Signatures of kinetic and magnetic helicity in the CMBR

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$$\langle \vec{A} \cdot \vec{\nabla} \times \vec{A} \rangle \neq 0$$

EW baryogenesis

Achúcarro & Vachaspati Phys. Rep. (2000)

$$\Delta Q_B = \Delta N_{CS}$$

$$N_{CS} = \frac{N_f}{32\pi^2} \int d^3x \, \varepsilon_{ijk} \left\{ g^2 W^{aij} W^{ak} - \frac{g^3}{3} W^{ai} W^{bj} W^{ck} \varepsilon_{abc} - g'^2 Y^{ij} Y^k \right\}$$

"non-Abelian helicity"

e.g. if only $Z_\mu \neq 0$

$$\varepsilon_{ijk} W^{ija} W^{ka} \propto \vec{Z} \cdot (\vec{\nabla} \times \vec{Z})$$

issues

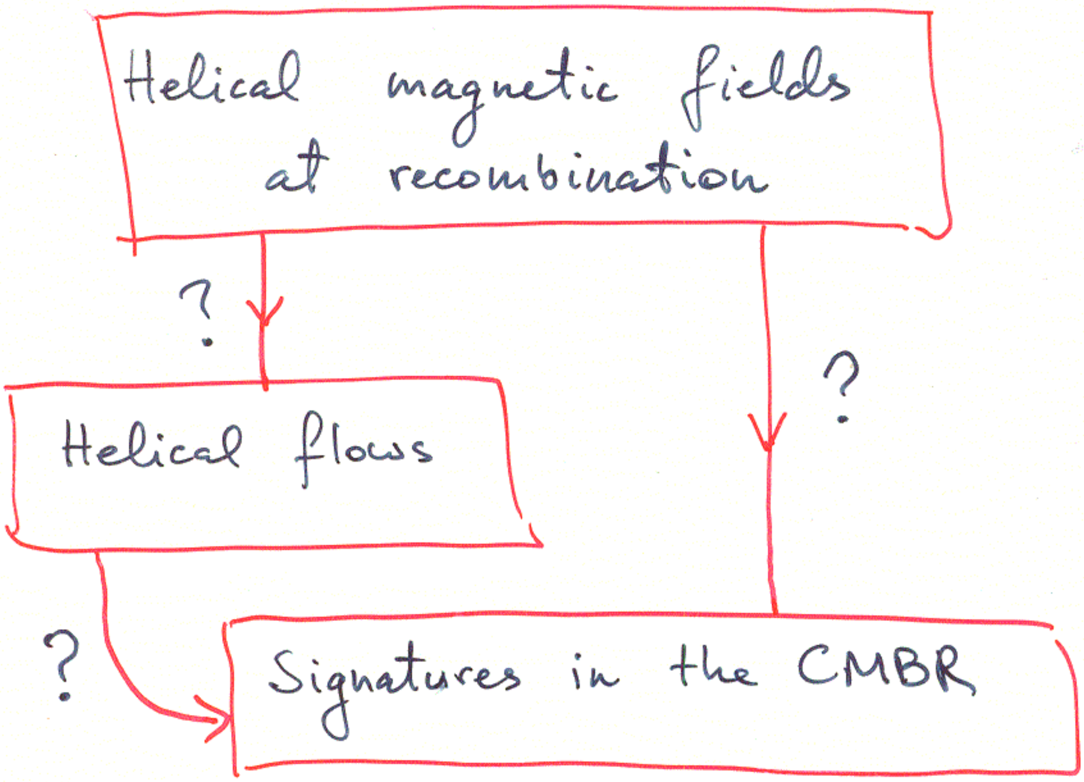
T. Vachaspati PRL (2001)
linked loops of Z-strings
see also J.M. Cornwall PRD (1997)

helical magnetic fields immediately after EW P.T.

issues

inverse cascades
MHD ...

helical magnetic fields at recombination



1. Does $\langle \vec{B} \cdot (\vec{\nabla} \times \vec{B}) \rangle \neq 0$
   imply $\langle \vec{v} \cdot (\vec{\nabla} \times \vec{v}) \rangle \neq 0$?

2. If $\langle \vec{v} \cdot (\vec{\nabla} \times \vec{v}) \rangle \neq 0$
   what is CMBR ?

3. If $\langle \vec{B} \cdot (\vec{\nabla} \times \vec{B}) \rangle \neq 0$
   can it be seen in the CMBR ?

Statistically homogeneous and isotropic random $\vec{B}$ field

$$\langle b_i(\vec{k})\, b_j(\vec{k}')\rangle = (2\pi)^3\, \delta(\vec{k}+\vec{k}')\, P_{ij}(\vec{k})$$

- reality condition: $P_{ij}^*(\vec{k}) = P_{ij}(-\vec{k}) = P_{ji}(\vec{k})$
- divergence-less : $k^i P_{ij} = 0$
- transformations under rotations

$$P_{ij}(\vec{k}) = \underbrace{S(k)\left[\delta_{ij} - \hat{k}_i \hat{k}_j\right]}_{\text{symmetric, P-even}} + \underbrace{i\, A(k)\, \varepsilon_{ijl}\, \hat{k}^l}_{\text{anti-symmetric, P-odd.}}$$

Helical $\vec{B} \xrightarrow{?}$ helical $\vec{v}$

- statistically homogeneous & isotropic $\vec{B}(\vec{x})$:

$$\langle b_i(\vec{k})\, b_j(\vec{k}')\rangle = (2\pi)^3 \delta(\vec{k}+\vec{k}') \cdot$$
$$\cdot \left\{ (\delta_{ij} - \hat{k}_i \hat{k}_j)\, S(k) + i\varepsilon_{ijl} \hat{k}_l\, A(k) \right\}$$

- Helicity:

$\langle \vec{B} \cdot (\vec{\nabla} \times \vec{B}) \rangle$ depends on $A(k)$,

does NOT depend on $S(k)$

- Lorentz force: (one-fluid approx)

$$F_L = \vec{j} \times \vec{B}, \quad \vec{j} = \vec{\nabla} \times \vec{B}$$

$\langle \vec{F}_L \rangle = \langle \vec{B} \times (\vec{\nabla} \times \vec{B}) \rangle$ depends on $S(k)$ <u>ONLY</u>

- One-fluid → no flows

Two-fluid approximation :

(e^-, γ) + baryons

$$|\vec{v}_\gamma - \vec{v}_e| \ll |\vec{v}_e - \vec{v}_B| \ll |\vec{v}_B|$$

- initially $v_e = 0$
- $\vec{B}$ is helical
- $\rho_i \frac{d\vec{v}_i}{dt} = \vec{F}_i - \sum_{j \neq i} \vec{P}_{ij}$; $i = \gamma, e^-, p$

+

Maxwell eqns

$\downarrow$

$$\boxed{|\vec{\nabla} \times \vec{v}_e| = \frac{1}{e n_e} \nabla^2 \vec{B}}$$

$|\nabla^2 \vec{B}| \sim \frac{B}{L^2}$

$|\vec{v}_e| \sim L |\vec{\nabla} \times \vec{v}_e|$

$$|\vec{v}_e| \sim 10^{-18} \left(\frac{B_0}{10^{-9} G}\right) \left(\frac{1\,\text{kpc}}{L_0}\right)$$

compare to 10^{-5}

Kinetic helicity

- $\langle \vec{v} \cdot (\vec{\nabla} \times \vec{v}) \rangle \neq 0$

- $\vec{v}$ is small

- Doppler shift is the main effect

$$v_j(\vec{x}) = \int \frac{d^3k}{(2\pi)^3} \, e^{-i\vec{k}\cdot\vec{x}} \cdot \left\{ i\hat{k}_j v^{(0)} + Q_j^{(1)} v^{(1)} + Q_j^{(-1)} v^{(-1)} \right\}$$

$$\vec{Q}^{(\pm 1)}(\vec{k}) \equiv -i \, \frac{\hat{e}_1 \pm i\hat{e}_2}{\sqrt{2}} \quad ; \quad \hat{e}_3 \equiv \hat{k}$$

$$i\hat{k} \times \vec{Q}^{(s)} = s \, \vec{Q}^{(s)} \quad ; \quad s = \pm 1$$

$$v^{(\pm 1)}(\vec{k}) \xrightarrow{P} - v^{(\mp 1)}(-\vec{k})$$

$$\int d^3x \; \vec{v}(\vec{x}+\vec{y}) \cdot \left(\vec{\nabla} \times \vec{v}(\vec{x})\right) =$$

$$= \int \frac{d^3k}{(2\pi)^3} \, k \, e^{i\vec{k}\cdot y} \left\{ v^{(1)}(\vec{k})\, v^{(1)}(-\vec{k}) - v^{(-1)}(\vec{k})\, v^{(-1)}(-\vec{k}) \right\}$$

$$\boxed{\langle v^{(1)} v^{(1)} - v^{(-1)} v^{(-1)} \rangle \neq 0}$$ ← P-odd

$$\Updownarrow$$

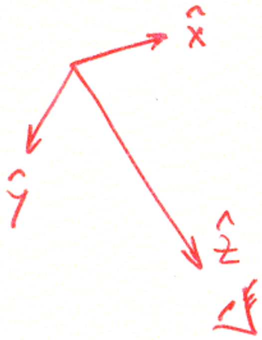
$$\boxed{\langle \vec{v} \cdot (\vec{\nabla} \times \vec{v}) \rangle \neq 0}$$

P-odd →

$$\boxed{C_\ell^{TB} \neq 0 \;, \quad C_\ell^{EB} \neq 0}$$

CMBR

Stokes parameters



$$E_x = a_x(t)\cos(\omega_0 t - \vartheta_x(t))$$

$$E_y = a_y(t)\cos(\omega_0 t - \vartheta_y(t))$$

$$I \equiv \langle a_x^2 \rangle + \langle a_y^2 \rangle$$

$$Q \equiv \langle a_x^2 \rangle - \langle a_y^2 \rangle$$

$$U \equiv \langle 2 a_x a_y \cos(\vartheta_x - \vartheta_y) \rangle$$

$$V \equiv \langle 2 a_x a_y \sin(\vartheta_x - \vartheta_y) \rangle$$

$$\frac{\Delta T(\hat{n})}{T} = \sum_{\ell m} a^{T}_{\ell m} Y_{\ell m}(\hat{n})$$

$$(Q \pm iU)(\hat{n}) = \sum_{\ell m} a^{\pm 2}_{\ell m} \; {}_{\pm 2}Y_{\ell m}(\hat{n})$$

$${}_{s}Y_{\ell m} \rightarrow (-1)^{\ell} \; {}_{-s}Y_{\ell m}$$

$$a^{E}_{\ell m} = -\frac{1}{2}\left(a^{(2)}_{\ell m} + a^{(-2)}_{\ell m}\right)$$

$$a^{B}_{\ell m} = -\frac{1}{2i}\left(a^{(2)}_{\ell m} - a^{(-2)}_{\ell m}\right)$$

$$a^{T,E}_{\ell m} \rightarrow (-1)^{\ell} a^{T,E}_{\ell m}$$

$$a^{B}_{\ell m} \rightarrow -(-1)^{\ell} a^{B}_{\ell m}$$

$$C_\ell^{XY} = \frac{1}{2\ell+1} \sum_{m=-\ell}^{\ell} \langle (a_{\ell m}^X)^* a_{\ell m}^Y \rangle$$

$$C_\ell^{TT}, \quad C_\ell^{TE}, \quad C_\ell^{EE}, \quad C_\ell^{BB}$$

P-invariant

$$C_\ell^{TB}, \quad C_\ell^{EB}$$

P-violating

What can one learn from

C_ℓ^{TB} and C_ℓ^{EB} ?

$$\frac{\ell(\ell+1)}{2\pi} C_\ell^{TB} \sim 10^{-15} - 10^{-10}$$

Given $\langle v^{(1)} v^{(1)} - v^{(-1)} v^{(-1)} \rangle \propto v_0^2 \frac{k^n}{k_*^{n+3}}$

- fix n and k_*
  to constrain v_0^2

- the sign ?

Result: Reasonable constraints on $|v_0|$ only if $n < -3$!

$n = -3$: $|v_0| \lesssim 10^{-5}$

$n = -4$: $|v_0| \lesssim 10^{-11}$

A strategy to detect $\vec{B}$ - helicity

- Find an observable sensitive to both helical and non-helical $\vec{B}$
- C_ℓ^{TT} and C_ℓ^{BB} depend only on non-helical $\vec{B}$
- Combine the two and extract the helical part

Faraday rotation

$$\Theta = \frac{3}{2\pi e} \lambda_0^2 \int \dot{\tau}(\vec{x}) \, \tilde{\vec{B}} \cdot \vec{d\ell}$$

line of sight

observed wavelength

$\dot{\tau} = n_e \sigma_T a$

"comoving" $\tilde{B} = a^2 B$

- n_e low enough for polarization to be produced
- n_e high enough for Faraday rotation to occur

$$F = \frac{3}{2\pi e} \frac{B_0}{\nu_0^2} \approx 0.08 \text{ radians} \left(\frac{B_0}{10^{-9}}\right) \left(\frac{30\,\text{GHz}}{\nu_0}\right)^2$$

Harari, Hayward, Zaldarriaga (PRD, 1997)

Kosowsky & Loeb (Ap. J., 1996)

Rotation Measure (λ independent)

$$RM = \frac{3}{2\pi e} \int \dot{\tau}\, \vec{B} \cdot d\vec{\ell}$$

$$RR' \equiv \langle RM(\hat{n})\, RM(\hat{n}') \rangle =$$

$$= \left(\frac{3}{2\pi e}\right)^2 \int \frac{d^3k}{(2\pi)^3} \left\{ \underbrace{\alpha(\vec{k})}_{\text{known}} S(k) + \underbrace{\beta(\vec{k})}_{\text{known}} A(k) \right\}$$

- if $S(k)$ is known, use RR' to find $A(k)$
- need to assume a functional form

Seshadri and Subramanian (PRL, 2001)

C_ℓ^{BB} from tangled magnetic fields

B-type polarization

$$C_\ell^{BB} = \int d^3k \ldots \int d\eta \ldots I(S(k)) \ldots$$

for $\ell > 1500$

$$T_0 \sqrt{\frac{\ell(\ell+1)}{2\pi} C_\ell^{BB}} \simeq (0.93\,\mu K) \cdot I\left(\frac{\ell}{R_*}\right) \cdot \left[\frac{B_{rms}}{3 \cdot 10^{-9} G}\right]^2 \left[\frac{\ell}{1500}\right]^{-1/2}$$

- assume a functional form for $S(k)$ to extract it from C_ℓ^{BB}.

Summary

- if there is kinetic helicity at L.S. then $C_\ell^{TB} \neq 0$ and $C_\ell^{EB} \neq 0$ (useful only if $n \lesssim -3$)
- kinetic helicity induced by helical magnetic fields is too small
- helicity of primordial $\vec{B}$ can, in principle, be detected using polarization and RM maps of the CMBR.