Phenomenology from lattice QCD

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Apologies to everyone whose work I will not have time to cover

Objectives of lattice QCD

- Establish that QCD is theory of strong interactions also in NP domain (e.g. structure functions, hadronic spectra) (parallel talk by S Collins)
- Fix the fundamental parameters of QCD (e.g. quark masses, α_s) (see also parallel talk by S Collins)
- Determine the NP QCD corrections to weak processes involving quarks (e.g. B-$\bar{B}$ mixing and UT) (see also parallel talks by D Becirevic and J Simone) $\longrightarrow$ see e.g. S Ryan at Lattice 2001 for a recent review of some charm results
- Understand QCD at finite-T and/or density (plenary talk by M Alford)
- Make predictions for exotic hadrons (e.g. $b\bar{b}g$) (parallel talk by S Collins)
- Understand mechanism(s) of confinement and χSB

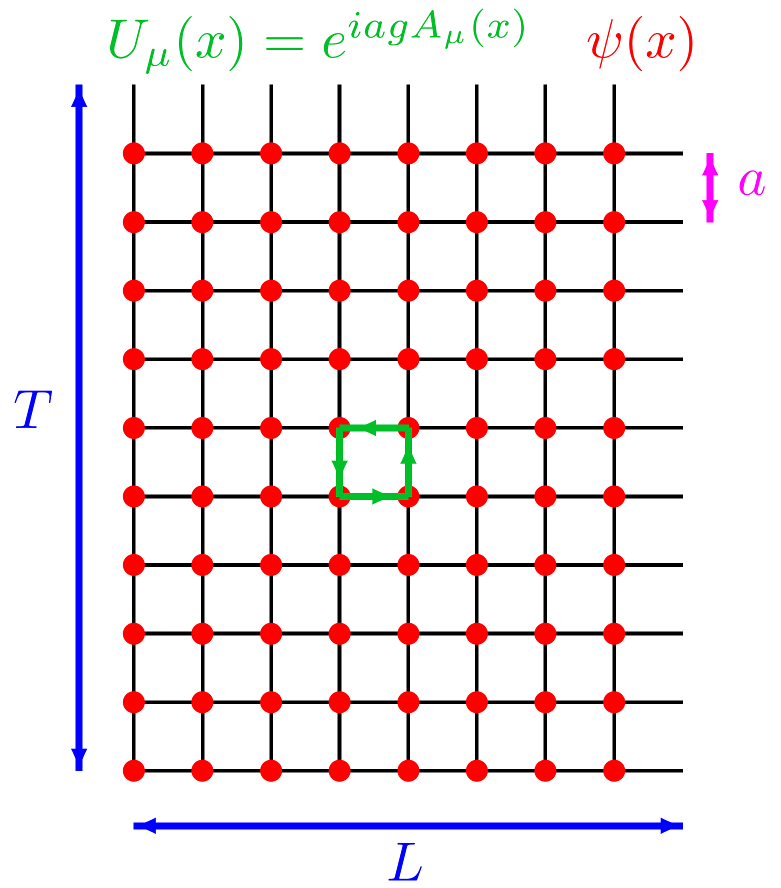
In particle phenomenology $\rightarrow$ contribute to and learn from rich experimental program of next few years: B-factories (constraining UT, rare decays, . . .), Tevatron Run II (ΔM_{B_s}, $\Delta\Gamma_{B_s}$, b-hadron lifetimes, . . .), CLEO-c (leptonic and semileptonic D decays, masses of quarkonia, hybrids, glueballs, . . .), LHC . . .

Why lattice QCD? (1)

Confinement $\longrightarrow$ fundamental quark (gluon) properties must be infered from properties of hadrons
$\Rightarrow$ need non-perturbative (NP) QCD tool to relate experiment to underlying theory

Lattice gauge theory $\longrightarrow$ mathematically sound definition of NP QCD:

- UV (and IR) cutoffs and a well defined path integral
- finite # of dof's + euclidean spacetime
  $\Rightarrow$ numerical evaluation of path integral using stochastic methods
- errors due to discretization $\sim (a\Lambda_{QCD})^n,\ (ap_\mu)^n$



Why lattice QCD? (2)

$\Rightarrow$ hadronic observables obtained directly from QCD lagrangian (i.e. *first principles*)

$\Rightarrow$ well defined and quantifiable statistical and systematic uncertainties (parametric size of errors known)

$\Rightarrow$ errors can be made arbitrarily small w/ stats $\to \infty$ and $a \to 0$ (in principle $\ldots$)

Lattice QCD in practice: cost and quenching

- Numerically very demanding:
  # of d.o.f. $\sim 32^3 \times 64 \times (4 \cdot 8 + 4 \cdot 3) \sim \mathcal{O}(10^8)$
- Cost of fully including sea quark effects is very high and increases rapidly as the sea quark mass is reduced

  $\Rightarrow$ "popularity" of quenched approximation ($N_f = 0$)–valence quarks treated exactly and sea quark effects treated as a mean field:

  $+$ large savings in computer time

  $-$ not a systematic approximation

  Quenching error on light hadron masses $\sim \mathcal{O}(10\%)$ (CP-PACS, 1998)
- More and more unquenched results, most partially quenched (2 light sea quarks instead of 3) $\longrightarrow$ we are still learning how to control systematics in these calculations

... *in practice: light quarks and the chiral limit*

Increase in numerical cost and in finite volume effects as m_q is reduced
$\Rightarrow N_f = 2$ calculations have $m_q \gtrsim m_s/2$ or $M_\pi^{lat} \gtrsim M_K^{expt}$ (exception: MILC with $N_f = 3$ and $m_q \gtrsim m_s/4$)

$\Rightarrow$ chirally extrapolate $m_q \to m_u,\ m_d$:

- extrapolation well controlled?
- M_π^{lat} in chiral regime? (i.e. can χPT be used?)

Chiral structure of quenched theory often very different from that of full theory
$\rightarrow$ how best to extrapolate to chiral limit?

. . . *in practice: heavy quarks and discretization errors*

With relativistic heavy quarks, discretization errors $\sim (am_Q)^n \Rightarrow$ need $m_Q \ll a^{-1}$
$\Rightarrow$ with $a^{-1} \sim 2 - 4\ \mathrm{GeV}$, b-quark cannot be simulated directly

- Relativistic quarks: (usually $O(a)$-improved Wilson)
  Charm region (and some) is accessible $\longrightarrow$ extrapolate to $1/M_B$ using HQ scaling
  - extrapolation can be significant
  - $(am_Q)^n$ errors may be amplified (if done at finite a)
- Effective theories: HQET, NRQCD, Fermilab
  Heavy quark mass is subtracted from the dynamics and quantities expanded in $1/m_Q$:
  + discretization errors $\sim a\Lambda_{\mathrm{QCD}}$, $a|\vec{p}|$ instead of am_b
  - need $1/m_b$ corrections $\Rightarrow$ perturbative uncertainties from "renormalon shadow" can be significant (Bernard, 2001)
  - continuum limit cannot be taken in NRQCD
- Combination of relativisitc and HQET results
  + interpolation rather than extrapolations
  Should be done with $a \to 0$ and NP renormalization

At current levels of implementation, can be viewed as complementary (for most quantities)

Heavy quark masses

Discretization errors are a worry and low order HQET not reliable
$\longrightarrow$ simulate relativistic QCD and take $a \to 0$

Bare masses tuned until the experimental value of an observable is reproduced (e.g. M_{D_s} or spin avg $c\bar{c}(1S)$)

Progress this year (still quenched)

- **Becirevic et al, 2002**
  - modern version of Gimenez et al, 1998 ($+$ Giusti et al, 2000)
  - quenched, $O(a)$-improved Wilson ($a^{-1} \sim 2.7$ GeV) instead of Wilson,
  - input: M_{D_s} $(M_{D_s^*}) \to m_c$ and $M_K \to m_s$ with $M_{K^*} \to a^{-1}$
  - NP renormalization (RI/MOM) at ~ 3 GeV and N^{3}LL conversion to RGI and $\overline{\text{MS}}$ masses `(Chetyrkin et al, 2000)` instead of NLL

- a number of systematics considered

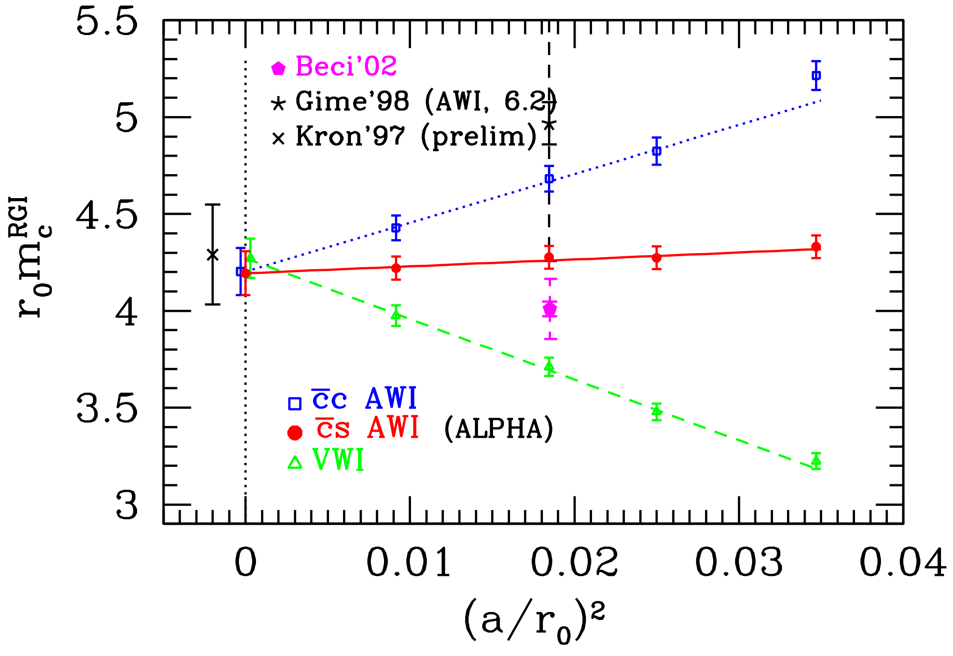
$$m_c^{\overline{\text{MS}}}(m_c) = 1.26(3)(12)\ \text{GeV}$$

• **Juge et al** are performing modern version of Kronfeld (preliminary, 1997): N^2LO renormalization instead of NLO, spin avg $c\bar{c}(1S) \to m_c$, $1P$-$1S$-splitting $\to a^{-1}$ and several lattice spacings

First continuum limit

• **Rolf and Sint (ALPHA)** (to appear)

- quenched, $O(a)$-improved Wilson, 4 lattice spacings ($a^{-1} \sim 2 \to 4$ GeV)
- input: $M_{D_s} \to m_c$ and $M_K \to m_s$, $m_s/\hat{m}$ from χPT$(r_0 = 0.5\ \text{fm})$
- 3 definitions of bare mass: VWI, $\bar{c}s$-AWI and $\bar{c}c$-AWI (non-singlet)
- NP renormalization *and* running à la ALPHA $\to m_c^{\text{RGI}} =$ RGI-mass fully nonperturbatively
- N^3LL conversion to $\overline{\text{MS}}$



Discretization errors $\sim (am_c)^2$

In the continuum limit

$$m_c^{\overline{\text{MS}}}(m_c) = 1.301(37)(7)\ \text{GeV}$$

with 1^{st} error from stat & a number of syst, and 2^{nd} from $3 \to 4$ loops

$N_f = 0$ scale ambiguity: $r_0 \to 1.1 r_0 \Rightarrow m_c^{\overline{\text{MS}}} \to 1.03 m_c^{\overline{\text{MS}}}$

(Not shown: result from combination of sumrule techniques and lattice computations (Bochkarev et al, 1996))

Summary

$$m_c^{\overline{\text{MS}}}(m_c^{\overline{\text{MS}}}) = 1.30(4)(20)\ \text{GeV}$$

with a 15% quenching error

Compare, PDG 2002: $1.0\ \text{GeV} \leq m_c^{\overline{\text{MS}}}(m_c^{\overline{\text{MS}}}) \leq 1.4\ \text{GeV}$ (without lattice)

$\to$ Need now an unquenched calculation

Reminder: for b, must use a heavy-quark expansion

$$\begin{array}{rcccccc} M_B & = & m_b^{\text{bare}} & + & \mathcal{E} & + & \mathcal{O}(\Lambda^2_{\text{QCD}}/m_b) \\ & = & m_b^{\text{pole}} - \delta m & + & \mathcal{E} & + & \mathcal{O}(\Lambda^2_{\text{QCD}}/m_b) \\ & & \uparrow & & \uparrow & & \\ & & 1/a & & 1/a & \leftarrow & \text{cancel} \end{array}$$

M_B : receives both short- and long-distance contributions
$\rightarrow$ would require QCD simulation on large and very fine lattice
$\rightarrow$ not possible at present without an EFT $(a^{-1} \sim 2 - 4 \text{ GeV})$

$\mathcal{E}$: binding energy contains long-distance part of M_B
$\rightarrow$ can be computed in HQET on large but only reasonably fine lattices

m_b^{bare}, δm : contain short-distance part of M_B

$\longrightarrow$ two strategies

1) “Perturbative”

a) $\mathcal{E}$ numerically with lattice HQET

b) δm in HQET PT, 3-loops ($N_f = 0$) and 2-loops ($N_f \neq 0$) (Martinelli et al, 1999; Di Renzo et al, 2001; Trottier et al, 2002)

c)
$$m_b^{\overline{\text{MS}}}(m_b^{\overline{\text{MS}}}) = c(m_b^{\overline{\text{MS}}})(M_B + \delta m - \mathcal{E}) + \mathcal{O}(\Lambda_{\text{QCD}}^2/m_b)$$

with $c(m_b^{\overline{\text{MS}}})$ at 3-loops (Chetyrkin et al, 1999; Melnikov et al, 2000)

Problems:

- $\mathcal{E} - \delta m \sim \alpha_s^4/a$, large as $a \to 0$
- δm has a renormalon ambiguity $\sim \mathcal{O}(\Lambda_{\text{QCD}})$ which cancels against the one in $c(m_b^{\overline{\text{MS}}})$ (Beneke et all, 1994; Bigi et al, 1994)

$\Rightarrow$ must go to highest possible order in PT ($\delta m_b^{\overline{\text{MS}}} \sim 200$ MeV at NLO, ~ 100 MeV at N^{2}LO and ~ 50 MeV at N^{3}LO, for $N_f = 0$)

$\Rightarrow$ $a \to 0$ impossible and left with $(a\Lambda_{\text{QCD}})^n$ and α_s^4 errors

2) Non-perturbative (Heitger + Sommer, Lattice 2001/2002)

New method for matching QCD and HQET fully non-perturbatively and in the continuum limit $\Rightarrow$ solves the problems of "perturbative" method

A sketch:

a) Observe that ($M(L_0, m_b^{\text{RGI}}, a) \xrightarrow{L_0\to\infty,\, a\to 0} M = M_B$)

$$m^{\text{bare}}(m^{\text{RGI}}, a) = M(L_0, m^{\text{RGI}}, a) - \mathcal{E}(L_0, a) + \mathcal{O}(1/L_0^2 m, \cdots)$$

is independent of $L_0 \Rightarrow$ for $m \sim m_b$, can also consider on small $(1/\Lambda_{\text{QCD}} \geq L_0 \gg 1/m_b)$ and fine-grained $(am_b \ll 1)$ lattices where discretization errors are small

b) Equate $m^{\text{bare}}(m^{\text{RGI}}, a)$ from small (L_0) and large (L) boxes and ...

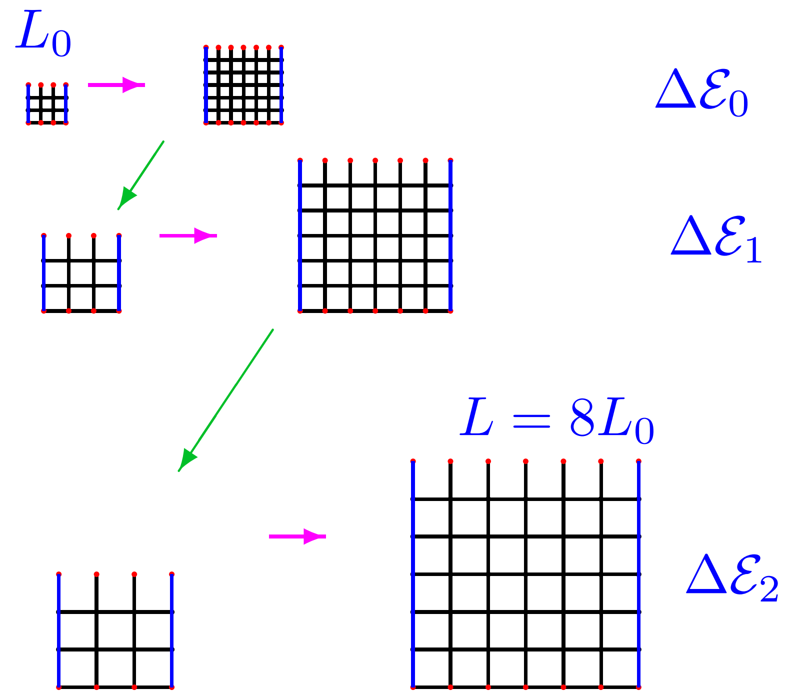
$$M \simeq \underbrace{\mathcal{E}(L, a_n) - \mathcal{E}(L_n, a_n)}_{\Delta\mathcal{E}_n(a_n)} + \cdots - \mathcal{E}(L_1, a_1) + \underbrace{\mathcal{E}(L_1, a_0) - \mathcal{E}(L_0, a_0)}_{\Delta\mathcal{E}_0(a_0)} + M(L_0, m^{\text{RGI}}, a)$$

where one takes $L > L_n = 2^n L_0 > \cdots > L_0$ (allowing $a \leq a_0 \leq \cdots \leq a_n$)

c) For each $\Delta\mathcal{E}_i$:

- range of scales is not too large $\rightarrow$ discretization errors can be kept under control with a same a_i
- $1/a_i$ divergence cancels $\rightarrow \Delta\mathcal{E}_i$ is finite

$\Rightarrow$ compute them on reasonable lattices using lattice *HQET* and take $a_i \rightarrow 0$



d) Take continuum limit of $M(L_0, m^{\mathrm{RGI}}, a)$ in lattice *QCD* in small physical volume for a number of m^{RGI} around m_b^{RGI} ($\Rightarrow$ range of scales reasonable) and interpolate to value which solves (they choose $L_0 \simeq 0.2\ \mathrm{fm}$, $n = 2$, $L \simeq 1.5\ \mathrm{fm} \sim 2^3 L_0$ and M_{B_s} instead of M_B)

$$M(L_0, m^{\mathrm{RGI}}) = M_B - \sum_{i=0}^{n} \Delta\mathcal{E}_i$$

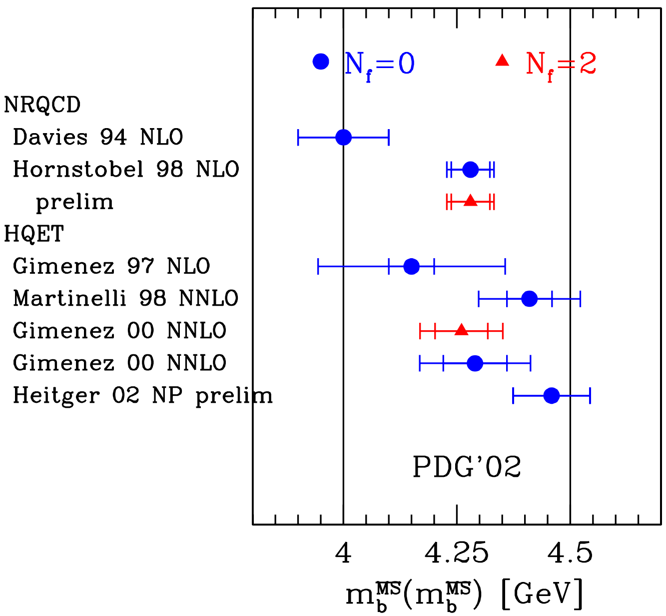
$\longrightarrow m^{\mathrm{RGI}} = m_b^{\mathrm{RGI}} + \mathcal{O}(1/L_0^2 m_b, \cdots)$

$\longrightarrow$ convert to $m_b^{\overline{\mathrm{MS}}}$ using $\mathrm{N}^3\mathrm{LL}$ running

$\Rightarrow$ No more discretization nor perturbative errors (but still $1/m_b$ corrections)

$\rightarrow$ Generalizable to other $1/m_b$ corrections in HQET

Results



- NRQCD results are obtained from M_Υ at finite b mass (all at NLO)
- Collins et al and Gimenez et al have preliminary quenched N^{3}LO results compatible with N^{2}LO Gimenez et al but with smaller perturbative uncertainty ~ 50 MeV
- $N_f = 2$ result shows no significant change wrt $N_f = 0$ (Gimenez et al, 2000), but only two $m_{sea} \sim m_s/2 \rightarrow m_s$; should be checked with further unquenched studies

NP result a bit on high side $\longrightarrow$ difference should be investigated

Summary

$$m_b^{\overline{\mathrm{MS}}}(m_b^{\overline{\mathrm{MS}}}) = 4.38(9)(10)\ \mathrm{GeV}$$

where second error is an estimate of the remaining quenching uncertainty (10% on B_s binding energy)

Lattice QCD for the unitarity triangle

$$\Delta m_d = C_B M_{B_d} f_{B_d}^2 \hat{B}_{B_d} A^2 \lambda^6 [(1-\bar\rho)^2 + \bar\eta^2]$$

$$\frac{\Delta m_d}{\Delta m_s} = \frac{M_{B_d}}{M_{B_s}} \xi^{-2} \lambda^2 [(1-\bar\rho^2) + \bar\eta^2]$$

$$|\epsilon_K| = C_K \hat{B}_K A^2 \lambda^6 \bar\eta [A^2 \lambda^4 (1-\bar\rho) S_{tt} + S_{tc}]$$

$$|\frac{V_{ub}}{V_{cb}}| = \lambda/(1-\lambda^2/2)\sqrt{\bar\rho^2 + \bar\eta^2}$$

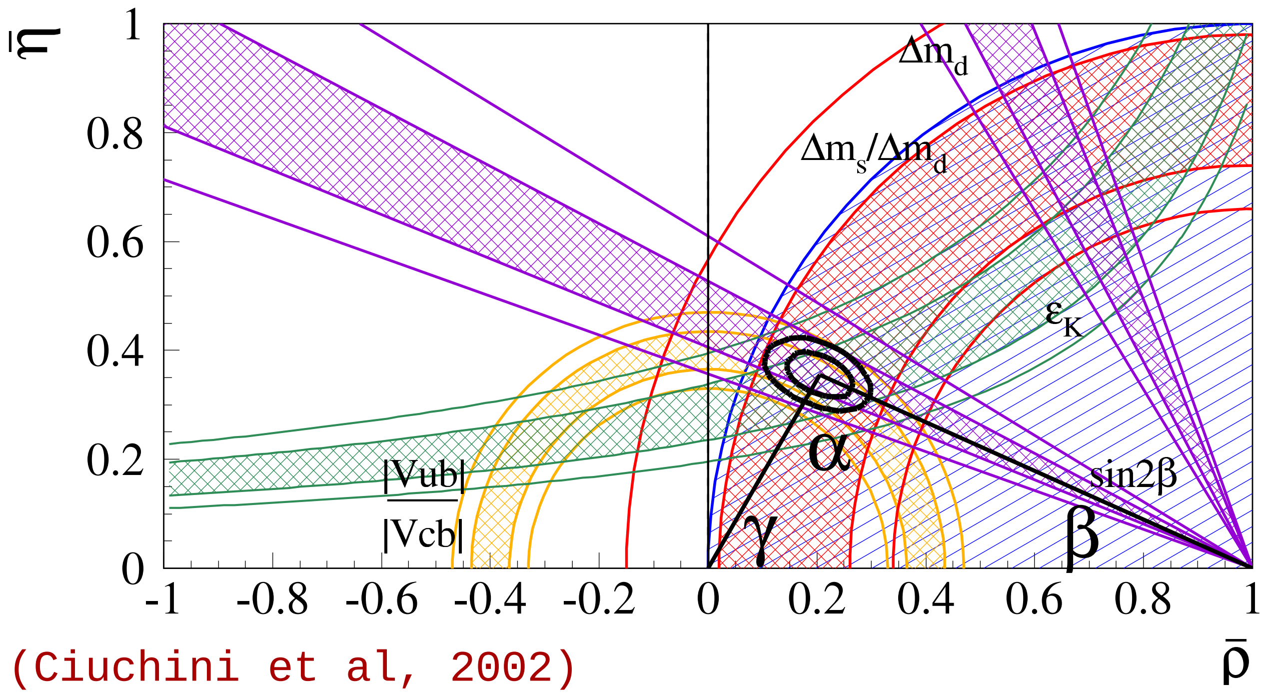
Lattice QCD $\rightarrow f_{B_d}\sqrt{\hat{B}_{B_d}},\ \xi,\ \hat{B}_K$

$$\langle \bar{B}_q | (\bar{b}q)_{V-A} (\bar{b}q)_{V-A}(\mu) | B_q \rangle = \frac{8}{3} M_{B_q}^2 f_{B_q}^2 B_{B_q}(\mu)$$

and similarly for $B_K(\mu)$

$$\xi = \frac{f_{B_s}\sqrt{B_{B_s}}}{f_{B_d}\sqrt{B_{B_d}}}$$

$$\langle 0 | \bar{b}\gamma_\mu \gamma_5 q | B_q(p) \rangle = i p_\mu f_{B_q}$$



(Ciuchini et al, 2002)

$|V_{cb}|$ can be obtained from $B \rightarrow D^*(D)\ell\nu$

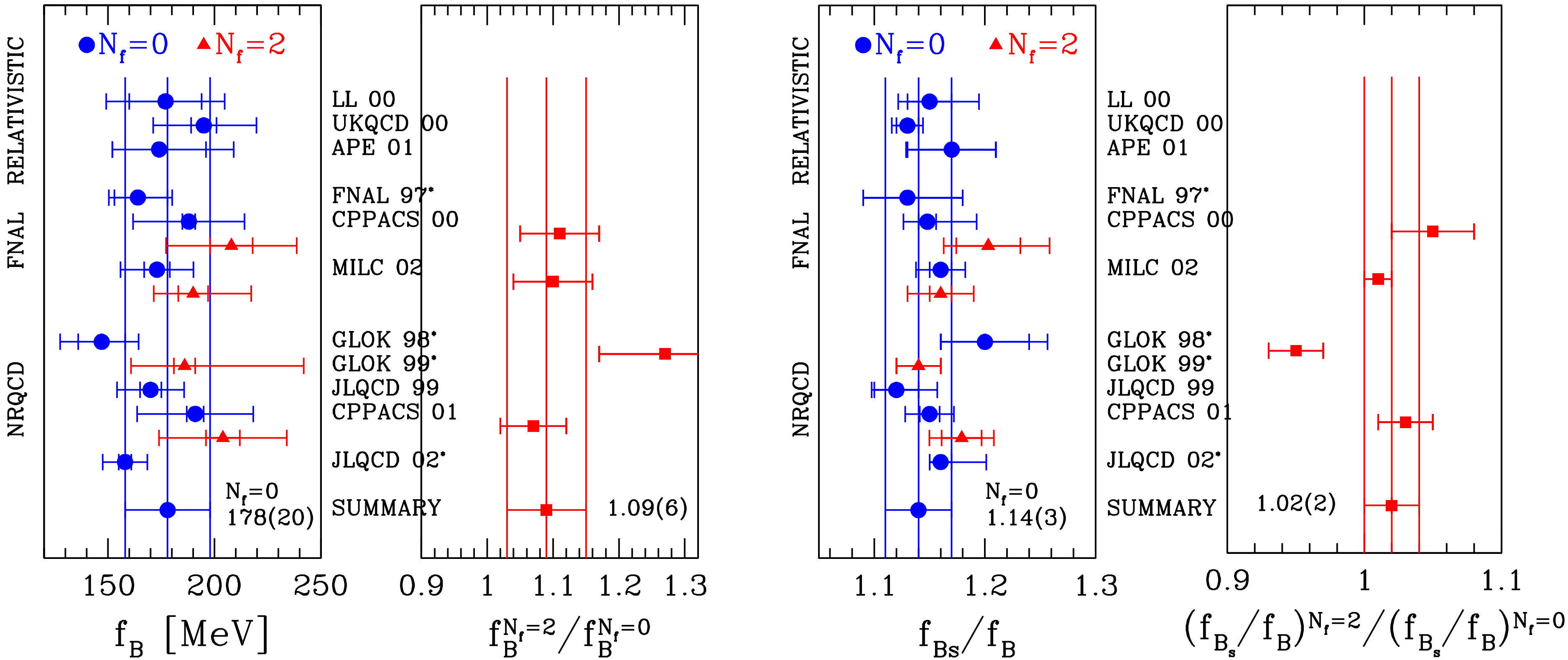
$|V_{ub}|$ from $B \rightarrow \pi(\rho)\ell\nu$

Lattice QCD $\rightarrow$ form factors

B-$\bar{B}$-mixing: f_B

Though they are correlated, it is useful to separate the study of $\Delta B = 2$ matrix element into f_B and B_B.

Recent results for f_B ...



- Results obtained w/ different HQ approaches broadly agree (relativistic vs EFT) $\Rightarrow$ HQ mass dependence appears to be under control
- W.A. of quenched results have remained stable in the past 6 years (e.g. $f_B: \ 165(23)\ \mathrm{MeV} \rightarrow 178(20)\ \mathrm{MeV}$)

- $N_f = 2$ calculations show increase in f_B over $N_f = 0$ of up to $\mathcal{O}(15\%)$ but only small effects in f_{B_s}/f_B
- MILC (2001), first $N_f = 3$ results (preliminary)

$$f_B^{N_f=3} > f_B^{N_f=2}, \quad f_B^{N_f=3}/f_B^{N_f=0} = 1.23(4)(6), \quad (f_{B_s}/f_B)^{N_f=3} = 1.18(1)^{+4}_{-1}$$

- New method (finite volume technique) put forward by `Guagnelli et al, 2000` and first quenched result: $f_B = 170(11)(??)\ \mathrm{MeV}$

All results assume mild extrapolations (mostly linear) of f_{B_q} in m_q from $m_q \sim m_s/2$ to $m_{u,d}$

$\Rightarrow$ is the chiral behavior under control? (serious issue for UT fits, due to importance of $\Delta m_s/\Delta m_d$ constraint)

Light quark mass dependence from (PQ)χPT (Grinstein et al, 1992; Booth, 1994; Sharpe and Zhang, 1996) ($N_f = 2$) ($m_{val} = m_{sea}$ for f_B)

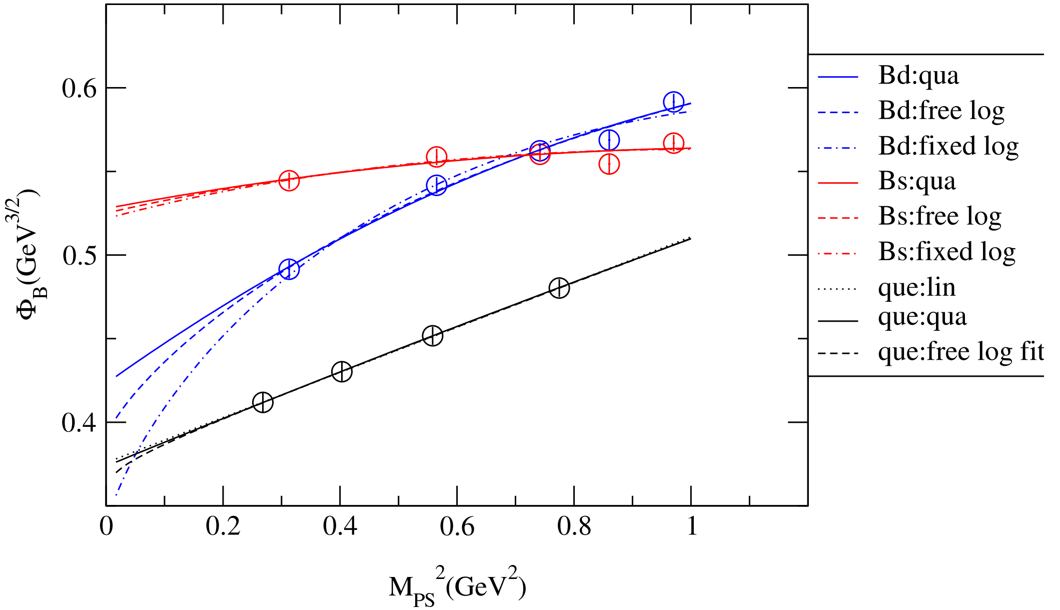
$$\frac{f_B\sqrt{M_B}}{\phi_B^{(0)}} = 1 - \frac{3}{8}(1+3g^2)\left(\frac{M_{sea,sea}}{4\pi F}\right)^2 \ln\frac{M^2_{sea,sea}}{\Lambda^2_{f_B}} + \text{h.o.t} + \mathcal{O}(\frac{1}{M_B})$$

and a similar expression for f_{B_s}, with g heavy-quark limit $B^*B\pi$ coupling

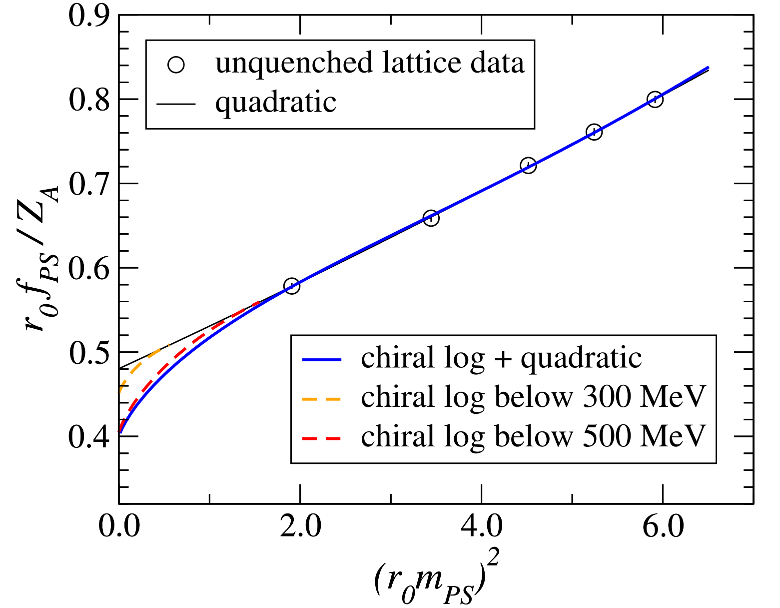
CLEO (2002) determines $g_c = 0.59(7)$ using its $\Gamma_{D^{*+}}$ (consistent with recent lattice calculation of $g_{D^*D\pi}$ (Abada et al, 2002))

$\rightarrow$ consider $g \sim 0.6$ in what follows (predictions $0.2 < g < 0.7$ (Colangelo et al, 2002))

$f_{B_{d(s)}} \sqrt{M_{B_{d(s)}}}$ vs $M^2_{sea,sea}$



F_π vs M_π^2



(Hashimoto and Yamada (JLQCD), Lattice 2002, preliminary)

They conclude, comparing quadratic and log fits: $(\delta f_B)_{\text{chi. extrap.}} = -17\%$
(see also Kronfeld & Ryan, 2002)

Comments:

- Data not inconsistent with log, but cannot distinguish log from quadratic behavior (all the action happens below lightest point)
- Find NLO correction of $\mathcal{O}(60\%) \Rightarrow$ cannot ignore h.o.t.'s (NLO χPT not expected to hold up to $1\ \mathrm{GeV}$)
- F_π vs M_π^2 not consistent with log behavior, while coefficient is comparable

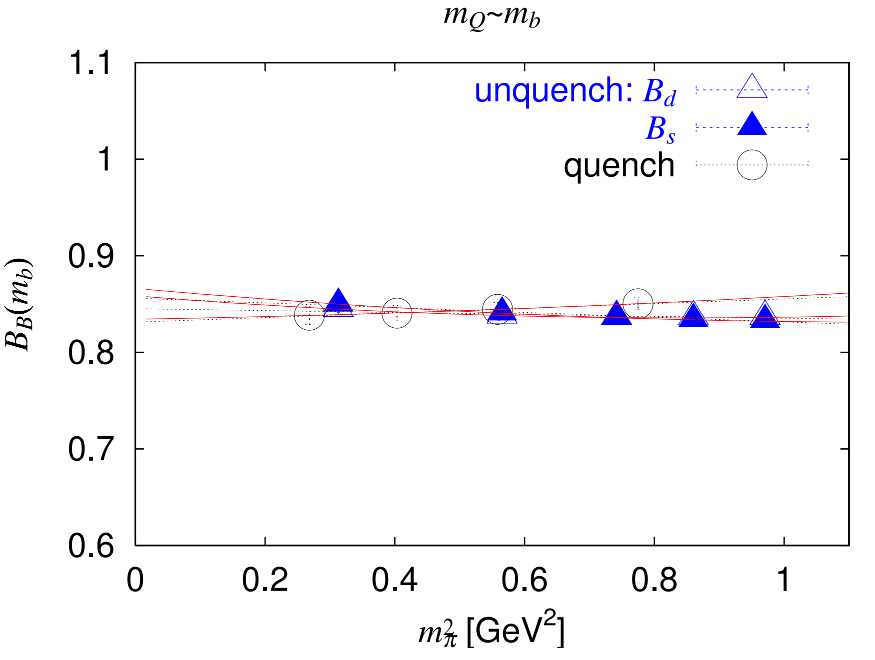
- χPT results obtained at leading order in $1/M_B$; however $1/M_B$ corrections may be significant for light-quark masses in the data region
- χPT at leading order in $1/M_B$ does not distinguish light-quark mass behavior of $f_B\sqrt{M_B}$ and f_B; f_B will have milder light-quark mass behavior and fits should show less variation
- setting $g \sim g_c$ is valid up to $1/m_c$ corrections
- Work at fixed $\beta \Rightarrow$ lattice shrinks by $\sim 25\%$ as $M^2_{sea,sea}$ goes from heaviest to lightest $\Rightarrow$ volume and a dependence which can be interpreted as $M^2_{sea,sea}$ dependence in fit
- Modelling of h.o.t.'s and allowing log to set in at lighter masses $\longrightarrow$ variations generically less than 10%

Conclusion:

- Data for $f_B\sqrt{M_B}$ exhibits significant liqht-quark mass dependence $\rightarrow$ more work is needed to understand extent to which it is physical
- Coefficent of log quite large when $g \sim 0.6 \Rightarrow$ effect could be significant (more work needed on g; to determine up to what masses NLO behavior may stay dominant; to determine to what extent $1/M_B$ corrections can modify picture)
- $(\delta f_B)_{\text{chi. extrap.}} \sim -10\%$ and $(\delta f_{B_s})_{\text{chi. extrap.}}$ negligible compared to other systematics seem reasonable

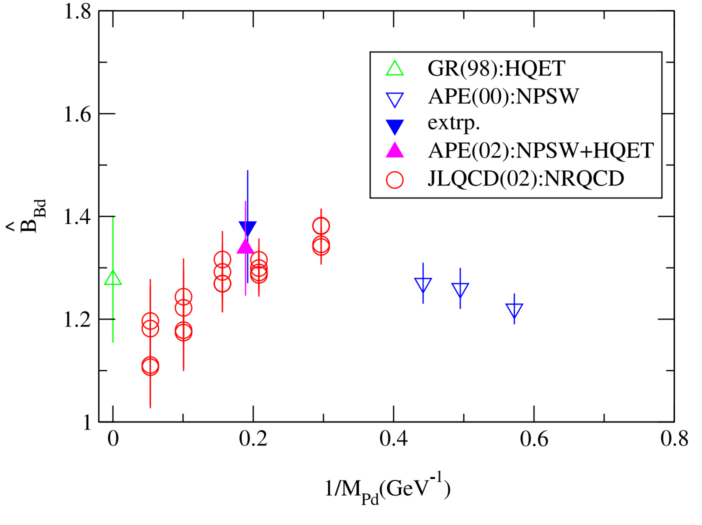
B-$\bar{B}$-mixing: B_B

Many fewer calculations, but situation with chiral extrapolation and unquenching appears to be much more favorable than with f_B

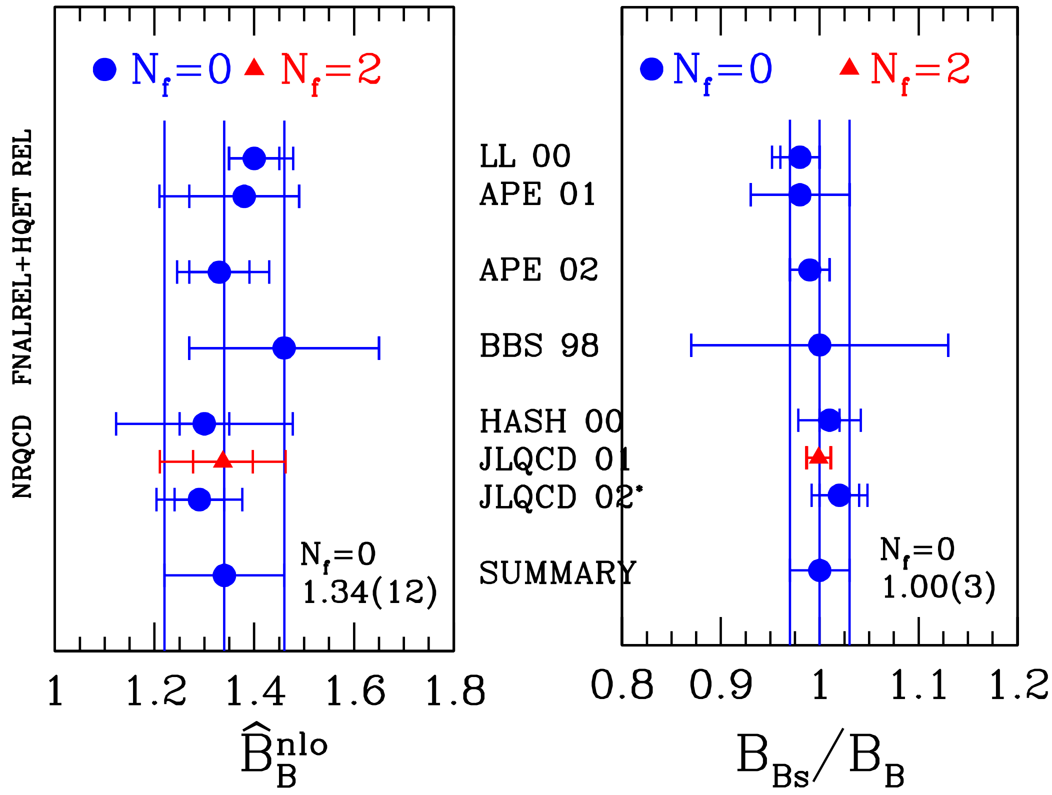


- Chiral log is $\sim (1-3g^2) \sim -0.1$ instead of $\sim (1+3g^2) \sim 2.1$, for $g \sim 0.6$
- Very little variation in going from $N_f = 0 \rightarrow N_f = 2$

(Figs from Yamada (JLQCD), CERN CKM-Workshop, 2002 and Lattice 2002)



$1/M_B$ dependence over full range of masses does not show clear trend (small residual systematics?) but is mild and different formulations agree at physical point



- All methods give fully consistent results: cancellation of errors in ratio
- No visible unquenching effects
- Mild light-quark mass dependence $\Rightarrow$ small error on $B_{B_s}/B_B \sim 3\%$
- $1/M_B$ dependence should be clarified

B-$\bar{B}$-mixing: summary

Keep only most recent calculations (> 1998 for $N_f = 0$ and > 1999 for $N_f = 2$) and omit those which have not yet made it into proceedings or papers

$N_f = 3$ estimates:

$$f_B = 203(27)^{+0}_{-20}\ \mathrm{MeV} \qquad f_{B_s} = 238(31)\ \mathrm{MeV} \qquad \frac{f_{B_s}}{f_B} = 1.18(4)^{+12}_{-0}$$

$$\hat{B}_B^{NLO} = 1.34(12) \qquad \hat{B}_{B_s}^{NLO} = 1.34(12) \qquad \frac{\hat{B}_{B_s}^{NLO}}{\hat{B}_B^{NLO}} = 1.00(3)$$

$$f_B\sqrt{\hat{B}_B^{NLO}} = 235(33)^{+0}_{-24}\ \mathrm{MeV} \qquad f_{B_s}\sqrt{\hat{B}_{B_s}^{NLO}} = 276(38)\ \mathrm{MeV}$$

$$\xi \equiv \frac{f_{B_s}\sqrt{B_{B_s}}}{f_B\sqrt{B_B}} = 1.18(4)^{+12}_{-0}$$

where asymmetric error is due to uncertainty in chiral extrapolation

Note: ξ obtained from $\Delta B = 2$ matrix elements directly comes out larger than through definition given above, but error bars are large (Bernard et al, 1998; LL et al, 2001)

K-$\bar{K}$-mixing: B_K

New $\mathcal{O}(a)$-improved Wilson results (SPQ$_{cd}$R, 2002) (preliminary) (talk by Becirevic):

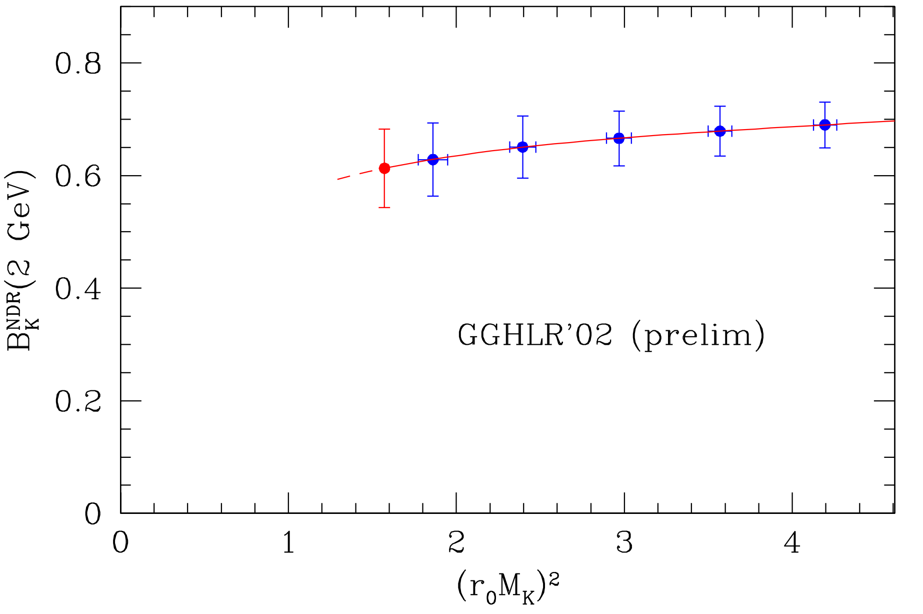
- high statistics
- 3 lattice spacings ($a^{-1} \sim 2.0 \to 3.4\,\mathrm{GeV}$) and continuum limit
- spurious mixing with wrong chirality 4-quark ops subtracted or eliminated through use of Ward identity (Becirevic et al, 2000)
- NP renormalization in RI/MOM scheme

Two studies with overlap fermions (have an exact chiral-flavor symmetry at finite a) (preliminary):

- First weak matrix element studies with overlap fermions
- Validate theoretically clean overlap fermions as a useful phenomenological tool
- Verify that explicit breaking of flavor symmetry (staggered) or chiral symmetry (Wilson) is controlled in standard calculations
- Check domain wall fermion (DW) calculations

1) Garron et al (GGHLR), Lattice 2002:

- 1 lattice spacing ($a^{-1} \sim 2.0$ GeV), $m_s/2 \lesssim m \lesssim m_s$
- NP renormalization in RI/MOM scheme



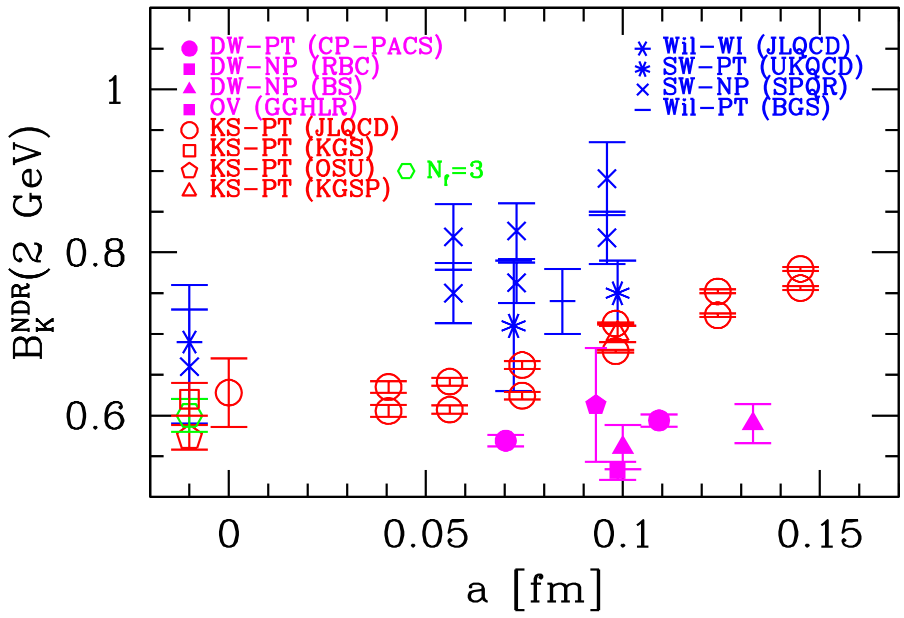
2) DeGrand, Lattice 2002

- 1 lattice spacing ($a^{-1} \sim 1.7$ GeV), $m_s \lesssim m \lesssim 2.5m_s$
- 1-loop renormalization

More overlap and DW calculations on the way

ALPHA is performing a high stat. calculation of $\hat{B}_K$ with tmQCD (twisted mass QCD)

K-$\bar{K}$-mixing: summary (1)



- Results consistent in continuum limit
- DW results are slightly lower (residual chiral symmetry breaking?)
- Reference result is still from quenched staggered JLQCD (1998) calculation (weak point: perturbative renormalization)
- Quenching: $\delta B_K \sim 15\%$ (OSU $N_f = 3$ and QχPT)
- $m_d = m_s \to m_d \neq m_s$: $\delta B_K \sim 5\%$ (χPT)

(Sharpe 1992, 1996)

Final number

$$B_K^{NDR}(2\,\mathrm{GeV}) = 0.628(42)(99) \longrightarrow \hat{B}_K^{NLO} = 0.86(6)(14)$$

with $\hat{B}_K^{NLO}$ two-loop RGI B-parameter

- Same result as in `LL, Lattice 2000`
- Clarify situation regarding DW results
- Need unquenched studies to reduce the 15% quenching error in order to maintain impact of indirect CPV in the kaon system on UT `(talk by Parodi)`

Semileptonic decays

$\mathcal{F}(1)$ **for** $B \to D^*\ell\nu$

$|V_{cb}|$ plays important rôle in constraining UT $\to$ must be determined precisely

- Can extracted from differential rate

$$\frac{d\Gamma}{d\omega} \sim |V_{cb}|^2 |\mathcal{F}_{D^*}(w)|^2$$

extrapolated to zero recoil, i.e. $w = v_B \cdot v_{D^*} = 1$

- HQET and Luke's theorem predict: $\mathcal{F}_{D^*}(1) = 1 + \mathcal{O}(1/m_Q^2)$, but precise measurement of $|V_{cb}|$ requires reliable determination of $\mathcal{F}_{D^*}(1) - 1$

- Through clever use of double ratios of matrix elements for $D^{(*)}, B^{(*)} \to D^{(*)}, B^{(*)}$ `Kronfeld et al (2001)` reconstruct, in a quenched calculation at 3 values of the lattice spacing `(parallel talk by Simone)`

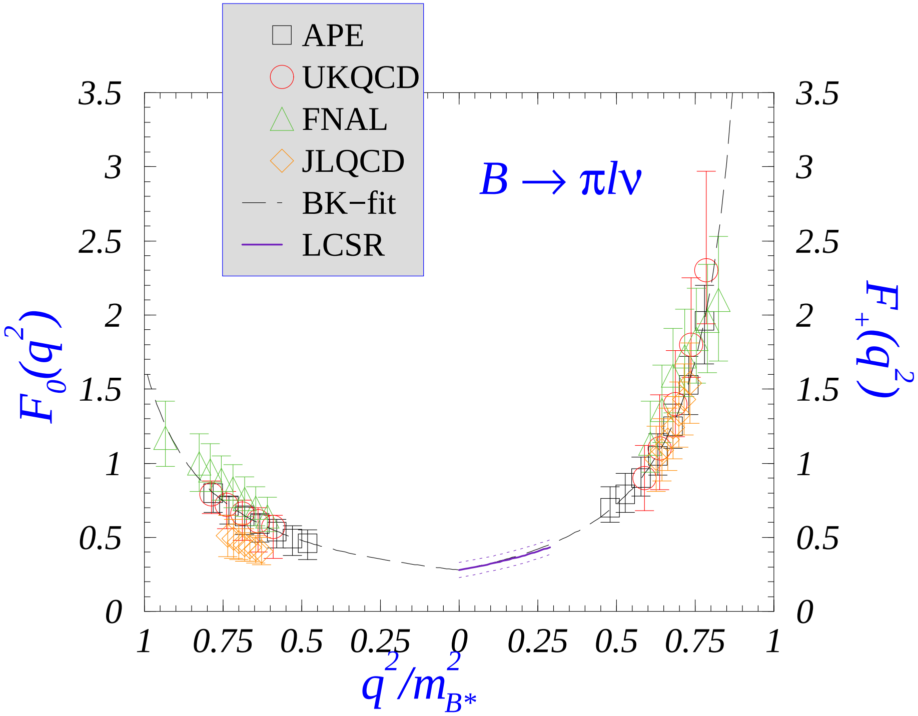
$$\mathcal{F}_{D^*}(1) = 0.913^{+24+17}_{-17-30}$$

This important calculation, which requires excellent control of statistical and systematic errors, should be performed by other groups

Enables measurement of $|V_{ub}|$ (no normalization by HQS here)

$$\langle \pi(k)|\bar{u}\gamma_\mu b|\bar{B}(p)\rangle \longrightarrow F_+(q^2),\ F_0(q^2)$$

Quenched calculations by four groups using relativistic, FNAL and NRQCD quarks:



(Becirevic, ICHEP 2002)

- good consistency on $F^+(q^2)$ which determines rate when $m_\ell \to 0$
- error is of $\mathcal{O}(20\%)$
- Fit of lattice results to BK parametrization (Becirevic et al, 2000) which incorporates most of known constraints on the form factors $\longrightarrow$ extrapolation consistent with LCSR (Khodjamirian et al, 2000)

Need now an unquenched calculation

Conclusion (1)

- Large range of quantities of central importance to particle physics is being computed in lattice QCD simulations, many of which could not be presented here
- For those that were, emphasis now on reduction of systematic errors (quenching, . . .)
- Not mentioned: potentially large reduction in uncertainties obtained by combining ratios of b-quark to equivalent charm quark matrix elements computed on the lattice with charm measurements from e.g. CLEO-c

Also not discussed:

- Major advances in the last few years associated with the formulation and implementation on the lattice fermions which have an exact continuum-like chiral-flavor symmetry at finite a (i.e. overlap, domain wall or fixed-point action fermions $\in$ Ginsparg-Wilson fermions)
- $\Rightarrow$ new possibilities for the calculation of weak matrix elements, in particular those associated with the $\Delta I = 1/2$ rule and direct CP violation in $K \to \pi\pi$ decays `(cf DWF calculations by CP-PACS and RBC, 2001; analytical work by Capitani et al, 2000)`

Conclusion (2)

- $\Rightarrow$ investigation of a numerically unexplored regime of QCD in which the correlation length of pion fields $\gg L$ (ϵ regime of Gasser et al (1987))
- has allowed, in the quenched approximation, the calculation of one of the low-energy constants (LECs) of the strong chiral lagrangian (Hernández et al, 1999; DeGrand, 2001; Hasenfratz et al, 2001)
- It is conceivable to generalize this approach to extract the LECs of the weak chiral Lagrangian by studying the weak interactions in the ϵ regime; a numerical investigation is under way (Giusti et al, in preparation)
- More generally, the range of approaches and the quantities studied in lattice QCD is constantly expanding $\longrightarrow$ should be exciting new results to present at ICHEP 2004