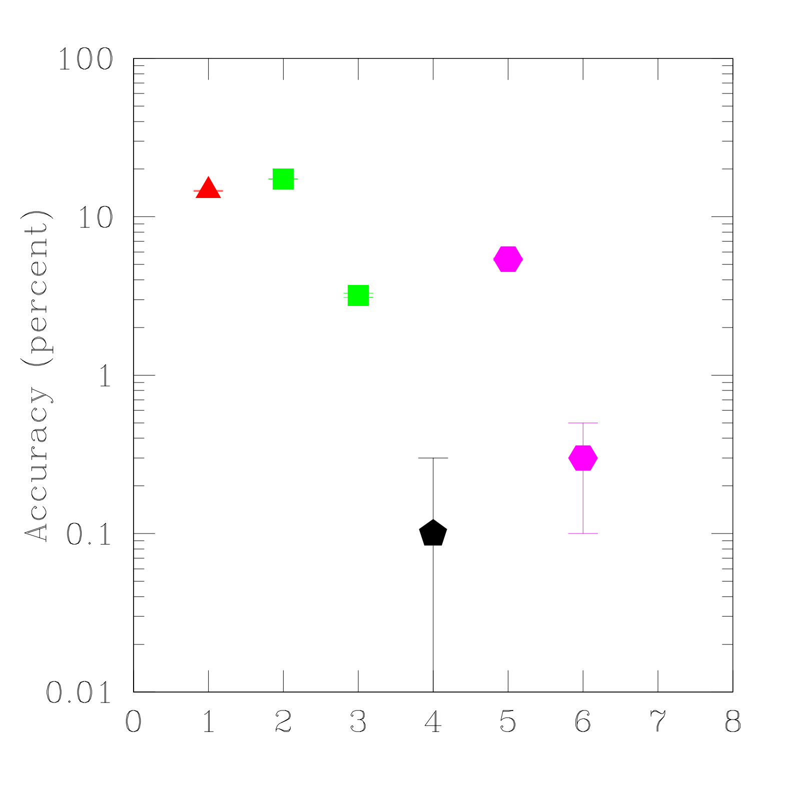
Charm Baryon Mass Hierarchy

Mass Splitting	$1/m_Q$	$1/N_c$	Flavor	Expt. $Q = c$
J_ℓ^2	1	$1/N_c^2$	1	$14.6 \pm 0.1\%$
T^8	1	$1/N_c$	ϵ	$17.3 \pm 0.1\%$
$J_\ell^i G^{i8}$	1	$1/N_c^2$	ϵ	$3.2 \pm 0.1\%$
$\{T^8, T^8\}$	1	$1/N_c^2$	ϵ^2	$0.1 \pm 0.2\%$
$J_\ell \cdot J_Q$	$1/m_Q$	$1/N_c^2$	1	$5.4 \pm 0.1\%$
$(J_\ell \cdot J_Q)\, T^8$	$1/m_Q$	$1/N_c^3$	ϵ	$0.3 \pm 0.2\%$

Define $\bar{M} = M - m_Q$. Expt. Accuracy = $\frac{|\sum M_i|}{\sum |\bar{M}_i|/2}$

Theory: $1/N_c^2$ Expt: $(14.6 \pm 0.1)\%$

Charm Baryon Mass Hierarchy

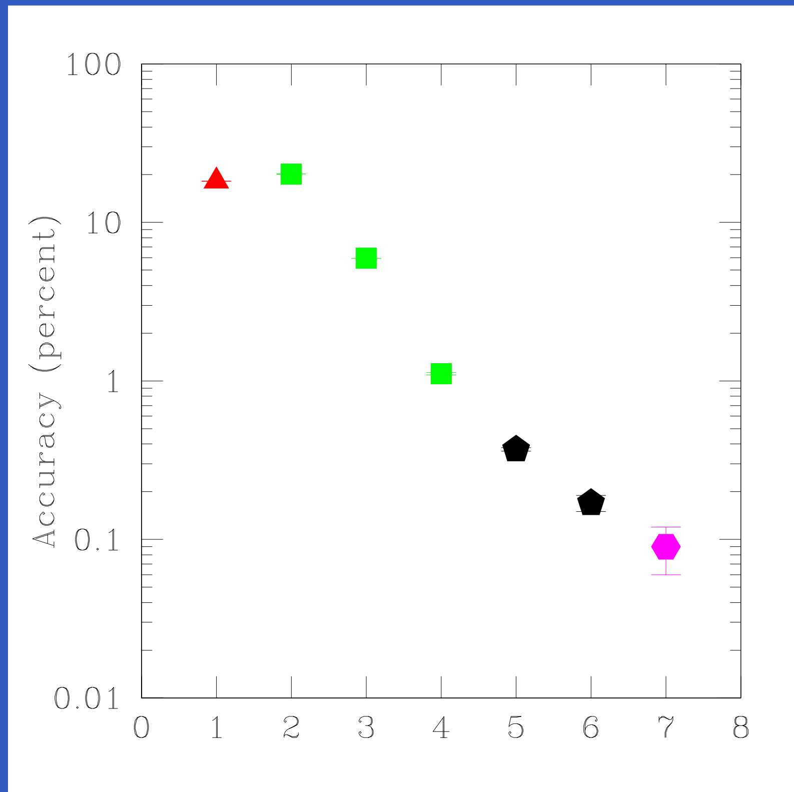


$$\frac{1}{N_c^2} : \frac{\epsilon}{N_c} : \frac{\epsilon}{N_c^2} : \frac{\epsilon^2}{N_c^2} : \frac{\Lambda}{N_c^2 m_Q} : \frac{\epsilon\Lambda}{N_c^3 m_Q}$$

(qqq) Baryon Mass Hierarchy

Mass Splitting	$1/N_c$	Flavor	Expt.
$\frac{5}{8}(2N + 3\Sigma + \Lambda + 2\Xi) - \frac{1}{10}(4\Delta + 3\Sigma^* + 2\Xi^* + \Omega)$	N_c	1	*
$\frac{1}{8}(2N + 3\Sigma + \Lambda + 2\Xi) - \frac{1}{10}(4\Delta + 3\Sigma^* + 2\Xi^* + \Omega)$	$1/N_c$	1	$18.21 \pm 0.03\%$
$\frac{5}{2}(6N - 3\Sigma + \Lambda - 4\Xi) - (2\Delta - \Xi^* - \Omega)$	1	ϵ	$20.21 \pm 0.02\%$
$\frac{1}{4}(N - 3\Sigma + \Lambda + \Xi)$	$1/N_c$	ϵ	$5.94 \pm 0.01\%$
$\frac{1}{2}(-2N - 9\Sigma + 3\Lambda + 8\Xi) + (2\Delta - \Xi^* - \Omega)$	$1/N_c^2$	ϵ	$1.11 \pm 0.02\%$
$\frac{5}{4}(2N - \Sigma - 3\Lambda + 2\Xi) - \frac{1}{7}(4\Delta - 5\Sigma^* - 2\Xi^* + 3\Omega)$	$1/N_c$	ϵ^2	$0.37 \pm 0.01\%$
$\frac{1}{2}(2N - \Sigma - 3\Lambda + 2\Xi) - \frac{1}{7}(4\Delta - 5\Sigma^* - 2\Xi^* + 3\Omega)$	$1/N_c^2$	ϵ^2	$0.17 \pm 0.02\%$
$\frac{1}{4}(\Delta - 3\Sigma^* + 3\Xi^* - \Omega)$	$1/N_c^2$	ϵ^3	$0.09 \pm 0.03\%$

(qqq) Baryon Mass Hierarchy



$$\frac{1}{N_c^2} : \frac{\epsilon}{N_c} : \frac{\epsilon}{N_c^2} : \frac{\epsilon}{N_c^3} : \frac{\epsilon^2}{N_c^2} : \frac{\epsilon^2}{N_c^3} : \frac{\epsilon^3}{N_c^3}$$

Conclusions

- $1/N_c$ expansion useful and predictive for QCD baryons
- $1/N_c$ hierarchy evident in (qqq) and (Qqq) baryon masses.
- $1/N_c$ expansion gives a quantitative understanding of spin-flavor symmetry breaking for baryons. Intricate pattern of spin-flavor breaking since $1/N_c$, $SU(3)$ breaking, and heavy quark symmetry breaking (Λ/m_Q) for $Q = c$ are all comparable.