Quark matter at high temperature/density

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Outline

I The phases of quark matter

Confined, quark-gluon plasma, color superconducting

II High density QCD

Color-superconducting quark matter, Color-flavor locking (CFL)

III High temperature QCD

Critical temperature, equation of state

lattice and weak-coupling calculations

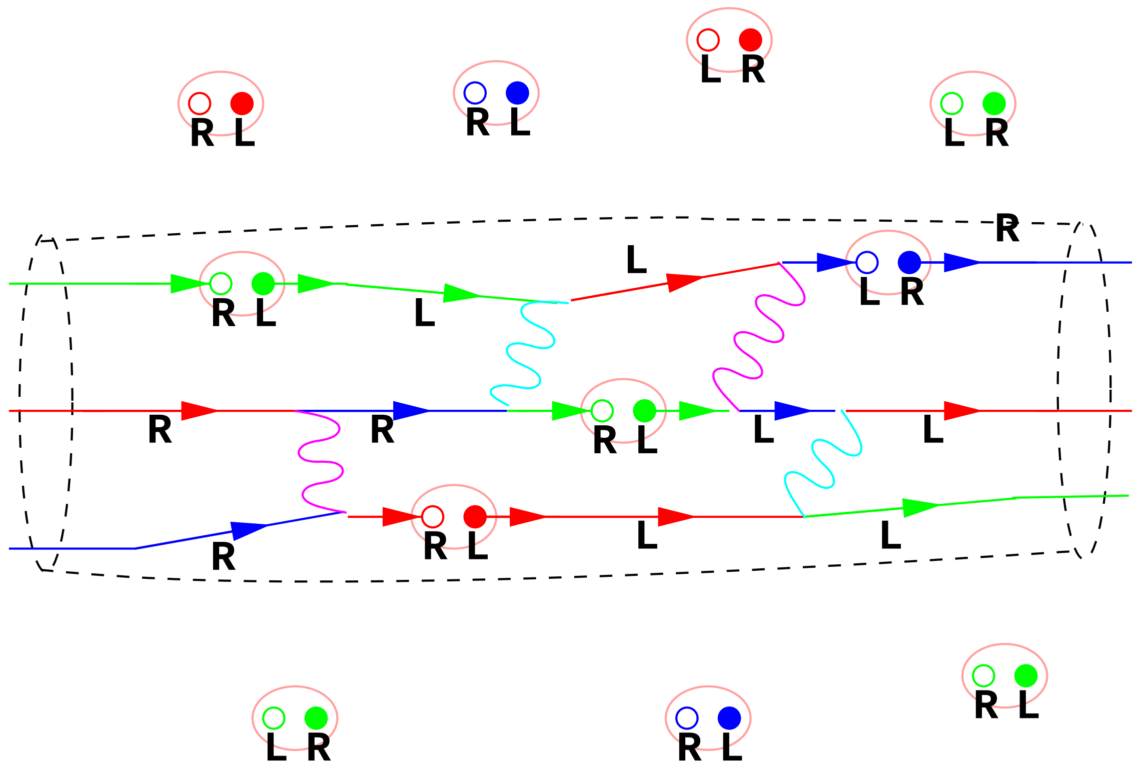
IV High temperature and moderate density

First lattice calculations of the chiral critical point

V Out of equilibrium

VI Looking to the future

Quantum Chromo Dynamics: more than the physics of a nucleon



QCD describes how quarks are bound together to form neutrons and protons. At ultra-high densities and pressures, matter enters a regime where QCD interactions are dominant.

I. The Phases of QCD

Low temperatures and densities: confined, broken chiral symmetry.

High temperatures ($T \gtrsim 150$ MeV): quark-gluon plasma (QGP)

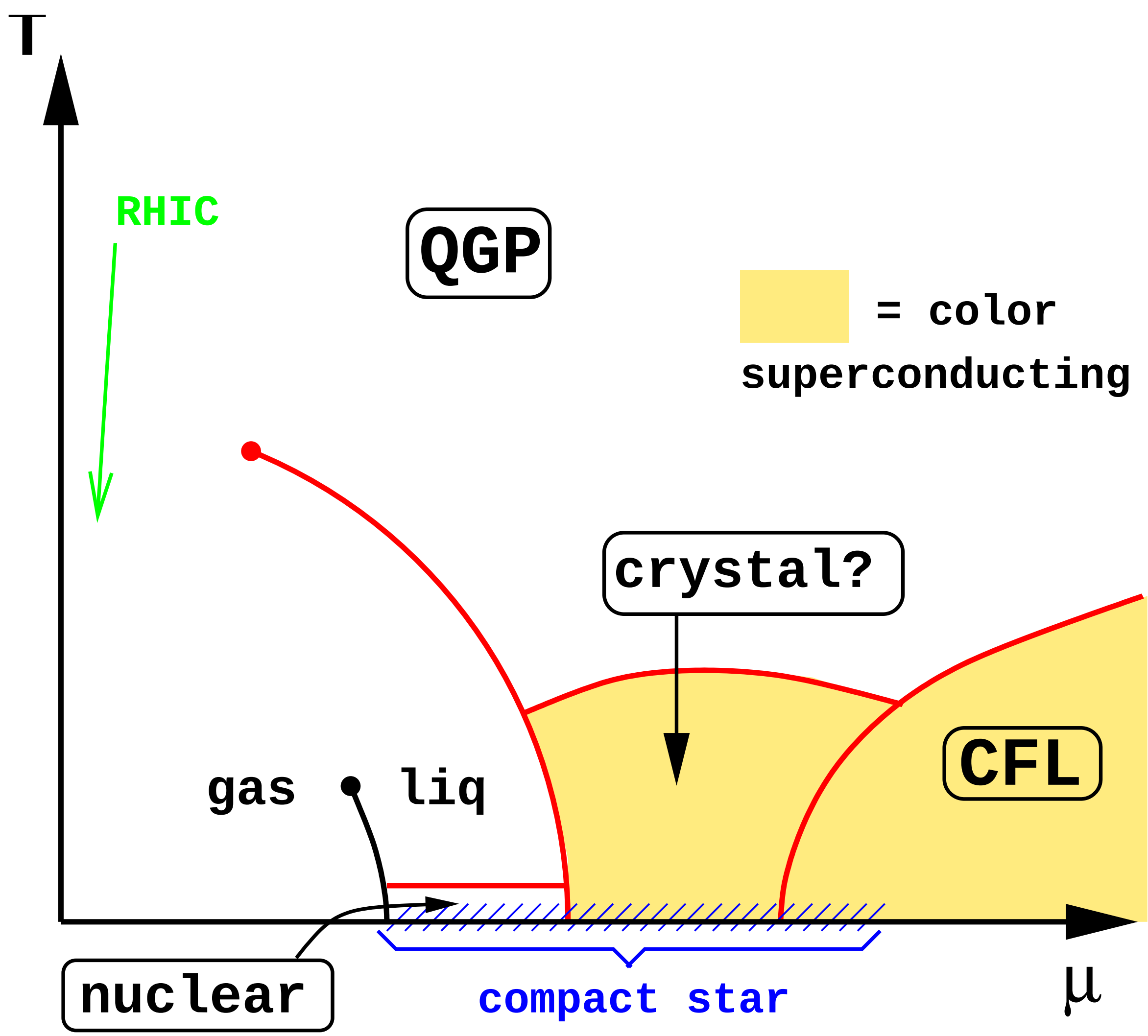
- chiral symmetry restored
- deconfinement
- signatures sought at heavy-ion colliders

High densities ($\varepsilon \gtrsim 300$ MeV/fm^3): color superconductivity

Quarks *pair* in color non-singlets.

Various phases depending on which colors and flavors participate.

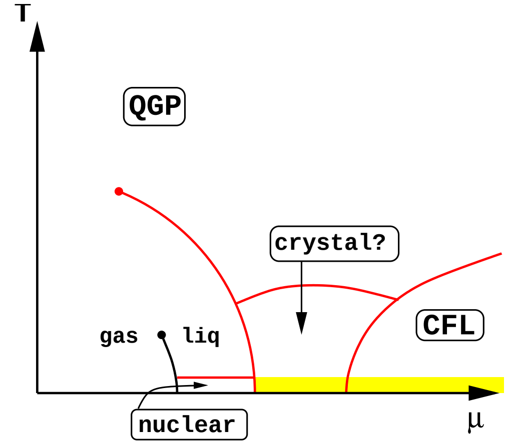
Conjectured QCD phase diagram



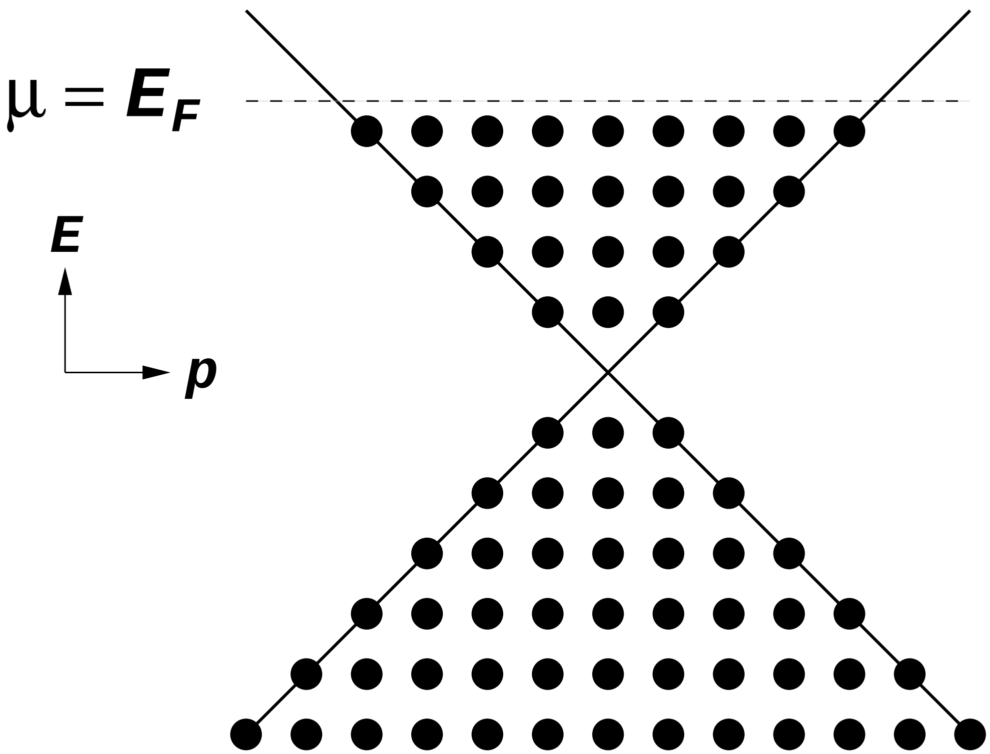
heavy ion collisions: chiral critical point and first-order line

compact stars: color superconducting quark matter core

II. High density QCD



At sufficiently high density and low temperature, there is a Fermi sea of almost free quarks.



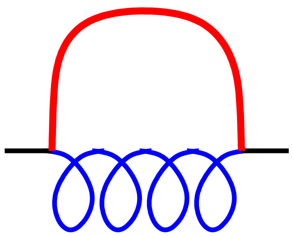
$$F = E - \mu N$$

But quarks have attractive QCD interactions.

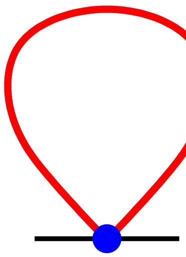
Any attractive quark-quark interaction causes pairing instability of the Fermi surface. This is the Bardeen-Cooper-Schrieffer (BCS) mechanism of superconductivity.

High-density QCD calculations

1. Weak-coupling methods. First-principles calculations direct from QCD Lagrangian, valid in the asymptotic regime, currently $\mu \gtrsim 10^6$ MeV.



2. Nambu–Jona-Lasinio models, ie quarks with four-fermion coupling based on instanton vertex, single gluon exchange, etc. This is a semi-quantitative guide to physics in the compact star regime $\mu \sim 500$ MeV, not a systematic approximation to QCD.



In both cases, you guess a color-flavor-spin pairing pattern P

$$\langle q^{\alpha}_{ia} q^{\beta}_{jb} \rangle_{1PI} = P^{\alpha\beta}_{ij\,ab} \Delta$$

then minimize the free energy wrt Δ to obtain a gap equation for Δ.

NJL gives $\Delta \sim 10{-}100$ MeV at $\mu \sim 500$ MeV.

Both methods agree on the favored pairing pattern.

Color superconductivity in three flavor quark matter: Color-flavor locking (CFL)

Equal number of colors and flavors gives a special pairing pattern
(Alford, Rajagopal, Wilczek, hep-ph/9804403)

$$\langle q_i^\alpha q_j^\beta \rangle \sim \delta_i^\alpha \delta_j^\beta + \kappa\, \delta_j^\alpha \delta_i^\beta$$

color α, β
flavor i, j

This is invariant under equal and opposite rotations of color and (vector) flavor

$$SU(3)_{\text{color}} \times \underbrace{SU(3)_L \times SU(3)_R}_{\supset U(1)_Q} \times U(1)_B \to \underbrace{SU(3)_{C+L+R}}_{\supset U(1)_{\tilde{Q}}} \times \mathbb{Z}_2$$

- Breaks chiral symmetry, but *not* by a $\langle \bar{q} q \rangle$ condensate.
- There need be no phase transition between the low and high density phases: ("quark-hadron continuity")
- Unbroken "rotated" electromagnetism, $\tilde{Q}$, photon-gluon mixture.

Compact stars

Where in the universe is color-superconducting quark matter most likely to exist? In compact stars.

A quick history of a compact star.

A star of mass $M \gtrsim 10 M_\odot$ burns Hydrogen by fusion, ending up with an Iron core. Core grows to Chandrasekhar mass, collapses $\Rightarrow$ supernova. Remnant is a compact star:

mass	radius	density	initial temp
$\sim 1.4 M_\odot$	$\mathcal{O}(10\ \mathrm{km})$	$\geqslant \rho_{\mathrm{nuclear}}$	~ 30 MeV

The star cools by neutrino emission for the first million years.

Signatures of color superconductivity in compact stars

Transport properties, mean free paths, conductivities, viscosities, etc.

1. Glitches and crystalline ("LOFF") pairing
   (Alford, Bowers, Rajagopal, `hep-ph/0008208`)
2. Cooling by neutrino emission, neutrino pulse at birth
   (Page, Prakash, Lattimer, Steiner, `hep-ph/0005094`; Carter and Reddy, `hep-ph/0005228`)
3. Gravitational waves: r-mode instability
   (Madsen, `astro-ph/9912418`)

Equation of state (mass-radius relationship)

Pressure of quark matter relative to hadronic vacuum

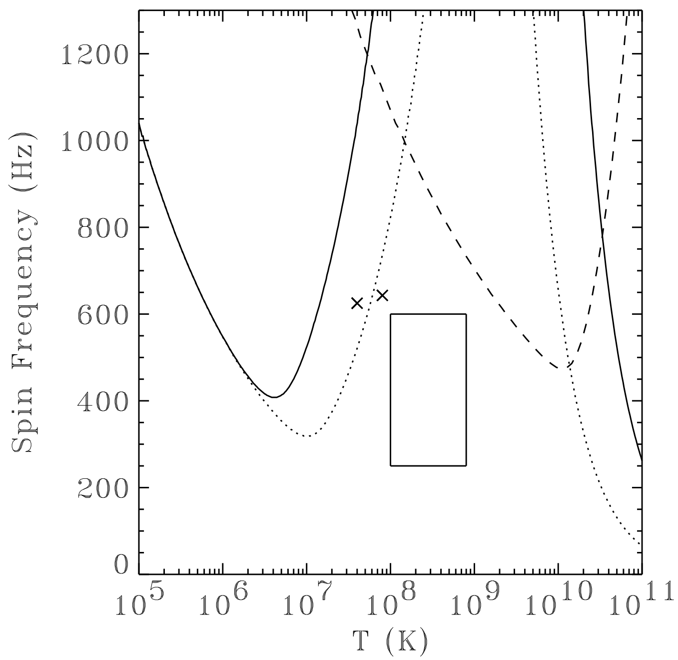
$$p \sim \mu^4 + \Delta^2\mu^2 - B$$

If bag constant is large enough to bring quark matter close to stability, a superconducting gap Δ may have large effects.

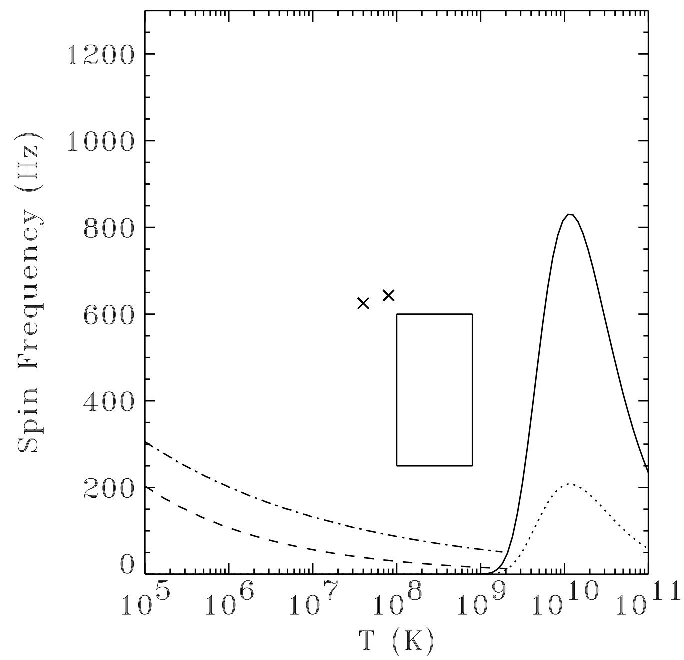
Critical frequency curves (Madsen, astro-ph/9912418)

Frequencies above curves are r-mode-unstable.

No pairing



CFL pairing



Solid line: quark matter

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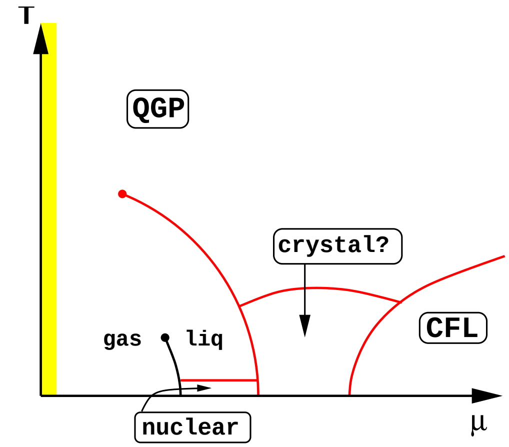
box: LMXBs

crosses: fastest pulsars

Pure CFL quark matter stars are ruled out.

III. High temperature QCD



<u>Calculational methods</u>

Lattice gauge:

Requires large computational resources;
need to extrapolate $a \to 0$, $V \to \infty$, $m_{u,d} \ll m_s$;
transport coeffs need analytic continuation
So far limited to $T \lesssim 4T_c$.

4D weak-coupling $\longrightarrow$ 3D lattice

Nonperturbative physics is in the lowest Matsubara mode. Reduce to a 3D theory of that mode, perturbatively integrating out higher modes, and do 3D lattice calculation.

Perturbation theory:

Perturbative series in $g(T)$, applicable at high temperature, converges slowly if at all. Resummation of hard thermal loops (HTL) helps for certain quantities.

Quantities calculated:

Lattice gauge

- Equation of state: pressure, energy density, entropy density

  (F. Karsch `hep-lat/0106019`)

- susceptibilities, screening lengths;

4D weak-coupling $\longrightarrow$ 3D lattice

- Debye mass of gluons. (Kajantie, Laine, Peisa, Rajantie, Rummukainen, Shaposhnikov, hep-ph/9708207)

- Upcoming: EoS of hot QCD. The contribution of the 3D theory has been calculated up to an integration constant e_0. If $e_0 = 10(2)$ then results will match to 4D lattice at $T \sim 5T_c$.

  (Kajantie, Laine, Rummukainen, Schröder, hep-ph/0007109)

Quantities calculated(cont):

Weak-coupling:

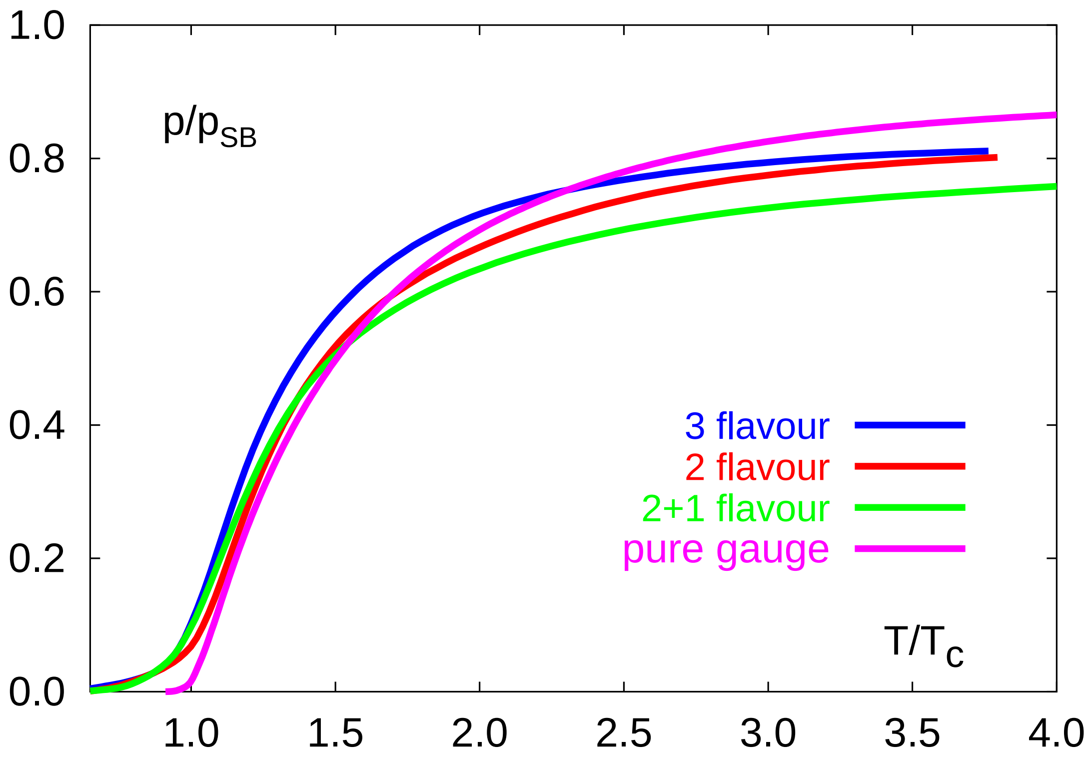
- Equation of state. Simple perturbative calculation to $\mathcal{O}(g^5)$ oscillates wildly. Resumming hard thermal loops (HTL) improves convergence; various recipes: (Blaizot, Iancu, Rebhan (hep-ph/0104033), Andersen, Braaten, Petitgirard, Strickland (hep-ph/0205085), Peshier (hep-ph/0011250))
- photon emission rate: full leading log calculation

  $$\text{Rate} \propto 1/g^2\big(1 + 1/\ln(g) + 1/\ln(g)^2 + \cdots\big)\big(1 + \mathcal{O}(g)\big)$$

  (Arnold, Moore, Yaffe, `hep-ph/0111107`)
- upcoming: dilepton rate, transport properties: shear viscosity, electrical conductivity, flavor diffusion.

Pressure of hot QCD: lattice calculations

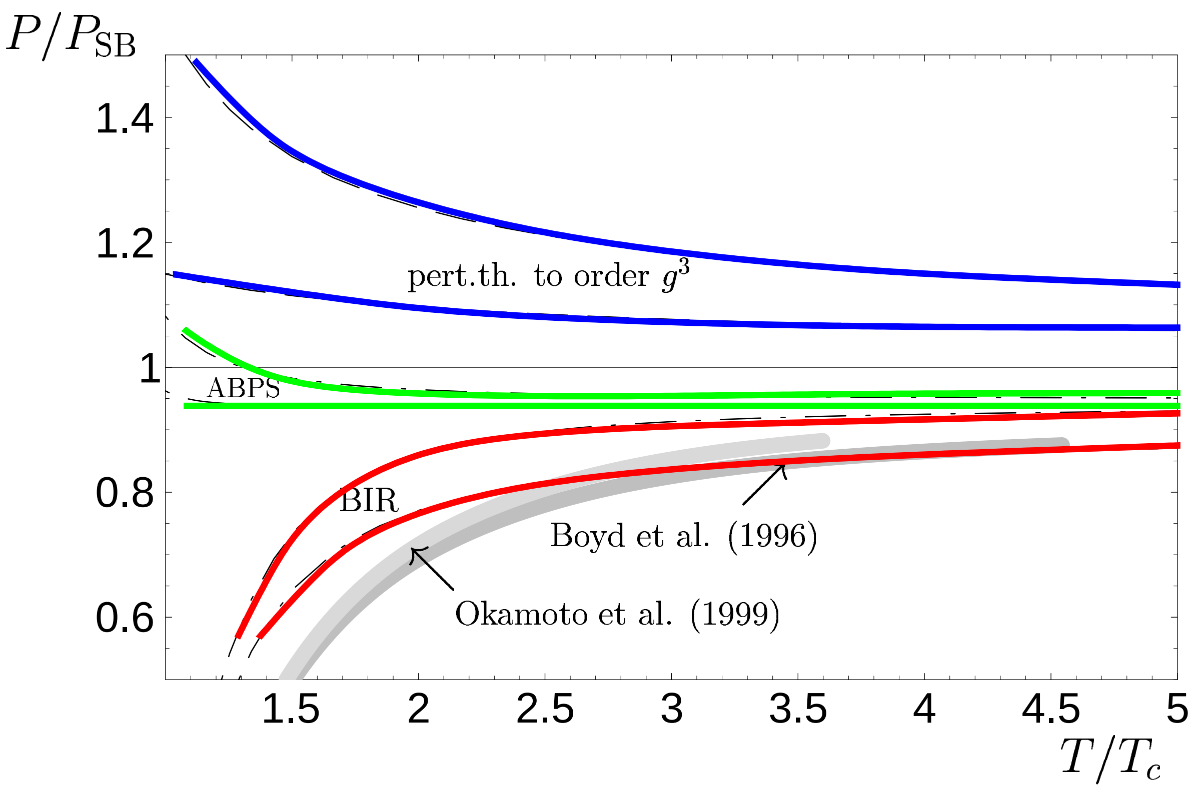
Improved actions have made it possible to obtain useful results from quite coarse lattices, lattice spacing $a \lesssim 0.2$ fm.



$T_c \sim 200$ MeV $\sim (1 \text{ fm})^{-1}$, so need $a_t \lesssim 0.2$ fm

(F. Karsch `hep-lat/0106019`)

Pressure of hot pure QCD: weak-coupling calculations



Grey bands: lattice calculations

blue lines: $\mathcal{O}(g^3)$ naive perturbation theory

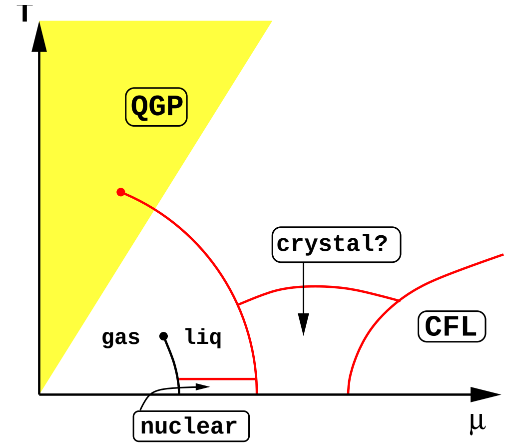
HTL resummations:

BIR: Blaizot, Iancu, Rebhan

ABPS: Andersen, Braaten, Petitgirard, Strickland

Appropriately resummed perturbation theory tells us that the pressure should slowly approach the free QGP value P_{SB} from below.

IV. High temperature and non-zero density



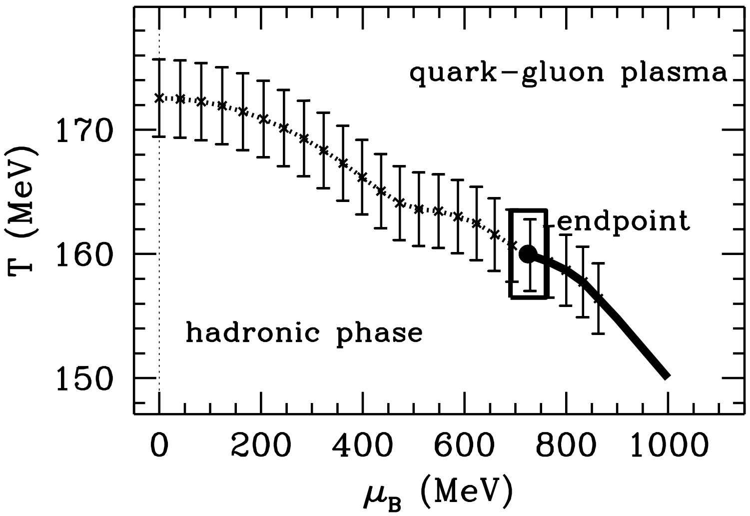
Lattice calculations at $\mu > 0, T = 0$ are stymied by the "sign problem".

Recently various groups have been exploring techniques applicable at $\mu/T \lesssim \frac{1}{2}$

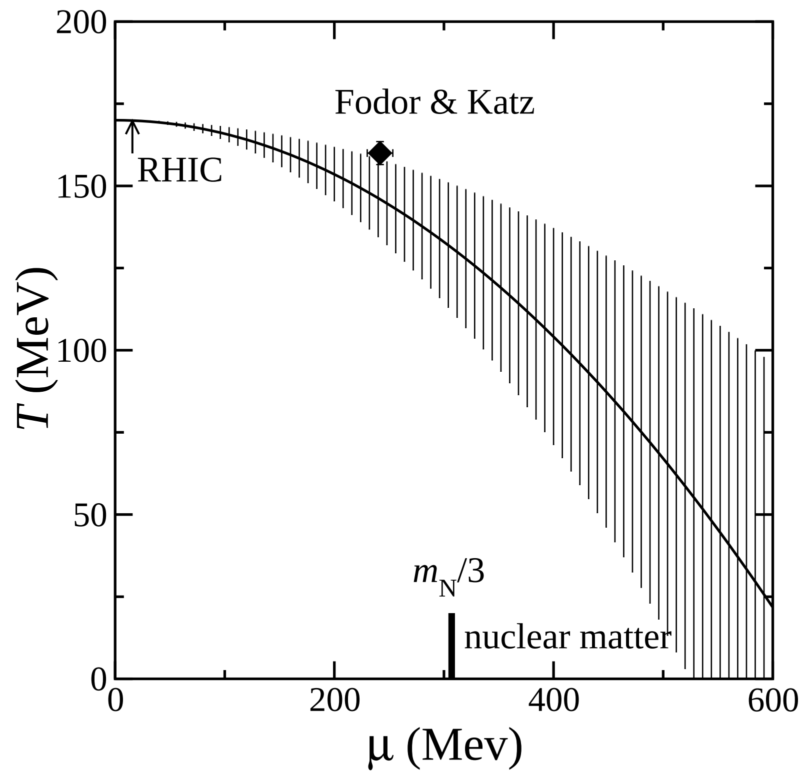
1. Multi-parameter reweighting. (Fodor and Katz, `hep-lat/0106002`)
   Reweight $\mu = 0$ ensemble to $\mu > 0$. Fails as $V \to \infty$. Currently using KS quarks at $a \sim 0.2$ fm, far from continuum or chiral limit.
2. Derivatives of observables wrt μ at $\mu = 0$. (Allton, Ejiri, Hands, Kaczmarek, Karsch, Laermann, Schmidt, Scorzato, `hep-lat/0204010`). Can extrapolate to $\mu > 0$, as long as observables are analytic.
3. Imaginary μ calculations. (de Forcrand and Philipsen `hep-lat/0205016`)
   Extrapolate to real μ, ie from negative to positive μ^2. Observables must be smooth in μ^2.

High temperature and non-zero density, results

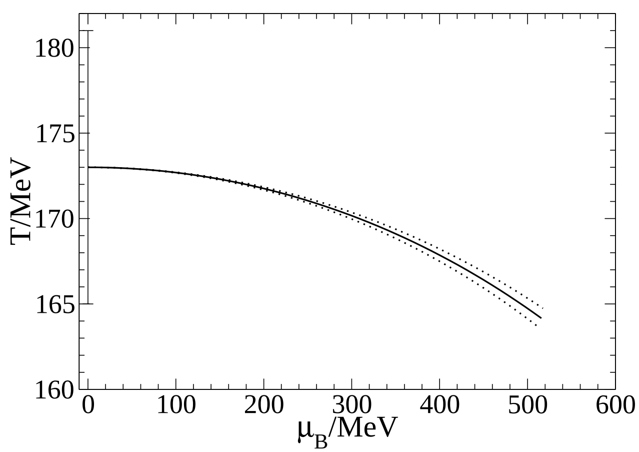
Reweighting:



$\partial^2/\partial\mu^2$ at $\mu = 0$:



Imaginary μ:



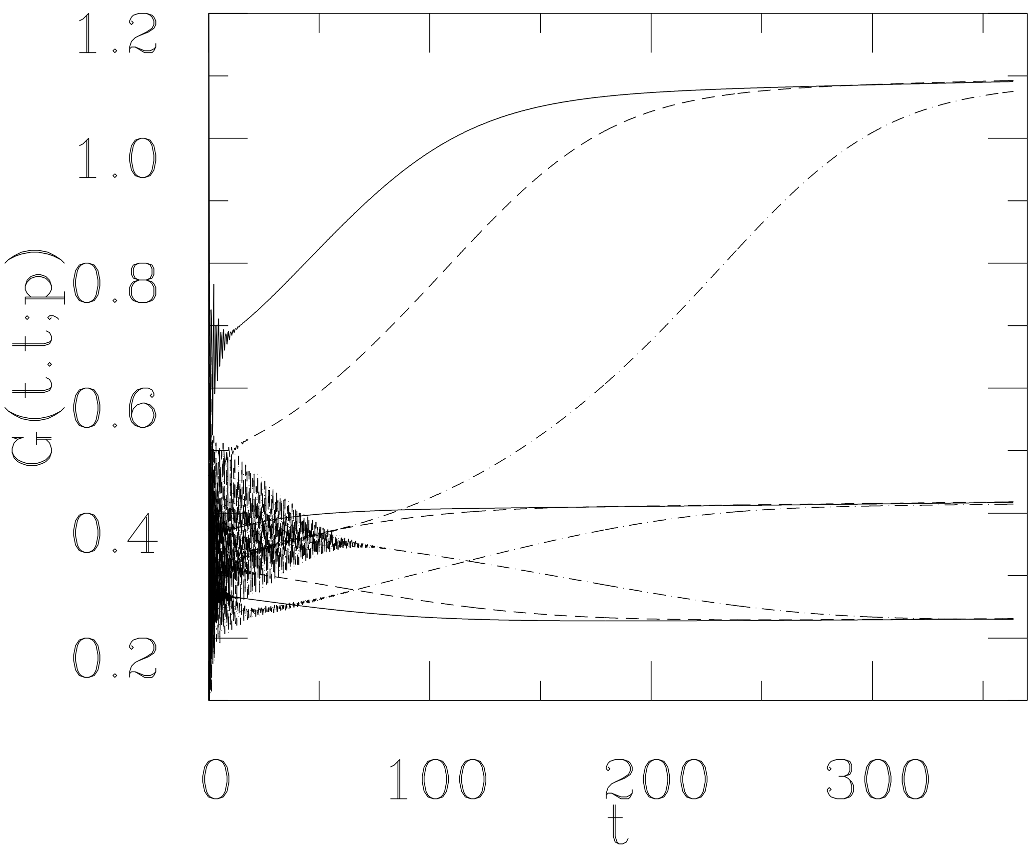
All agree on curvature of critical line at low chemical potential.

V. Far from equilibrium

Calculations of the QCD EoS and transport properties at high temperature may be applicable to heavy ion collisions at RHIC and future LHC. But such collisions occur in a few fm$/c$ and may be out of equilibrium.

Treating field theories out of equilibrium is much harder. Only recently has it been shown that interacting scalar field theory will equilibrate.

Time evolution of propagator G(p) in 1+1D scalar field theory, for $|p| = 0, 3, 5$.



Stages of evolution:

1. Fast damping of oscillations
2. Drifting
3. Approach to equilibrium

(Berges, `hep-ph/0201204`)

Systematic expansion in $1/N$, this is the 3-loop result.

VI. Looking to the future

- High density:
  - Compact-star phenomenology: glitches, conductivity/emissivity (neutrino cooling), shear/bulk viscosity (r-mode spin-down).
  - Other phases: "Kaon" condensation in CFL phase,
  - Better weak-coupling calculations, include vertex corrections
- High temperature:
  - transport properties to order g^2
  - Develop a working quasiparticle formalism
- High temp and density lattice QCD: more accurate results
- Out of equilibrium: 2PI for 4D theories, gauge theories