Cosmic Solutions in the Einstein-Weinberg-Salam Theory & the Generation of Large Electric and Magnetic Fields

(ABS 558)

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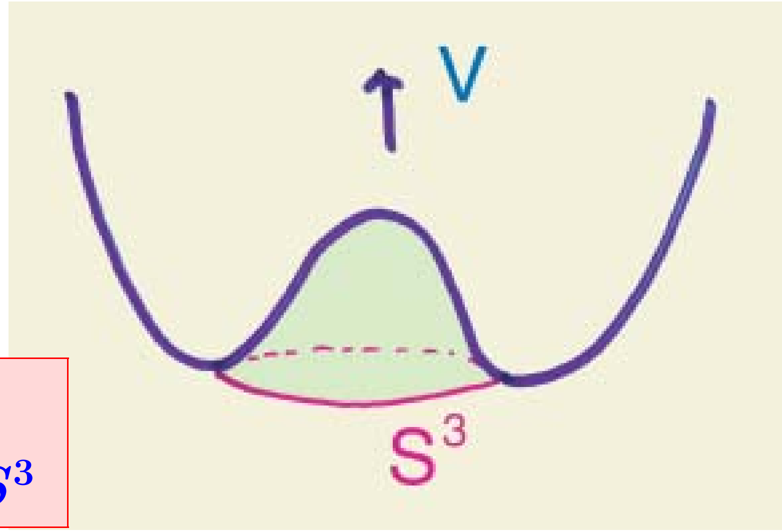
In the closed Robertson-Walker universe

evolution of topological solution **in** $SU(2) \times U(1)$: A^a_μ, B_μ, Higgs doublet Φ **⇒ large $\vec{E}$, $\vec{B}$**

Nontrivial mapping

$$V = \lambda \left(\Phi^\dagger \Phi - \frac{v_0^2}{2} \right)^2$$



$$\text{Space } S^3 \rightarrow \begin{cases} \text{Higgs vacuum } S^3 \\ \text{Yang-Mills } SU(2) \sim S^3 \end{cases}$$

Natural coordinates of $S^3 \sim SU(2)$:

Maurer-Cartan forms σ^j

$$d\sigma^j = \epsilon^{jkl} \sigma^k \wedge \sigma^l$$

$k = +1$ RW spacetime

$$ds^2 = -dt^2 + a(t)^2 \sigma^j \otimes \sigma^j$$

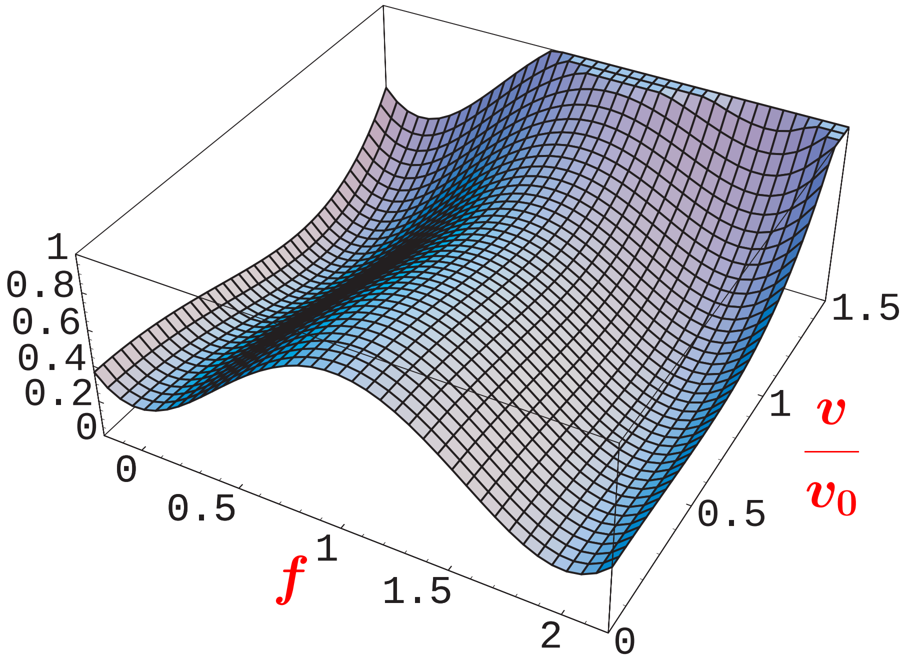
$\theta_W = 0$, $SU(2)$ theory

$$\Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix} \quad , \quad A = \frac{f(t)}{2g} \sigma^j \tau^j$$

$$V = \frac{\lambda}{4}(v^2 - v_0^2)^2 + \frac{3v^2 f^2}{8a^2} + \frac{3f^2(f-2)^2}{2g^2 a^4}$$

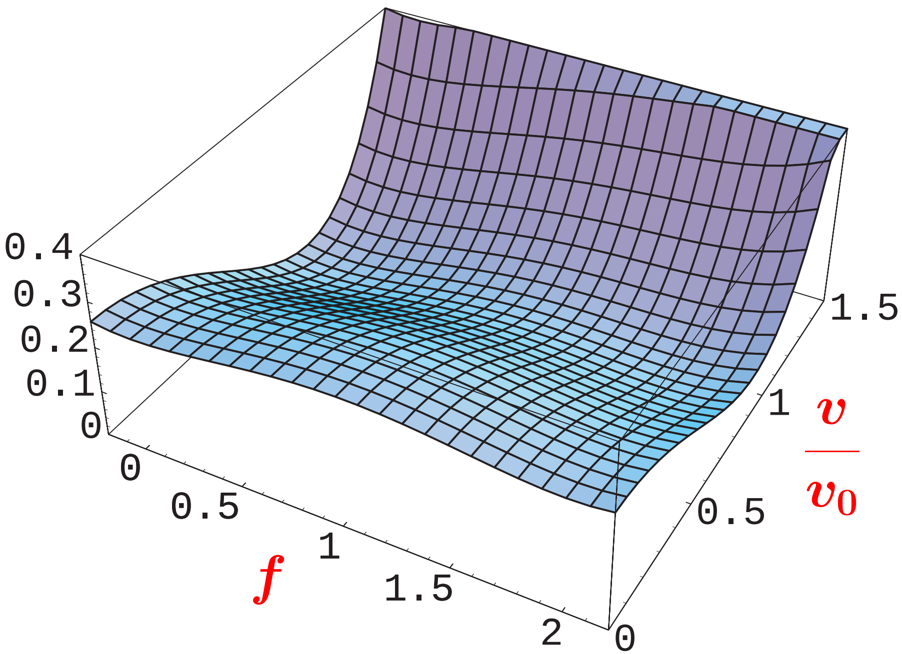
$gv_0 a < \sqrt{2}$

global min. $(v, f) = (v_0, 0)$
local $\qquad = (v_1, f_1)$



$gv_0 a \to$ large

Barrier disappears.
$\Rightarrow$ Starts to roll down.



For $SU(2) \times U(1)$ **Compact, anisotropic, homogeneous space**

$$ds^2 = -dt^2 + a_1(t)^2 \sigma_1 \otimes \sigma_1 + a_2(t)^2 \sigma_2 \otimes \sigma_2 + a_3(t)^2 \sigma_3 \otimes \sigma_3$$

$$R_{ab} = \begin{pmatrix} u_0(t) & & & \\ & u_1(t) & & \\ & & u_2(t) & \\ & & & u_3(t) \end{pmatrix}$$

$$A = \frac{1}{2g} \{ f_1(t) \sigma_1 \tau_1 + f_2(t) \sigma_2 \tau_2 + f_3(t) \sigma_3 \tau_3 \}$$

$$B = h(t) \sigma_3 \quad , \quad \Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v(t) \end{pmatrix}$$

$$T_{ab} = \begin{pmatrix} \rho(t) & & & \\ & p_1(t) & & \\ & & p_2(t) & \\ & & & p_3(t) \end{pmatrix}$$

$$a_1 = a_2, \; f_1 = f_2$$

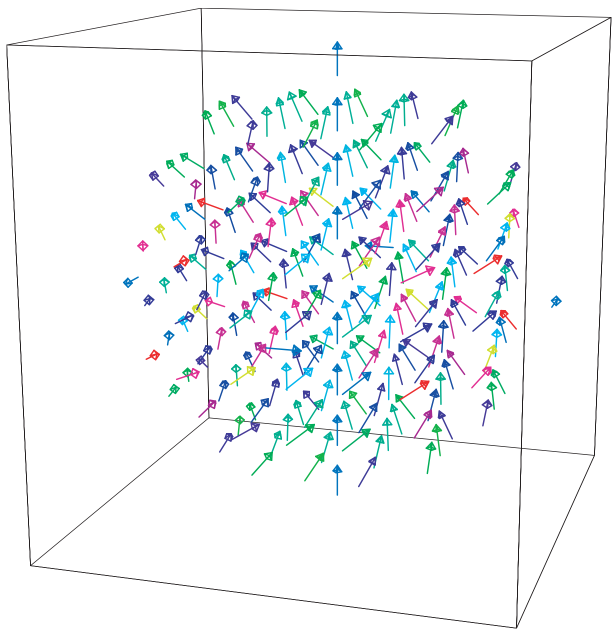
Direction in the $SU(2)$ breaking & in the $U(1)$ field : $\vec{x}$-dependent

Electromagnetic fields

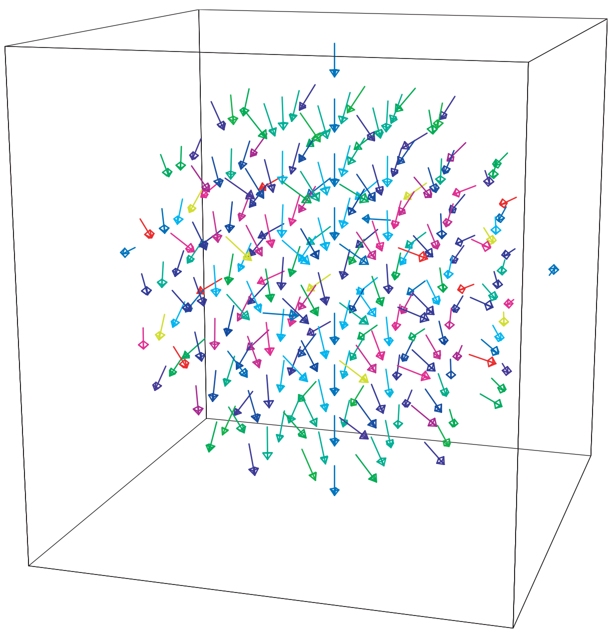
e^3-direction : $\vec{x}$-dependent
$|\vec{E}|$, $|\vec{B}|$: $\vec{x}$-independent

$$\vec{x} \longrightarrow \vec{E}\ ,\ \vec{B}$$
$$S^3 \longrightarrow S^2$$

Hopf map !



northern hemisphere of S^3



southern hemisphere of S^3

Time evolution

Einstein eqs. $\quad R_{ab} - \frac{1}{2}\eta_{ab}R = 8\pi G\, T_{ab} - \eta_{ab}\Lambda \quad$ & $\quad$ Field eqs.

Most interesting when

$$\Lambda \sim (gv_0)^2 \quad \leftrightarrow \rho = \frac{\Lambda}{8\pi G} \sim (10^{11}\mathrm{GeV})^4$$

Local minimum

$\Rightarrow$ expansion $\Rightarrow$ rolling down the hill

$\Rightarrow$ DOES NOT approach the global minimum !

$\Rightarrow$ Generates large $\vec{E}$ and $\vec{B}$.

Potential

relevant terms

$$V = \frac{\lambda}{4}(v^2 - v_0^2)^2 + \frac{v^2}{8}\left\{\frac{2f_1^2}{a_1^2} + \frac{(f_3 - g'h)^2}{a_3^2}\right\}$$

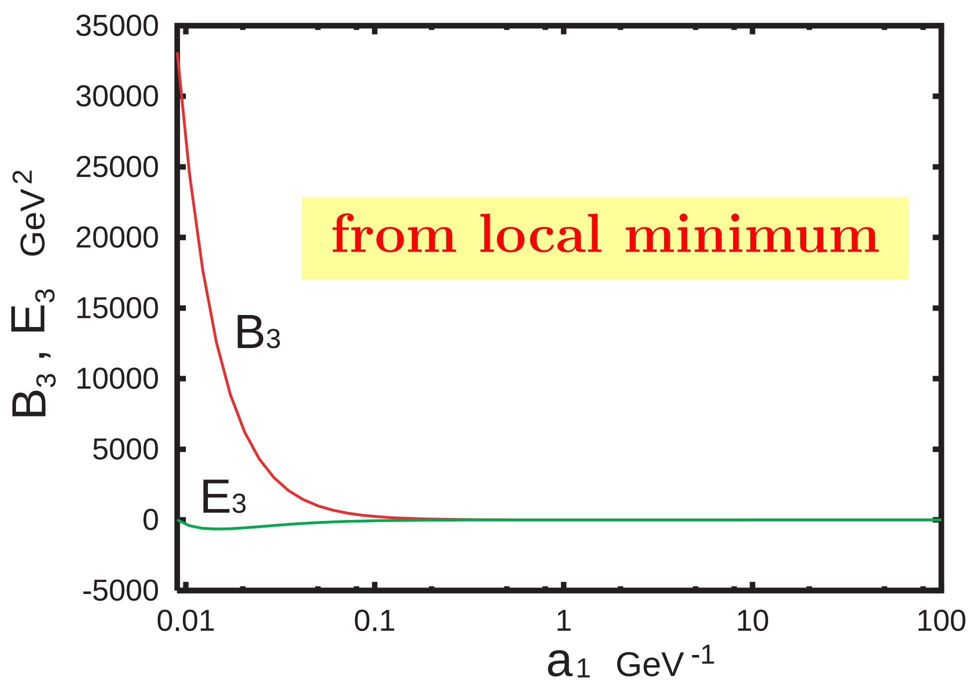
W Z

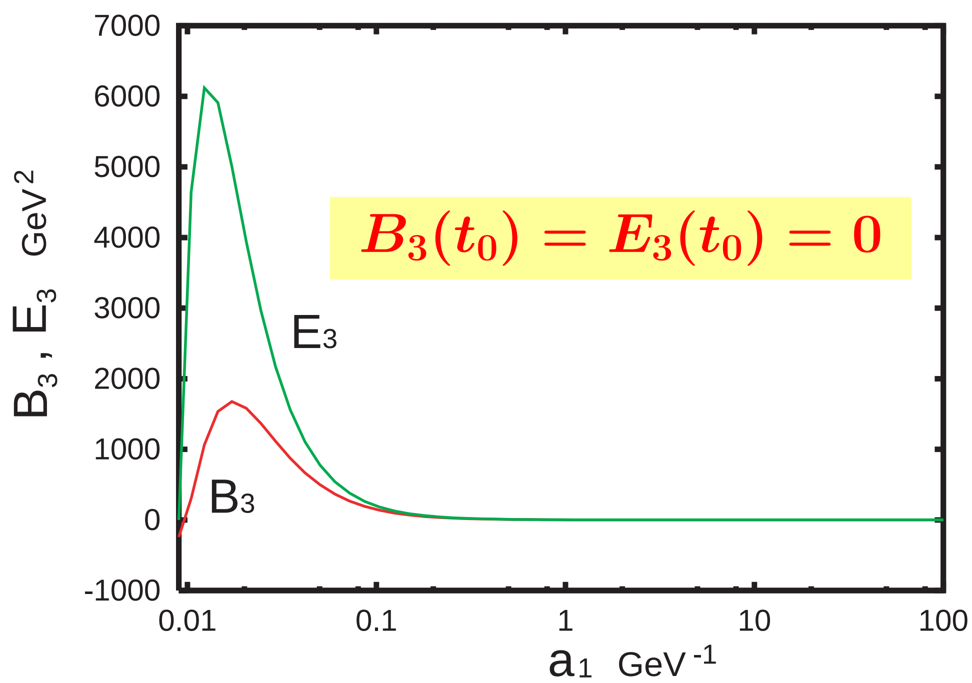
$$+ \frac{1}{2g^2}\left\{\frac{(2f_3 - f_1^2)^2}{a_1^4} + \frac{2f_1^2(f_3 - 2)^2}{a_1^2 a_3^3}\right\} + \frac{2h^2}{a_1^4}$$

flat direction $f_3 - g'h = 0$

Independent of the initial values

$\vec{E}$ and $\vec{B}$ are generated.





$\Lambda = 1.0 \times 10^5$ GeV^2 and $a(t_0) = 9 \times 10^{-3}$GeV^{-1}

Typically, $B \sim (50\text{GeV})^2$, $\hbar\omega_c = \dfrac{\hbar e B}{mc} \sim 10^6$ GeV

Birth of the Universe $\quad a \sim l_{\mathrm{Pl}} = \dfrac{1}{M_{\mathrm{Pl}}}$

$\Downarrow$

$$a \sim \frac{1}{M_{\mathrm{GUT}}} \ , \quad \frac{1}{10^{11}\mathrm{GeV}}$$

$\Downarrow$

inflation **EWS solution**

$\Downarrow$

reheating

$\Downarrow$

Present Universe

Conclusions

Topological configurations in EWS

Generation of $\vec{B}$ & $\vec{E}$

Anisotropic, but homogeneous

Before or during inflation?