# NEUTRINO OSCILLATIONS AND WARPED GEOMETRY

(with S. Huber (+ C.A. Lee))

- INTRODUCTION / MOTIVATION
- 5D WARPED MODEL → Dirac $\nu$ → Majorana $\nu$
- CONCLUSION

Q. Shafi

- Extra Dimensions may lead to 'deeper' unification than ordinary grand unification

(e.g. thru inclusion of gravity;

unification of families;

matter/higgs/gauge sector unification;)

resolution of the gauge hierarchy problem;

- Most promising approach so far is M-theory (whose low energy limit is presumably 11-D SUGRA)

contains all known 10-D superstring theories

- But new possibilities (e.g. warped dimension)

# Why 'Warped' Dimension ?

- Could provide resolution of the gauge hierarchy problem (Randall Sundrum)

  ($\text{how } M_W \ll M_{Planck}$),

  without invoking SUSY ;

  (may also generate other 'small' hierarchies, e.g. $\rho_v \sim 10^{-120} M_P^4$)

- Sheds new light on fermion mass hierarchies and mixings ;

- Allows one to accomodate the observed $\nu$ oscillations even within The framework of the SM ;

- May be consistent with grand unification

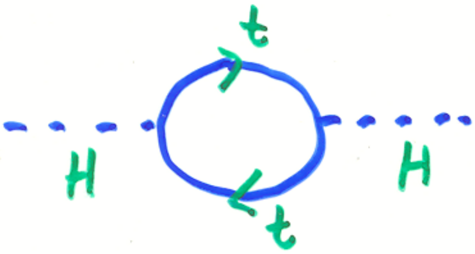
- Can be experimentally tested.
  (hopefully at the LHC)

# Gauge Hierarchy Problem

$$M_{planck} \gg M_W$$

$M_{planck}$: $\sim 10^{19}$ GeV

$M_W$: $\sim 10^{2}$ GeV

But 


$$\delta M_H^2 \sim \Lambda^2 \sim M_{Planck}^2$$

$\Rightarrow$ fine tuning needed to generate $M_W$

Proposed solutions include :

- SUSY (adds new diagrams, TeV partic
- Extra dimensions
- Use both (cf: strings)

# EXTRA DIMENSION(S)

## WHO NEEDS THEM?

- Unification of forces (Kaluza-Klein)

Consider 5 dimensional gravity

metric tensor $\rightarrow g_{AB}$ , $A, B = 0, 1, \ldots, 4$

Dimensional reduction to $M_4 \times S^1$:

$$g_{\mu\nu} \; , \quad g_{\mu 4} \sim A_\mu \, , \quad g_{44}$$

graviton ($\mu,\nu = 0,\ldots,3$) — EM field (!) — scalar

- Electric charge quantized ($e \propto \frac{1}{M_{pl} R}$
- Monopoles
- 'Higgs' mechanism
- Tower of new states (esp. graviton)

# WARPED GEOMETRY

Randall
Sundrum
⋮

$$S = M_5^3 \int d^5x \sqrt{-G}\,(R - \Lambda) + \int d^4x \sqrt{-g}\,(-V_b$$

5d gravity + cosm. const

Source term for domain wall at $x_5 = 0$

Ansatz: $ds^2 = e^{2A(x_5)} \eta_{\mu\nu} dx^\mu dx^\nu + dx_5^2$

$(g_{\mu\nu} = G_{MN}(x_5 = 0)\,\delta_\mu^M \delta_\nu^N$

One finds:

$$6A'^2 + \frac{1}{2}\Lambda = 0$$

$$3A'' + \frac{1}{2} V \delta(x_5) = 0$$

With $\Lambda < 0$, one finds

$$A = \pm k x_5 \ , \quad k = \sqrt{-\Lambda/12}$$

$$3\Delta(A') = -\frac{1}{2}V \ .$$

So

$$A = -k x_5 \ , \quad x_5 > 0$$

$$A = k x_5 \ , \quad x_5 < 0$$

$$V = 12k \ ,$$

& we have

AdS curvature scale

$$ds^2 = e^{-2k|x_5|}\eta_{\mu\nu}dx^\mu dx^\nu + dx_5^2 \ ;$$

peaked at $x_5 = 0$

Naive reduction to 4 dim by integrating over $x_5$ yields

$$M_4^2 = M_5^3 \int dx_5 e^{-2k|x_5|} < \infty$$

# Do we obtain 4d gravity? (on the brane)

Despite the gapless KK spectrum, observer on the brane effectively sees 4d gravity. This is $\because$ most of the bulk KK modes have wave functions with support far from $x_5 = 0$ $\Rightarrow$ brane fields & localized graviton couple only very weakly to the bulk continuum.

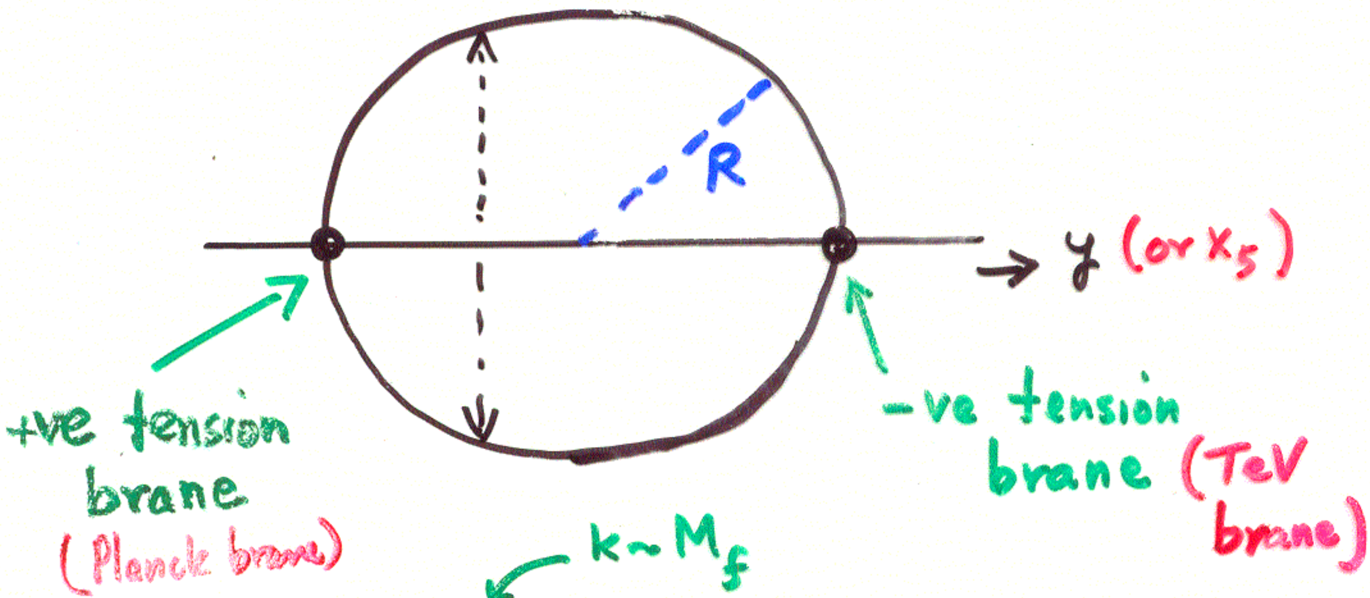
Thus, $V(r) \sim 1/r$ receives small $1/r^3$ corrections. $V \sim \frac{Gm}{r}\left(1 \pm \left(\frac{1}{k^2 r^2}\right)\right)$

# WARPED GEOMETRY

(MODEL BUILDING)

Randall
Sundrum
Gogberashvili
:

One extra dimension which corresponds to the orbifold $S^1/Z_2$




$k \sim M_5$

$$ds^2 = \underbrace{e^{-2k|y|}}_{\text{warp factor}} \eta_{\mu\nu} dx^\mu dx^\nu + dy^2$$

$$M^2_{Planck} = \frac{M_5^3}{k}\left(1 - e^{-2kR\pi}\right) \qquad \boxed{kR \sim 10}$$

$$v = v_0 e^{-kR\pi} \sim \text{TeV} \, (!)$$

vev in 4d effective theory

VEV in 5 dim

## Original Proposal

All SM fields reside on the TeV brane; only gravity feels the extra dimension

$\Rightarrow$ hierarchy problem under control!
(scale on TeV-brane ~ TeV)

BUT

Difficulty with non-renormalizabe operators:

$$\frac{1}{M_{Pl}^2}\bar{\psi}\psi\bar{\psi}\psi \longrightarrow \frac{1}{(TeV)^2}\bar{\psi}\psi\bar{\psi}\psi$$

- → Rapid p decay
- → Large FCNC
- → Large neutrino masses ($\gg$ eV)
- → Fermion Mass hierarchies & mixings ?

# WAY OUT ?

- Symmetries ?

- Permit SM fields to leave the brane ? (Higgs stays on the TeV brane)

unless SUSY invoked

Ghergetta
Pomarol
Hewitt et al
Huber + QS

5d Bulk field $\Longrightarrow$ Tower of 4d fields

KK ansatz: $\Phi(x^\mu, y) = \sum_{n=0}^{\infty} \Phi^n(x^\mu) f_n(y)$

wave fns from field eqns

# Zero modes

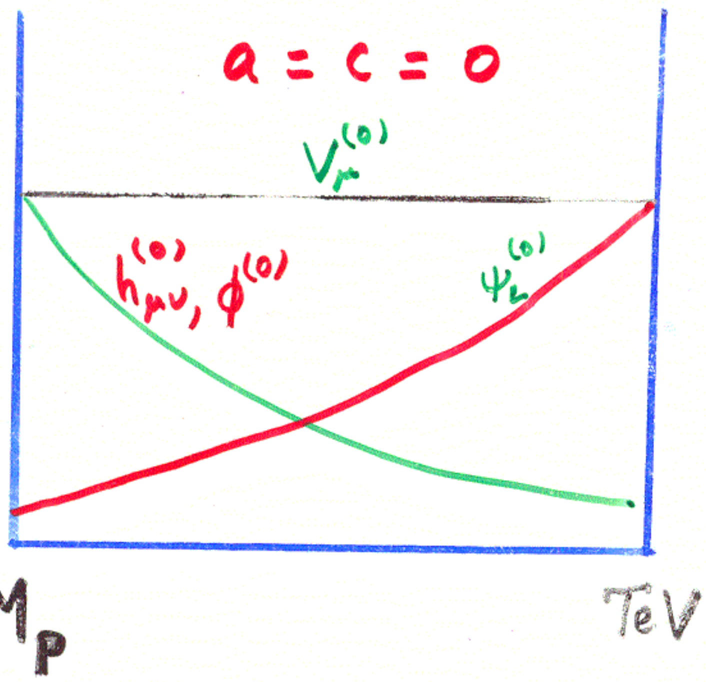
$$\phi^{(0)}(y) \sim e^{(1-\alpha)k|y|}$$ [ $m_\phi^2 = ak^2$ ; $\alpha = \sqrt{4+a}$ ]

$$\Psi_L^{(0)}(y) \sim e^{(\frac{1}{2}-c)k|y|}$$ [ $m_\Psi = ck\,\varepsilon(y)$ ]

5D bulk mass

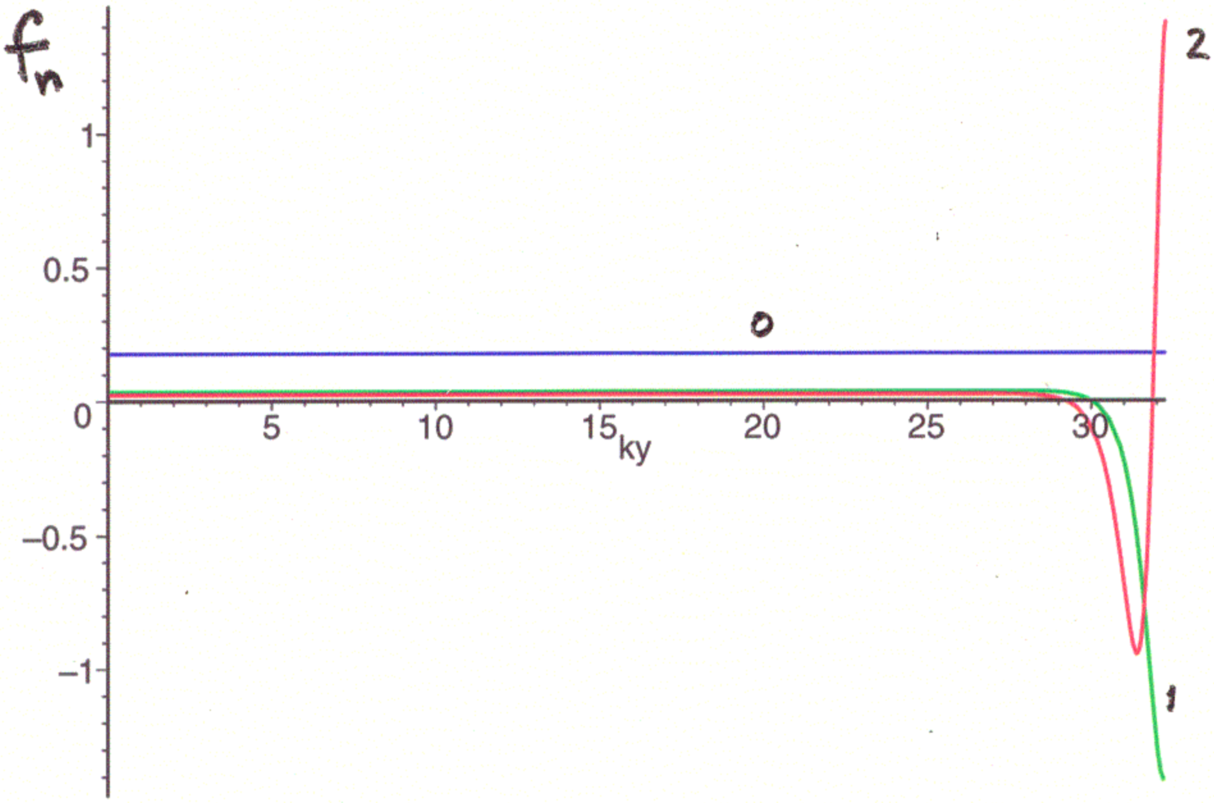
$$V_\mu^{(0)}(y) \sim \text{indep. of } y$$

$$h_{\mu\nu}^{(0)}(y) \sim e^{-k|y|}$$




$\phi^{(0)}$, $\Psi_L^{(0)}$ can localise on either brane (depends on a & c)

SM fields in Bulk (e.g. gauge bosons)




Planck brane

TeV-brane

(excited states near TeV brane!)

# Bounds on KK Gauge Bosons

(WARPED GEOMETRY)

- Higgs on the TeV-brane

- zero mode (gauge boson) becomes massive

→ wave function $f_0$ becomes $y$ dependent

($m_0 \neq 0 \Rightarrow f_0 \neq$ const)

In particular, $f_0$ reduced at the TeV-brane → $M_Z = m_0 \not\propto g^{(5)} \langle H \rangle$

SM relationship between $M_Z$, $M_W$ and coupling modified (deviations from SM $< 10^{-3}$ $\Rightarrow \langle H \rangle / k \lesssim 10^{-2}$)

⇒ bound on the TeV-brane KK scale

$$m_1 \sim k e^{-\pi k R} \gtrsim 10 \text{ TeV} \quad (\langle H \rangle / k \lesssim 10^{-2})$$

(could be somewhat smaller)

$\sim$ 7 TeV ← Huber, Lee, Q.S

(7 TeV)

# NEUTRINO OSCILLATIONS

Challenges:

(i) Origin of mass ?

(ii) Dirac or Majorana?

(iii) Relations to Quark masses/mixings?

'Bimaximal' mixing versus $\theta_{quark} << 1$;

Third lepton mixing angle small ;

Lepton mass ratios not as large as in quark sector;

<u>SM fermions</u> arise as 'zero' modes of the KK decomposition.

5d Dirac mass $m_\psi = c k \varepsilon(y)$

Grossmann
Neubert
Huber + QS
Gherghetta
Pomarol


(Note: $\psi(-y) = \pm \gamma_5 \psi(y)$)

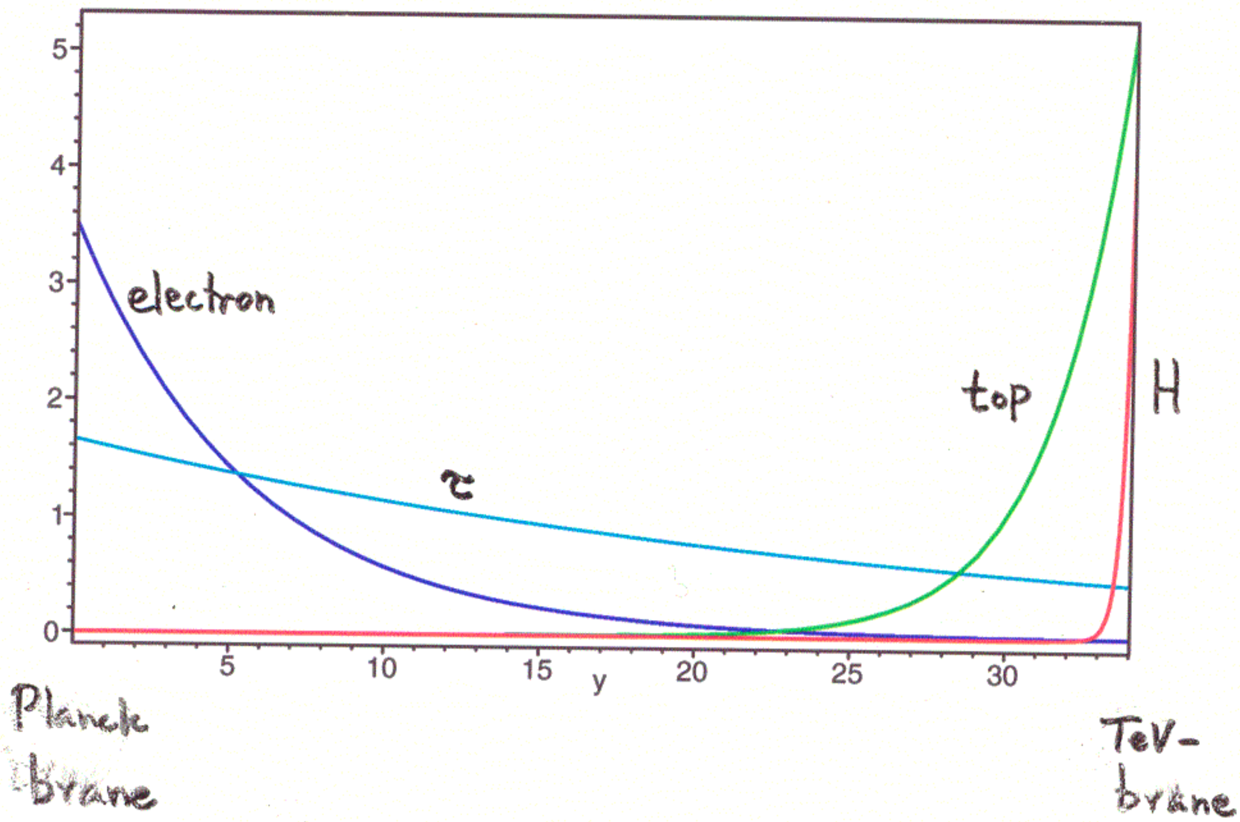
$m_n \sim n \pi k e^{-\pi k R}$ ← excited states

For $c > \frac{1}{2}$: zero mode localized (exponential) → Planck brane

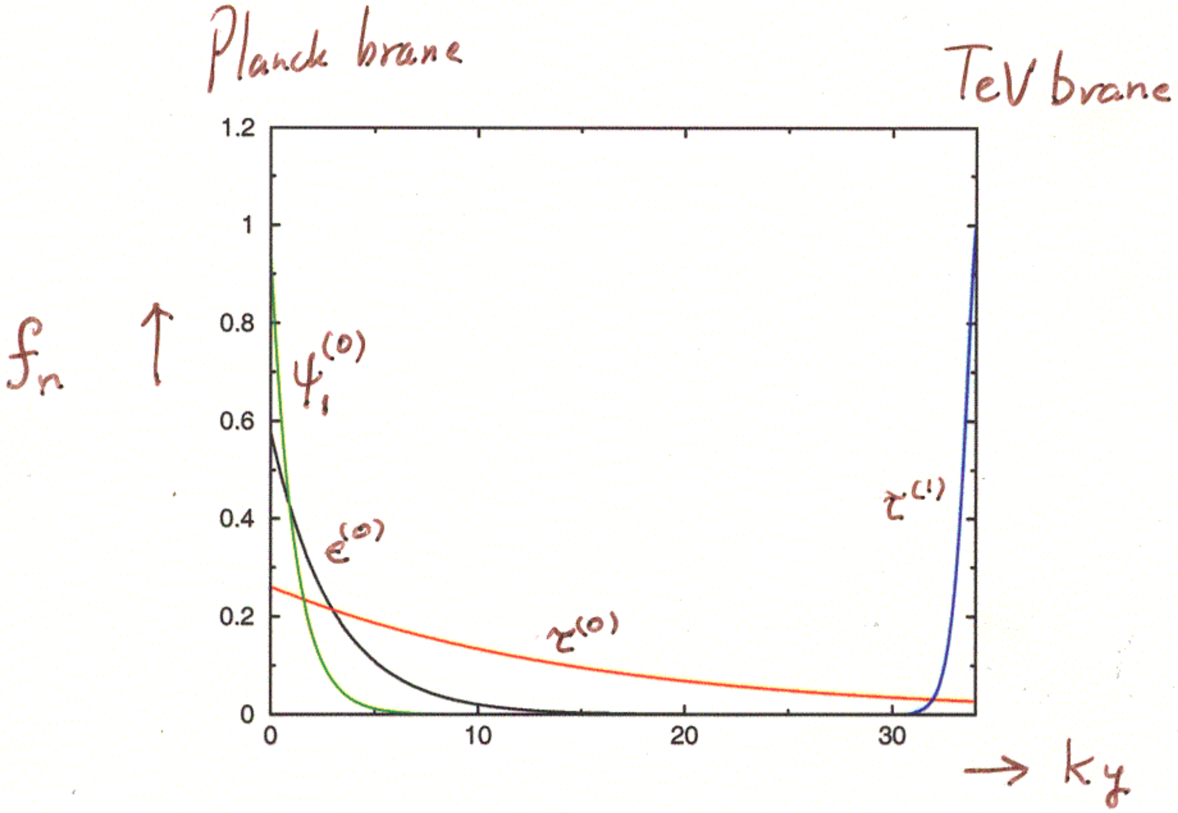
$c < \frac{1}{2}$: zero mode → TeV-brane

Fermion mass hierarchy: Determined by the overlap between the fermion wave functions and Higgs (on TeV brane)

E.g: $c(e) = 0.68$, $c(\mu) = 0.59$, $c(\tau) = 0.54$



5
4
3
2
1
0
5
10
15
20
25
30
y
electron
τ
top
H
Planck brane
TeV-brane



Planck brane
TeV brane
$f_n$
$\psi_1^{(0)}$
$e^{(0)}$
$\tau^{(0)}$
$\tau^{(1)}$
$k y$
1.2
1
0.8
0.6
0.4
0.2
0
0
10
20
30

## Neutrino Oscillations in RS

- 3 right-handed $\nu$'s in bulk (Dirac)
- Dirac mass from

$$\int d^4x \int dy \sqrt{-G}\ \frac{\lambda^{(5)}_{ij}}{\sqrt{k}}\ H\ \bar{L}^i\ \psi^j$$

$H$: TeV-brane

$\bar{L}^i$: 'Bulk'

$\psi^j$: RH-$\nu$ near Planck brane

!! (Dirac masses suppressed)

## Spectrum (flavor index suppressed)

SM $\nu$'s : KK-tower

$$\left\{ \begin{array}{llll} \nu_L^{(0)}, & \nu_L^{(1)}, & \nu_L^{(2)}, & \dots \\ -, & \nu_R^{(1)}, & \nu_R^{(2)}, & \dots \end{array} \right.$$

Sterile $\nu$'s (Right-handed)

$$\left\{ \begin{array}{llll} -, & \psi_L^{(1)}, & \psi_L^{(2)}, & \dots \\ \psi_R^{(0)}, & \psi_R^{(1)}, & \psi_R^{(2)}, & \dots \end{array} \right.$$

# Neutrino Mass Matrix

$$\begin{array}{cc} & \begin{array}{cccc} \psi_R^{(0)} & \nu_R^{(1)} & \psi_R^{(1)} & \cdots \end{array} \\ \begin{array}{c} \nu_L^{(0)} \\ \nu_L^{(1)} \\ \psi_L^{(1)} \\ \vdots \end{array} & \begin{pmatrix} \langle H \rangle & 0 & \langle H \rangle & \cdots \\ \langle H \rangle & M & \langle H \rangle & \cdots \\ 0 & \langle H \rangle & M & \\ \vdots & \vdots & & \ddots \end{pmatrix} \end{array}$$

KK scale

For solar $\nu$ both small and large mixing angle solutions can be realized, depending on how we choose the 'c' parameters.

# NEUTRINO MIXINGS

- SM neutrinos at different locations
  (e.g. to generate charged lepton masses)

  → 'expect' nearest neighbor-type mixing, small mixing angles;

  - Allow order 10 hierarchy in $\lambda_N$ to realize large atmospheric mixing small ∡ MSW solution;

- SM neutrinos at the same location;
  order unity $\lambda_N$

  → bimaximal scenario possible;

  (also LOW & vacuum solutions can be realized)

# Consider bimaximal mixing

$\nu_L$: $c_{e_L} = c_{\mu_L} = c_{\tau_L} = 0.57$ } ← Explains $m_\tau \gg m_\mu \gg m_e$

$(c_{e_R} = 0.79,\ c_{\mu_R} = 0.62,\ c_{\tau_R} = 0.50)$

$\psi_R$: $c_{\psi_1} = 1.43,\ c_{\psi_2} = 1.36,\ c_{\psi_3} = 1.30$

Large MSW

$$\lambda_{ij} = \begin{pmatrix} -2.0 & 1.5 & -0.5 \\ -1.8 & -1.1 & 1.9 \\ 0.5 & 1.9 & 1.7 \end{pmatrix}$$

5-d coupling

$\Delta m^2_{atm} = 5 \cdot 10^{-3}\ eV^2 \rightarrow$ atm $(\sim m_{\nu_1})$

$\Delta m^2_{sol} = 1. \ 10^{-4}\ eV^2 \rightarrow$ Solar $(\sim m_{\nu_2})$

$\sin^2 2\theta_{atm}$ $0.98 \rightarrow$ atm

$\sin^2 2\theta_{sol} \simeq 0.90 \rightarrow$ Solar

$[U^2_{e3} = 0.03$ close to exp bound

# See Saw Mechanism & Warped Geometry

$$\mathcal{L}_{Maj} = M_N \, N N$$

$$\mathcal{L}_{Dirac} = \lambda_N \, \nu_L \, N H$$

$$\Rightarrow \quad M_\nu = \lambda_N^2 \frac{\langle H \rangle^2}{M_N}$$

With $\lambda_N \lesssim$ unity, decent neutrino mass are realized for $M_N \sim 10^{14} - 10^{12}$ GeV.

In models with warped geometry containing a large 5D Majorana mass, it turns out that the wave function of the lowest state of the singlet neutrino is modified. This state, because of the warped geometry, is localized at the TeV brane, where it acquires mass $\sim m_{KK}$ ($\sim$ few TeV) $\Rightarrow M_N \sim m_{KK}$ (need something more sophisticated)

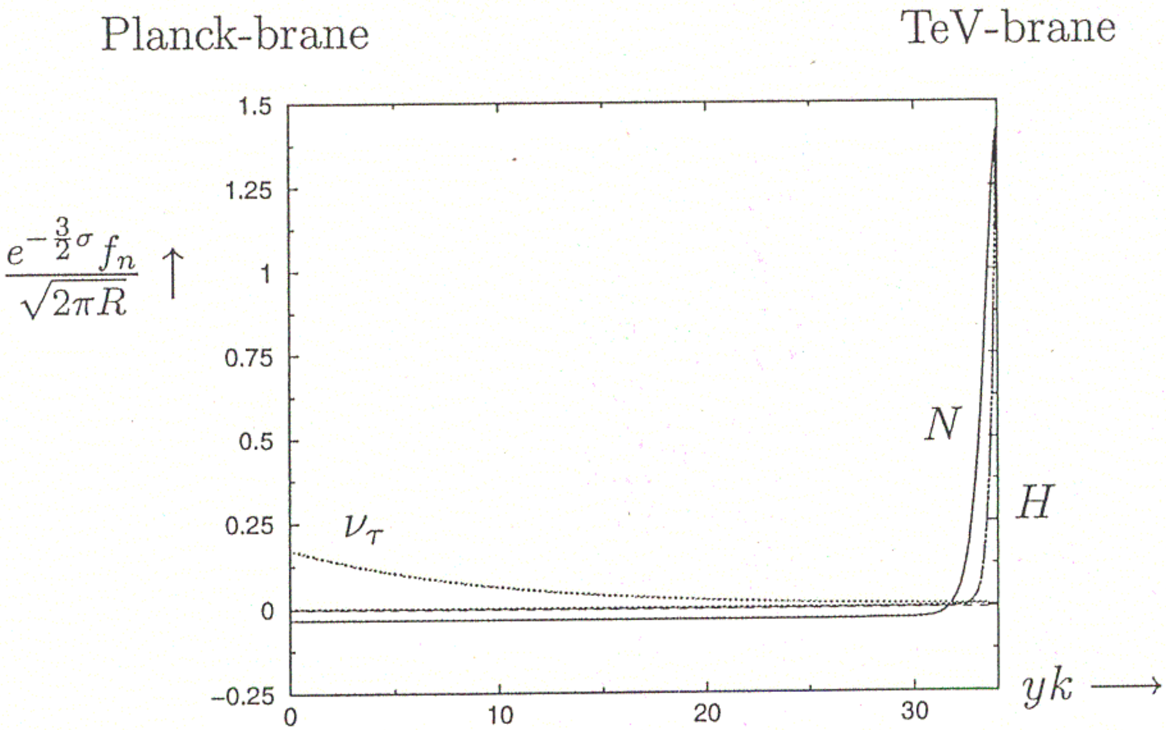



Figure 1: Wave functions of the $\nu_\tau$ and $N$ zero modes, together with a sketch of the Higgs profile.

Consider the dimension five operator

$$\int d^4x \int dy \sqrt{-g}\, \frac{l_{ij}}{M_5^2} H^2\, \Psi_i\, C\, \Psi_j$$

Charge conjugation

$$\equiv \int d^4x\; M_{ij}^{(\nu)}\; \Psi_i^{(0)}\; C\; \Psi_j^{(0)} \,,$$

$$M_{ij}^{(\nu)} = \int_{-\pi R}^{+\pi R} \frac{dy}{2\pi R}\, \frac{l_{ij}}{M_5^2}\, e^{-4\sigma(y)}\, H^2(y)\, f_{0i}^{(\nu)}(y)\, f_{0j}^{(\nu)}(y) \,.$$

With SM fields in the bulk, dim 5 $\nu$ masses can become unacceptably large if they stray too close to the TeV brane.

A careful choice of $k/M_5$ ($\sim 0.01$) and $l$ ($\sim 10^{-3}$) can yield neutrino masses in the desired range.

Resolution of Neutrino Puzzles follows the 'Neutrino Mass Anarchy' Models : (Hall, Murayama, Weiner)

Mass Matrices with randomly chosen entries of order unity have a large probability to fit the neutrino data.

Consider, for example, the matrix

$$M^{(\nu)} \sim \begin{pmatrix} \epsilon^2 & \epsilon & \epsilon \\ \epsilon & 1 & 1 \\ \epsilon & 1 & 1 \end{pmatrix},$$

$$\epsilon \sim f^{(\nu)}_{0,e}(\pi R) / f^{(\nu)}_{0,\mu}(\pi R) \sim \exp\{-(c_{e,L} - c_{\mu,L}) \cdot \pi k R\}$$

To generate 'maximal' $\nu_\mu - \nu_\tau$ mixing, $c_{\mu,L} = c_{\tau,L} = 0.72$

With $c_{e,L} = c_{\mu,L}$, get $\epsilon = 1$ ;

With $c_{e,L}$ slightly smaller ($\approx 0.79$), $\epsilon \approx 0.15$ .

better choice for explaining why $|U_{e3}|^2 < 0.05$.

# SUMMARY ($\nu$ Oscillations & Warped Geometry)

- Bulk SM fields;
- Higgs on the TeV Brane;
- Neutrinos could be Dirac or Majorana;

Dirac → introduce SM singlet fermion (eliminate dim 5 Majorana masses by imposing some symmetry, say lepton number) $\Rightarrow p$ stable; But $n$-$\bar{n}$ possible; Also $\mu \to e\gamma$, etc.

Majorana → dim 5 operators (no new particles introduced) → $\beta\beta 0\nu$ possible; Lepton no. parity would allow dim 5 ops. but suppress $p$ decay.

- SMOKING GUN: FIND K-K excitations of SM gauge fields (at LHC?)

(• GRAND UNIFICATION?)