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QCD at high energy

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QCD is the theory of strong interactions

That said, for the next 35 minutes I have two possible choices:

- 1 Show how amazingly well it works: α_s runs, the colour factors are just right, PDFs evolve according to DGLAP, jets are a spectacular proof of some very fundamental ideas, ...
- 2 Discuss where it could fail:
 - Phenomenological issues which need a better understanding
 - New techniques and results, which will play a major role at TeV-scale colliders

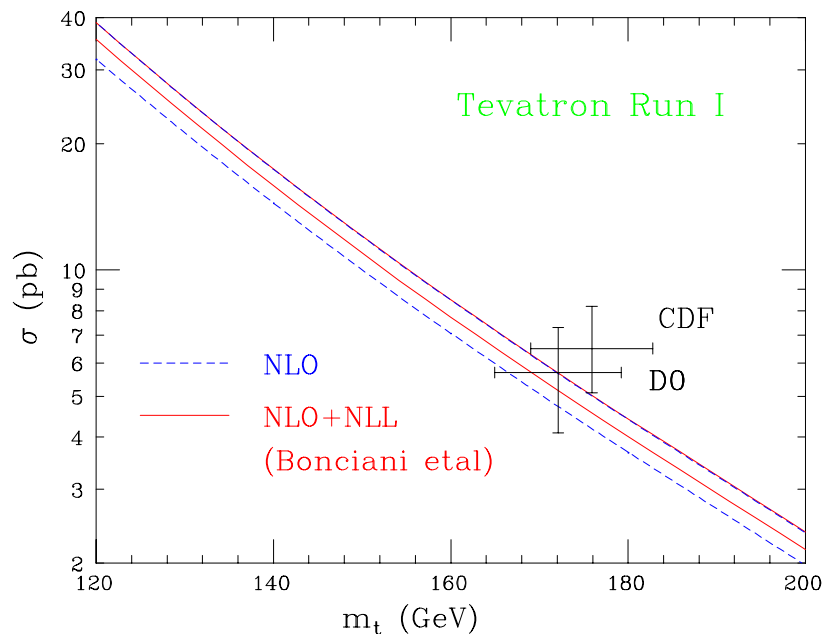
I shall follow option 2. A lot on the outstanding achievements in QCD will be presented by K. Long in the next talk. More on the theory side in Z. Bern's talk

Do we understand heavy flavour (= t, b, c) production?

The breakthrough: $Q\bar{Q}$ rates at the NLO (Nason, Dawson & Ellis; Beenakker, Kuijf, van Neerven, Smith, Meng, Schuler) NLL resummations are also available, for many classes of logs (threshold, large p_T , ...)

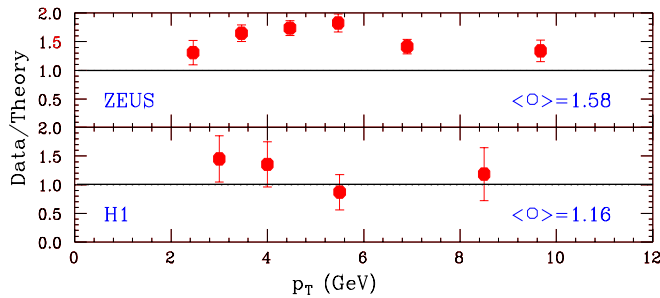
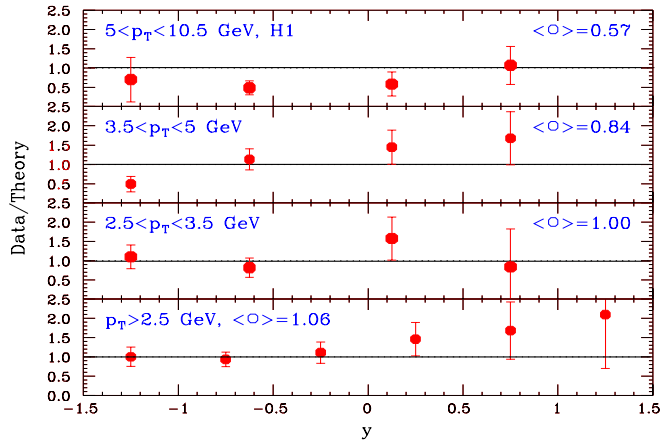
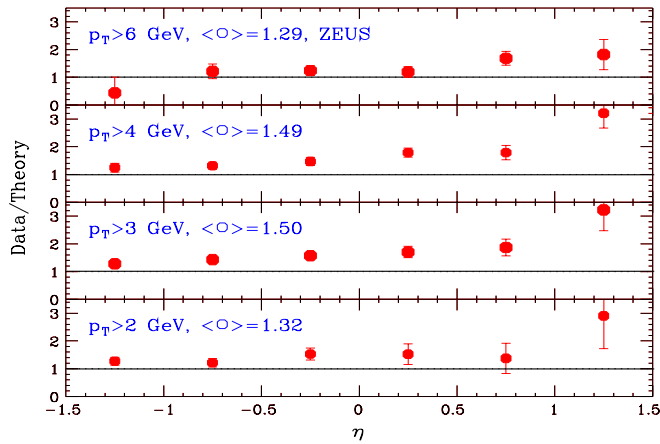
However:

- NNLO corrections are expected to be rather large for c and b ($\alpha_s(m_c) \simeq 0.35$, $\alpha_s(m_b) \simeq 0.2$), moderate for t ($\alpha_s(m_t) \simeq 0.1$, $K_{NLO} \simeq 1.3$ at the Tevatron)
- Non-perturbative effects play a significant role at small masses and small E_{CM}

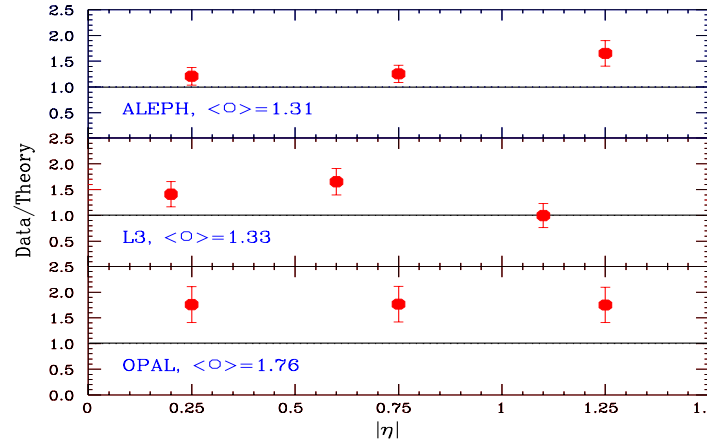
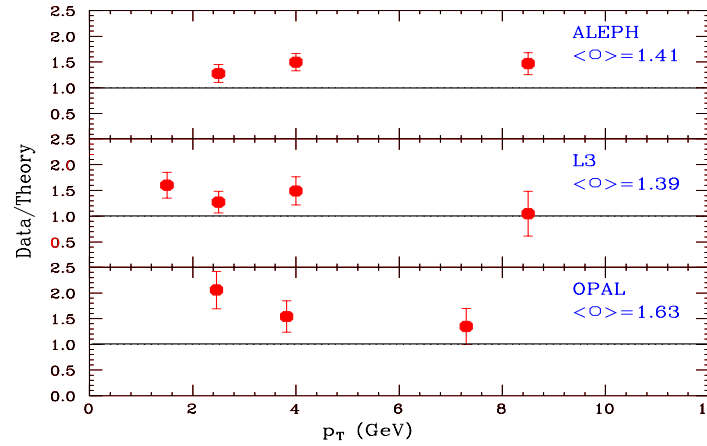


- $t\bar{t}$ production OK. Soft gluon resummation gives a very moderate increase wrt the NLO rate.
- Run II: reduce the errors on mass and rate, measure single-top production
- The picture emerging from the study of b and c production is less clear \longrightarrow

The good news



HERA & LEP



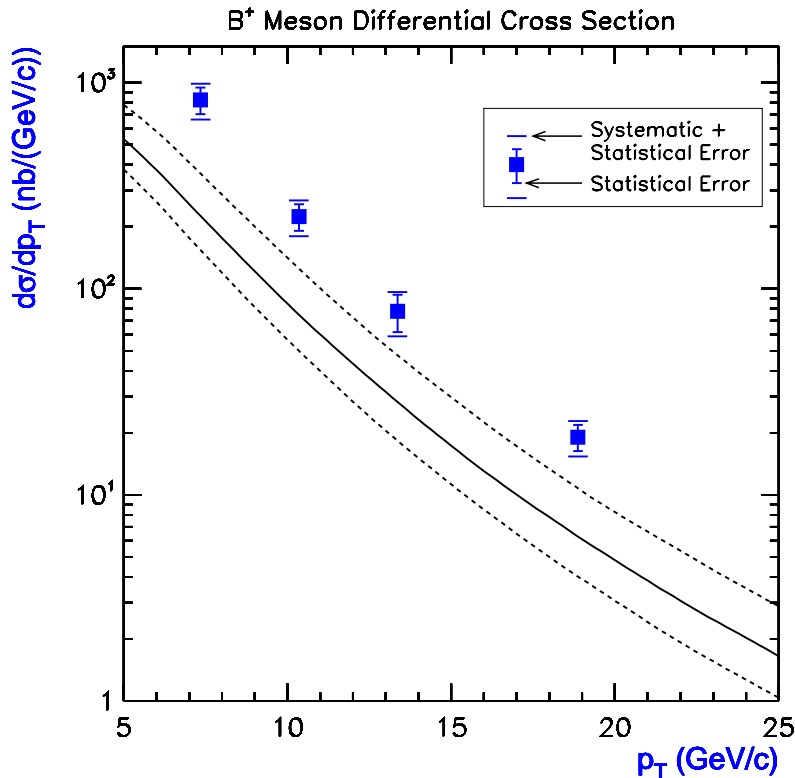
LEP: shapes OK, norm on the upper side of QCD

HERA: H1 OK, ZEUS so-and-so (harder p_T , excess at $\eta > 0$). DIS OK

Fixed-target data are too numerous to summarize (studies of $c\bar{c}$ correlations by E791 and E831). Agreement can be obtained by supplementing NLO with k_T -kick effects

QCD does surprisingly ($m_c/\Lambda_{QCD} \sim 4$) well

The usual bad news



← CDF: Data/Theory = $2.9 \pm 0.2 \pm 0.4$

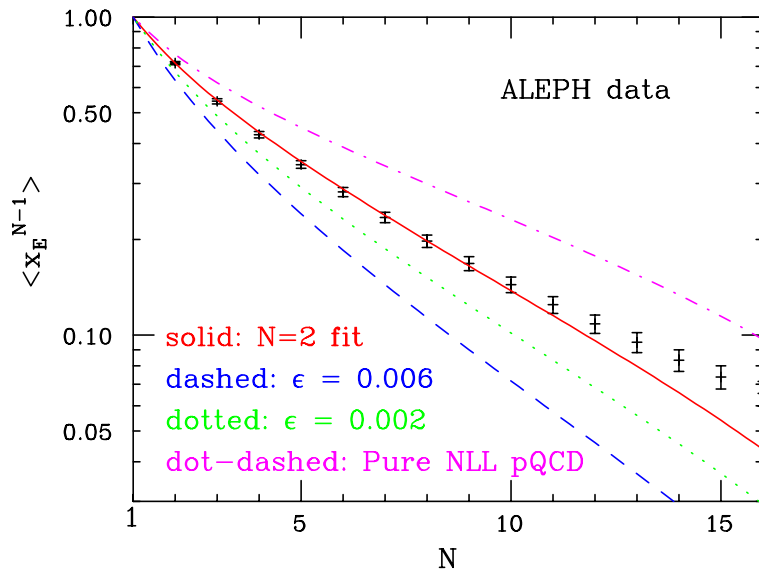
The theory is the same as for charm. Problems with the NP part?

$$\frac{d\sigma_B}{dp_T} = \int dz d\hat{p}_T D(z; \epsilon) \frac{d\sigma_b}{d\hat{p}_T} \delta(p_T - z\hat{p}_T)$$

WARNING : ϵ is non-physical!

- ϵ values obtained from e^+e^- fits: LO \rightarrow 0.007; NLO \rightarrow 0.003; FONLL \rightarrow 0.002
FONLL = NLO + NLL, combines NLO ($p_T \sim m_Q$) with NLL resummation ($p_T \gg m_Q$) (Cacciari, Greco & Nason); also available in e^+e^- , γp
- Mandatory to use at the Tevatron (and anywhere else) the same theoretical approximation used to fit the NP parameter(s)
- The persisting discrepancy prompted investigations on beyond-the-SM effects (Berger&al). However, let's insist that QCD is correct

A very good, but still useless, fit



- The p_T spectrum is power-like

$$\frac{d\sigma_b}{d\hat{p}_T} \simeq \frac{C}{\hat{p}_T^N} \implies \frac{d\sigma_B}{dp_T} = \frac{C}{p_T^N} D_N$$

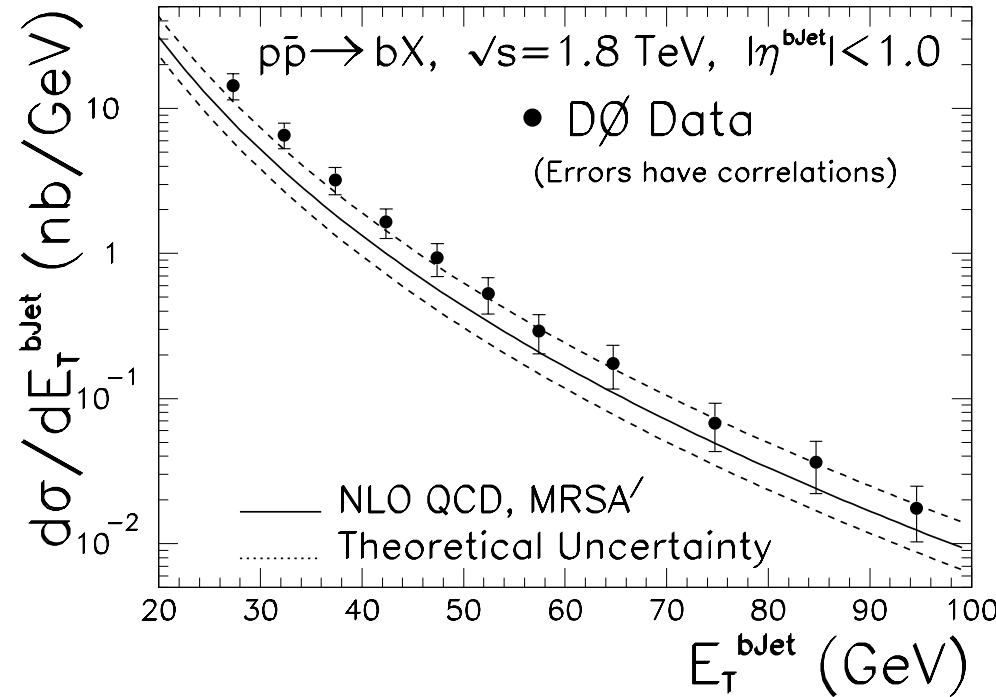
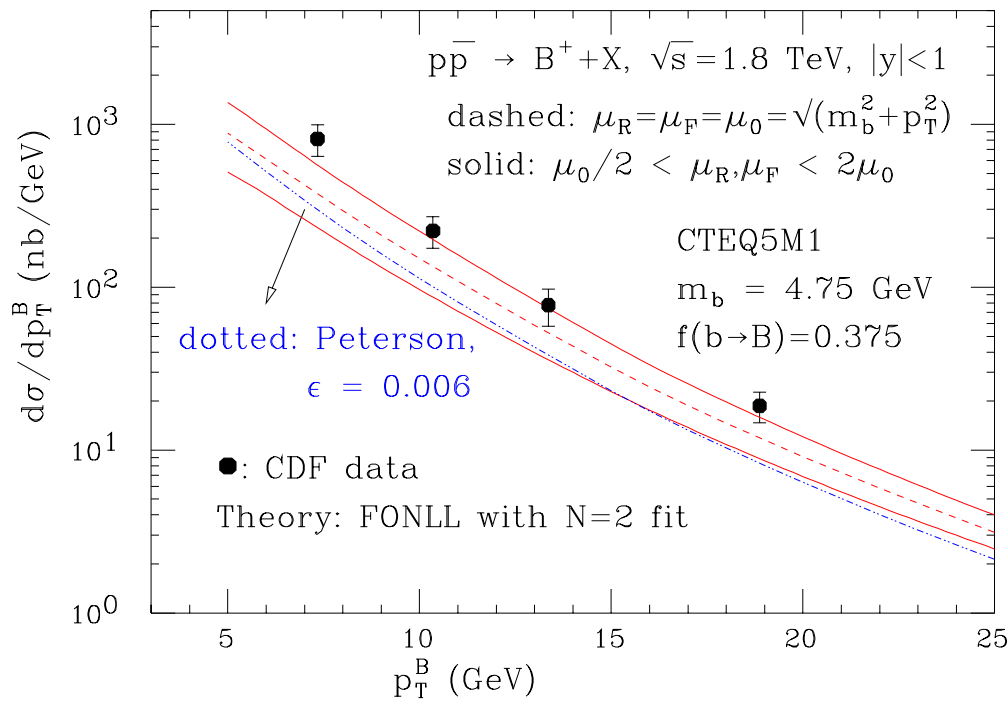
$$D_N = \int dz z^{N-1} D(z; \epsilon)$$

This approximates $d\sigma_B$ fairly well (Mangano)

- Fitted $D(z; \epsilon)$ must agree with data for small Mellin moments – not true for present fits. With FONLL: $\epsilon_{N=2} = 0.0003$ (Cacciari&Nason), $\epsilon_{standard} = 0.002$
Lots of b -fragmentation data submitted to ICHEP

Standard and $N = 2$ fits **are not equivalent**: beyond-LO cross sections **are negative** at large z 's, and this region is not included in standard fits. Unfortunately, the large- z region gives important contributions to the normalization (old FONLL fits with $d\sigma/dz > 0$ gave $\epsilon \simeq \epsilon_{N=2}$! (Nason&Oleari))

It gets much better



Data/Theory = $1.7 \pm 0.5(th) \pm 0.5(exp)$ (Cacciari&Nason)

- Improvement due to FO \rightarrow FONLL (20%), and to the correct treatment of the fragmentation (45%). Data are consistent with the upper end of the QCD band
- Further improvements: small- x and threshold resummation (a $\sim 20\%$ each?), NNLO contributions (probably large/very large)
- QCD does also well for **b-jets** (Mangano&SF). It is probably wise to reconsider former $B \rightarrow b$ deconvolutions

Is everything OK now?

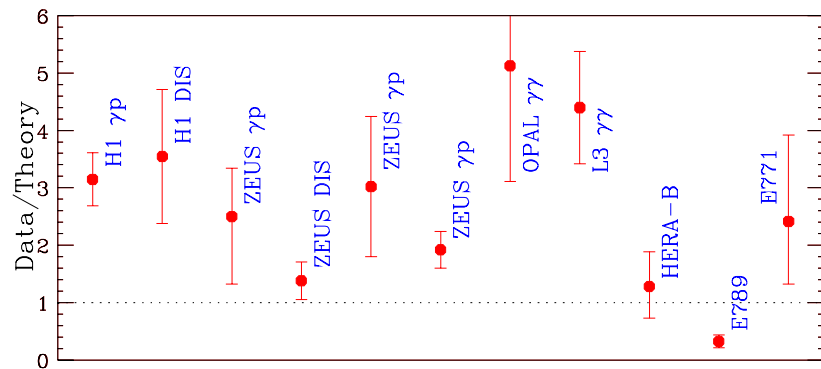
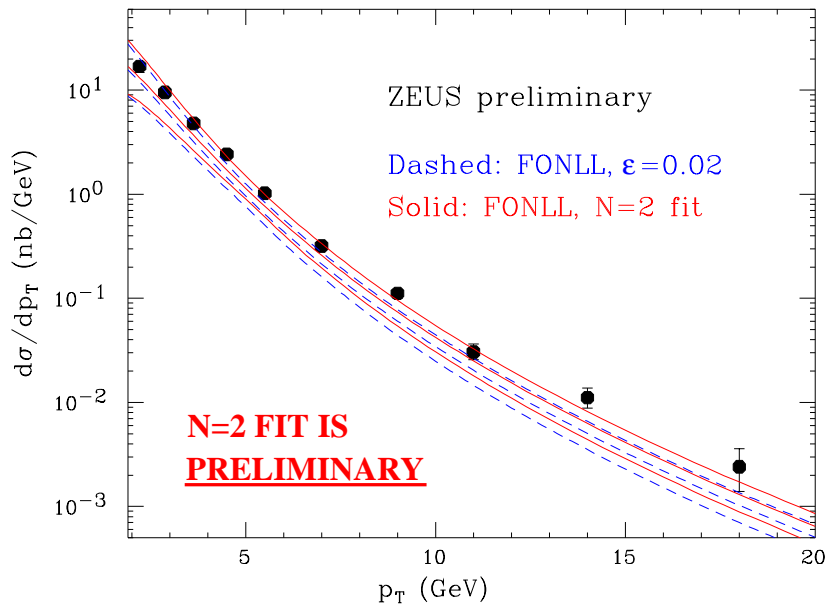
A preliminary N=2 fit gives a better comparison with preliminary Zeuss measurements for D^* production wrt standard fits. However:

- Why pure NLO does a bit better at large p_T ?
- There is still a discrepancy in the positive η region

All in all, charm production data are in good agreement with pQCD predictions

But here comes the b again: $\gamma\gamma$, γp , and DIS rates show **VERY** large discrepancies with pQCD

- Best bet for $1.7 \rightarrow 1.0$ for b at the Tevatron: **NNLO corrections**
- Best bet for b at LEP and HERA: **????**
Be **very** careful with extrapolations



Power corrections: the quest for universality

The problem: understanding hadronization corrections without using MC's

The assumption: the ambiguities of pQCD determine the form of the HC, since pQCD+npQCD=data. Universality \equiv HC's may be different for different observables, but in a calculable manner; they must depend on the same (set of) np parameter(s)

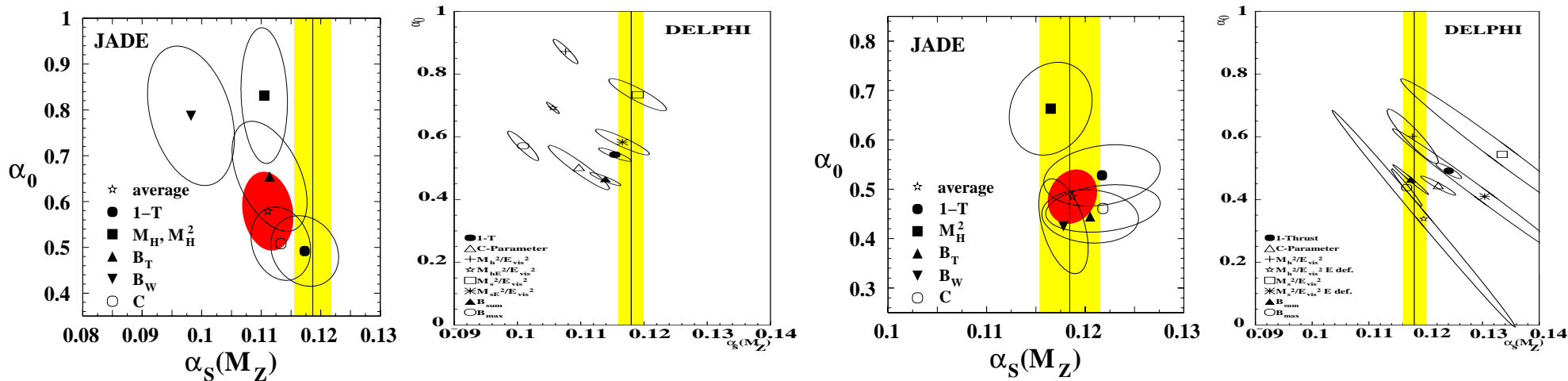
- Universality is related to the behaviour of α_s in the IR (Dokshitzer, Marchesini & Webber; Korchemsky & Sterman). With DMW:

$$\langle \mathcal{T} \rangle = \langle \mathcal{T} \rangle_{pert} + c_{\mathcal{T}} \mathcal{P} \qquad \frac{d\sigma}{d\mathcal{T}}(\mathcal{T}) = \frac{d\sigma}{d\mathcal{T}} \Big|_{pert} (\mathcal{T} - c_{\mathcal{T}} \mathcal{P})$$

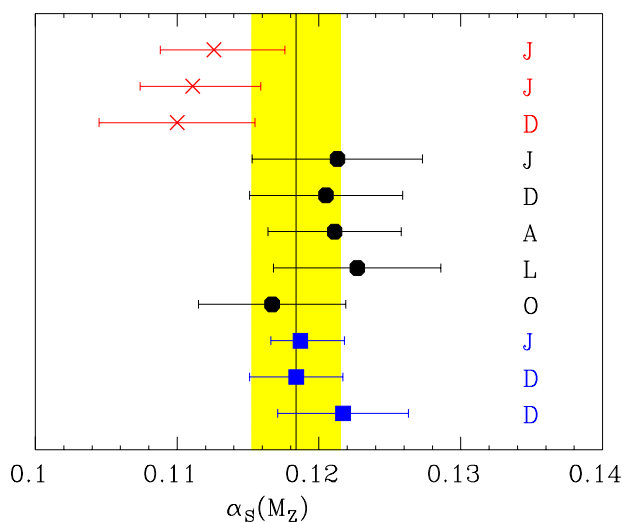
$$\mathcal{P} = \frac{4C_F}{\pi^2} \mathcal{M} \frac{\mu_I}{Q} \left[\alpha_0(\mu_I) - \alpha_s(Q) + \mathcal{O}(\alpha_s^2(Q)) \right] \qquad \mu_I \alpha_0(\mu_I) = \int_0^{\mu_I} dk \alpha_s(k)$$

- \mathcal{T} not inclusive in g decay + ambiguities due to the gluon mass $\implies \mathcal{M}$ parametrizes the inclusion of these subleading corrections (DM&Lucenti&Salam, Dasgupta&W)
- Will \mathcal{M} get large contributions from higher orders? If so, universality is an empty concept. If not, fitting (α_s, α_0) to data should give sensible results

α_s versus α_0 : the results



$\langle \alpha_0 \rangle \sim 0.5$ as expected (DW). Universality holds at $\sim 25\%$ ($1-2 \sigma$, depending on the observables). Distributions are worse than means, the more so when they are “exclusive” (such as B_W and M_H), and for small Q



Crosses: distributions with PC; Blobs: distributions with MC
Boxes: means with PC

- MC's have “more hadronization” than DMW; in a fit to the latter, α_s is driven to small values to compensate for the lack of non-perturbative effects
- How can we improve the description of distributions?

A more refined treatment of non perturbative effects

In the limit of two narrow jets $\mathcal{T} \rightarrow 0$, the emission of soft gluons factorizes wrt the hard process (Korchemsky&Sterman), the two phenomena being incoherent

$$\frac{d\sigma}{d\mathcal{T}}(\mathcal{T}) = \int_0^{\mathcal{T}Q} d\varepsilon f_{\mathcal{T}}(\varepsilon) \left. \frac{d\sigma}{d\mathcal{T}} \right|_{pert}(\mathcal{T} - \varepsilon/Q) \implies f_{\mathcal{T}}^{DMW}(\varepsilon) = \delta(\varepsilon - Qc_{\mathcal{T}}\mathcal{P})$$

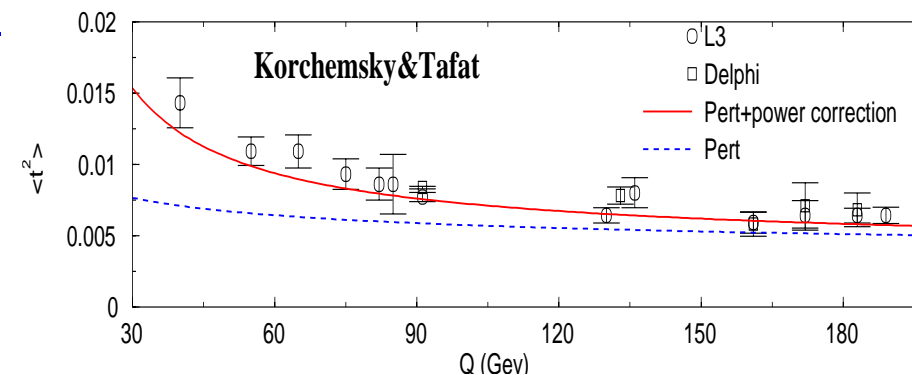
Roughly, the shape function $f_{\mathcal{T}}$ is related to the IR region of the Sudakov

$$\int_0^{\infty} d\varepsilon e^{-\nu\varepsilon/Q} f_{\mathcal{T}}(\varepsilon, \mu_I) = e^{-S_{NP}(Q/\nu, \mu_I)} \equiv DMW + \mathcal{O}((\nu/Q)^2)$$

- K&S use the standard NLL $d\sigma_{pert}$. In DGE (Gardi&Rathsman) the renormalon chain is exponentiated (Beneke&Braun). Renormalon ambiguities \Leftrightarrow form of $f_{\mathcal{T}}$
- With DGE, results for T and M_H are in better agreement in the (α_S, α_0) plane. But:

$$\alpha_S(M_Z) = 0.1086 \pm 0.0004(exp)$$

Theory error is about 5%



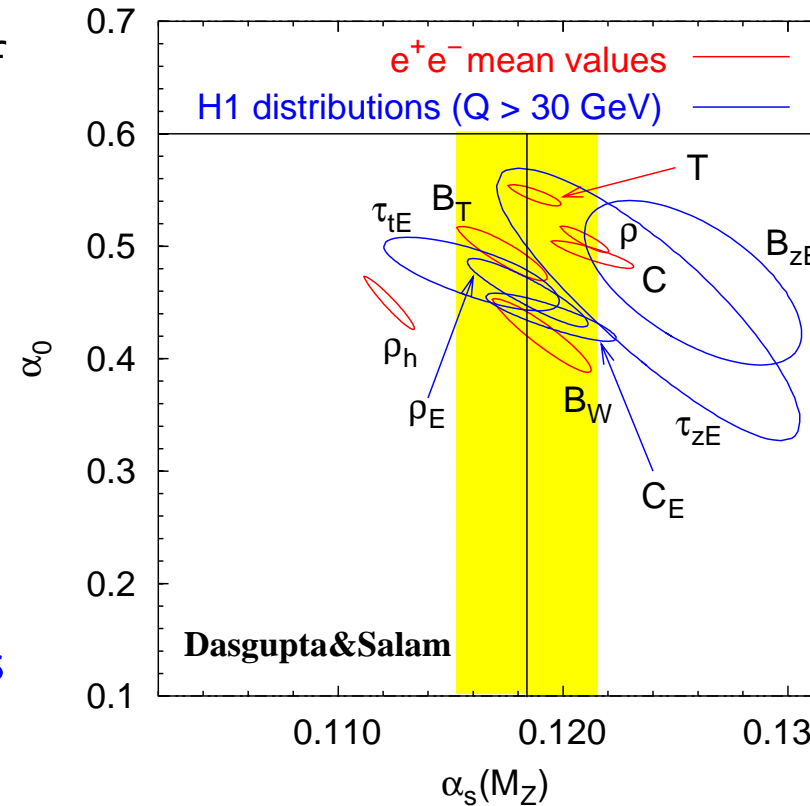
A cross check in DIS

Progress has been recently made for resummation of event shapes in DIS (Antonelli, Dasgupta & Salam).
More data needed to reduce errors

Furthermore:

- Beware of mass effects (Salam&Wicke), which can contribute $(\log Q)^{1.6}/Q$: some of them can be eliminated using the E-scheme
- Extend the investigation to 3-jet-like quantities (Banfi&al), such as K_{out} , with g at Born
- Minimal sensitivity and RGI analyses by DELPHI show that one can live without power corrections

A tentative conclusion: universality is supported by data, but distributions display unpleasant features. Minimal sensitivity + RGI + large theoretical uncertainties
 \implies compute NNLO

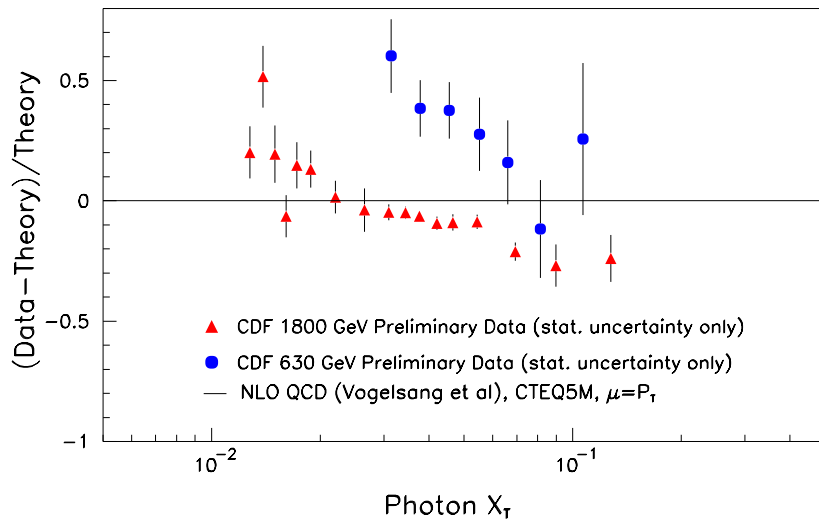


Why bother (apart from principle reasons)?

A few issues would surely benefit from a better understanding of power corrections

We still don't understand well prompt photon production at the Tevatron (mainly at CDF)

- Must also consider: 1) Isolation cuts (theory and experiments should be consistent) 2) Joint re-summation (Laenen, Sterman & Vogelsang)

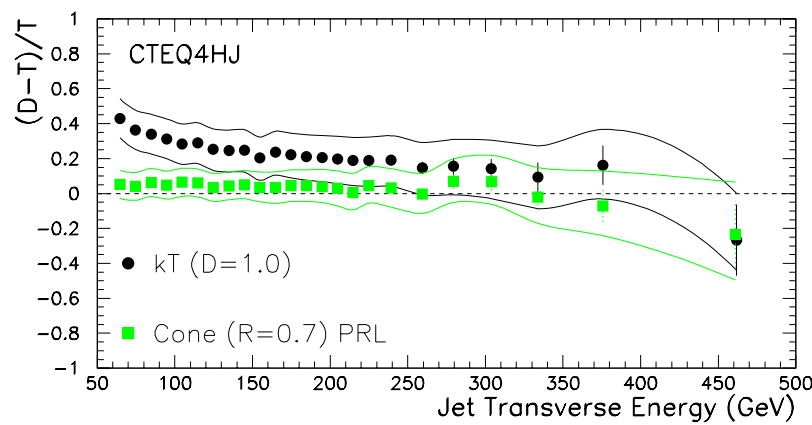


- Recent CDF and D0 results do constrain PDF at large $E_T(\text{jet})$: "default" set is HJ-like, and the agreement with QCD is very satisfactory in the whole range

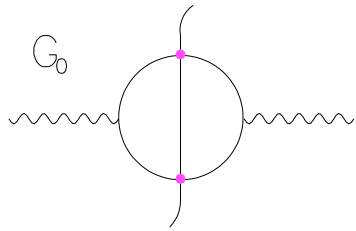
Why does the k_T algorithm display discrepancies?

- MC studies show that k_T and cone algorithms are affected differently by hadronization

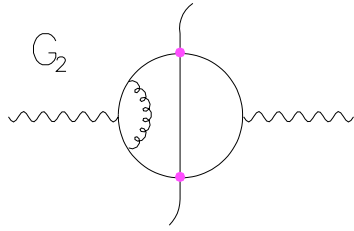
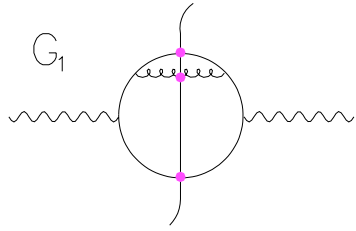
To add to the puzzle, k_T alg works OK at HERA



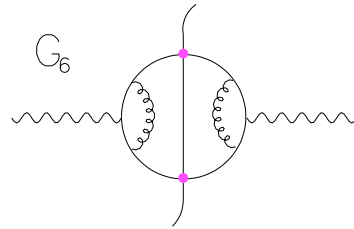
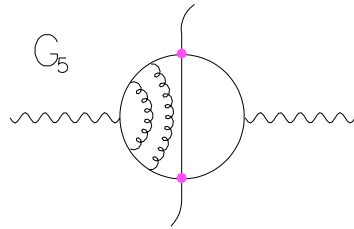
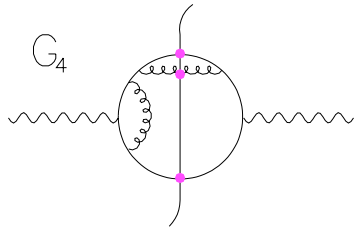
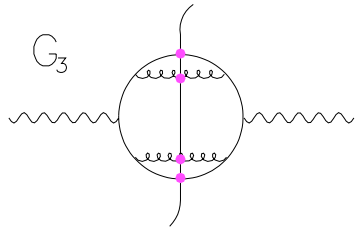
NNLO computations



$\mathcal{O}(\alpha_S^0)$ LO



$\mathcal{O}(\alpha_S^1)$ NLO

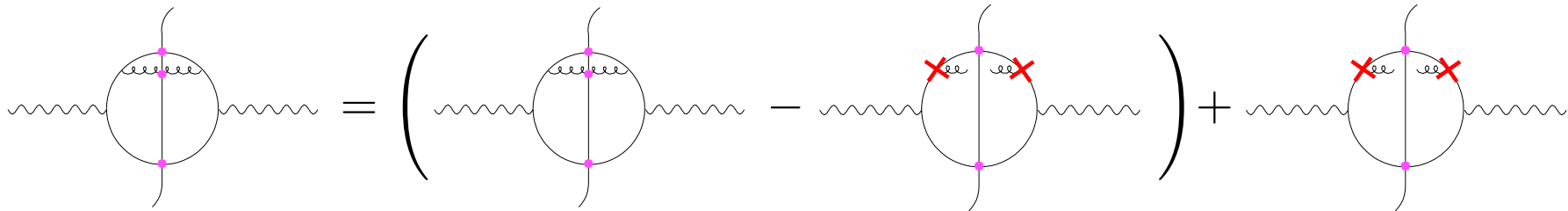


$\mathcal{O}(\alpha_S^2)$ NNLO

- At the NLO, the extra parton can be: G_1) real; G_2) virtual
- At the NNLO, the **two** extra partons can be: G_3) both real; G_4) one real and one virtual; G_5+G_6) both virtual
- *The problem* is the same: how to cancel analytically the divergences without performing a complete analytical computation, which is impossible in general
- **A notable exception**: fully-inclusive quantities. With simple kinematics, analytical integrations possible (but tough): DY K-factor, DIS CF, Higgs (new!)

Non-inclusive cross sections

At the NLO, add and subtract the **most singular part** of the real matrix element, using reduced kinematics to compute the observable (Ellis, Ross & Terrano)



The diagram shows an equation for a one-loop correction to a cross section. On the left is a circle with two external wavy lines and two internal wavy lines, with two pink dots on the vertical line. This is equal to the difference of two similar diagrams in parentheses, plus a third diagram. The first diagram in parentheses is identical to the left one. The second diagram in parentheses has red 'X' marks on the two internal wavy lines. The third diagram on the right also has red 'X' marks on the two internal wavy lines.

If we use the same strategy at the NNLO, we have to:

- A:** Compute 2-loop integrals ($2 \rightarrow 2$ and $1^* \rightarrow 3$ now available: see Bern's talk)
- B:** Compute the most singular terms of double-real and real-virtual diagrams (done!)
- C:** Use the result of B to construct the **IR counterterms**, avoiding **overlapping divergences**. Integrate the counterterms analytically, factoring out the phase space of hard partons (to be integrated numerically)

At variance with **B** and **C**, **A** has to be carried out for each and every new process (as usual, the computation of finite parts is harder than that of divergences). Available results will allow the computation of $e^+e^- \rightarrow 3 \text{ jets}$ and $H_1 H_2 \rightarrow 2 \text{ jets}$

BUT ONLY WHEN **C** WILL BE SOLVED

NNLO computations: worth the effort?

- 1) Better estimates for total production rates
- 2) Reduced theoretical uncertainties (dependences on mass scales)
- 3) More realistic kinematical features

In hadronic collisions, one needs **NNLO-evolved PDFs**. NNLO MRST set is only an approximation, since:

- AP kernels are only known at three loops (vNeerven&Vogt) through their lowest Mellin moments (Larin, Nogueira, Retey, Ritbergen, Vermaseren), and small- x behaviour; exact computation under way (Moch, Vermaseren, Vogt)
- The only genuine NNLO result used are DIS CFs (in DY, MRST use $x_F(NLO) \times K_{NNLO}/K_{NLO} \leftarrow$ need for less-inclusive results)

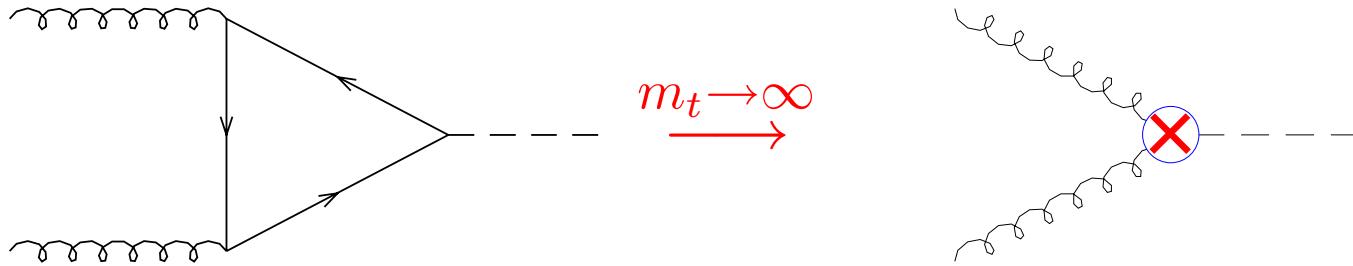
PDFs uncertainties must be carefully considered: a lot of activity in the field recently (Giele, Keller & Kosower, Alekhin, Botje, CTEQ)

\Rightarrow Certainly worth the effort, but NNLO phenomenology still awaits the solution of difficult technical problems. **Precision** physics requires a better understanding of the interplay between perturbative and non-perturbative/soft physics. **Processes with large K-factors must be high in the priority list**

SM Higgs at the NNLO

$K_{NLO} \sim 2 \implies$ we better compute NNLO corrections. This is feasible since:

A) gg channel dominates; B) the top is very heavy;



C) the effective ggH interaction is known to $\mathcal{O}(\alpha_s^4)$ (Chetyrkin, Kniehl & Steinhauser)

- The kinematics is Drell-Yan like
- The gluon is peaked towards small x 's: the process is dominated by threshold production $x_H = M_H^2/\hat{s} \rightarrow 1$

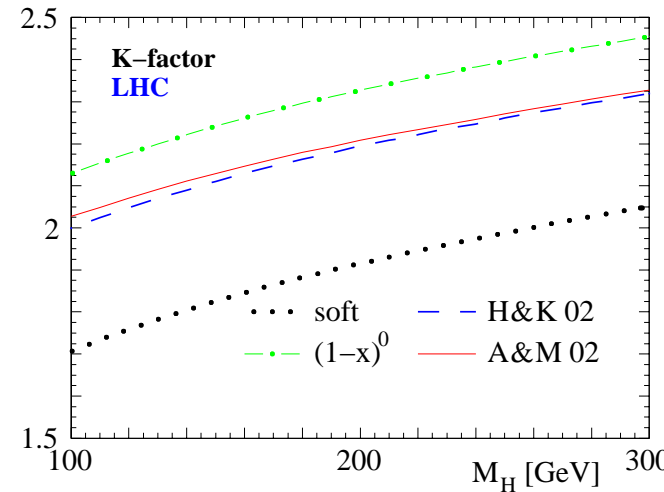
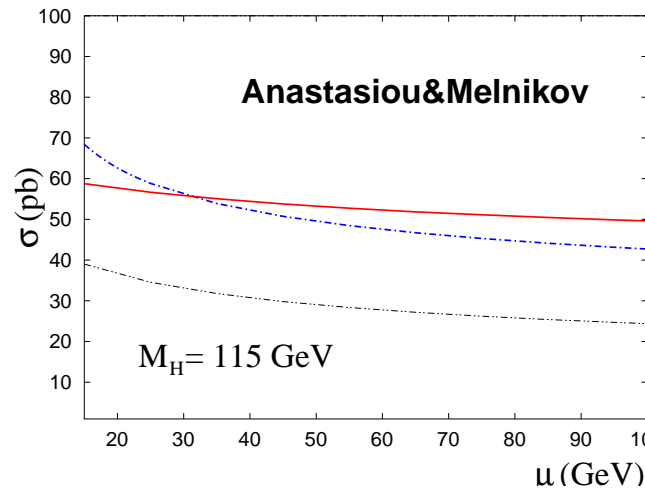
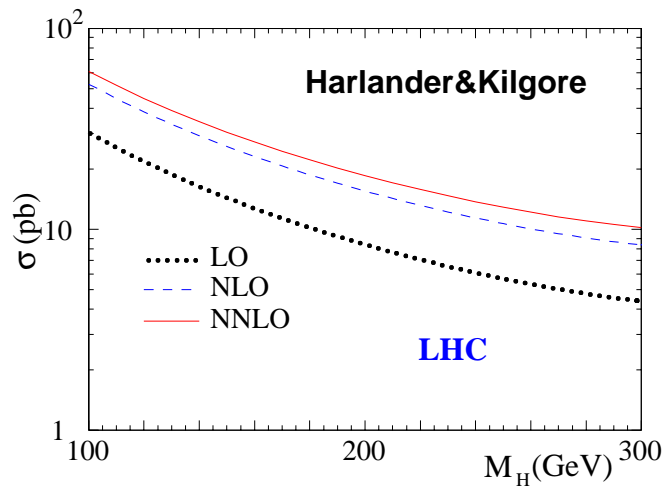
NLO \rightarrow NNLO took a couple of years!

- 0) The computation of $gg \rightarrow H$ to 2 loops (Harlander)
- 1) 2-loop+SC (Catani, deFlorian & Grazzini) or 2-loop+S+SL (Harlander&Kilgore) approximation of double-real ME's (most singular terms for $x_H \rightarrow 1$)
- 2) Double-real ME's expanded for $x_H \rightarrow 1$, up to $(1 - x_H)^{16}$, the rest exact (H&K)

Not the end of the story

The expansion around $x_H = 1$ is seen to converge very fast. But just in case:

- 3) Double-real ME's computed exactly (Anastasiou&Melnikov). Multi-loop-like techniques now applied to phase-space integrals (see Bern's talk)



A&M agree well with H&K. $\log^2 x_H$ terms agree with those obtained with resummation techniques (Hautmann). And there is more:

- Soft gluon resummation in progress (CdFG&Nason): at the LHC/Tevatron
 $NLL + NLO = NLO (1 + 20\%/30\%)$, $NNLL + NNLO = NNLO (1 + 9\%/16\%)$
- Higgs+jet, with jet veto (CdFG): $\sigma_{veto} = \sigma_{incl} - \Delta\sigma(p_T^{(jet)} > p_T^{(veto)})$. This allows bkg reduction to $H \rightarrow W^*W^*$ due to misidentified b -jets in $t\bar{t}$ or tW production

MC's have (at least) two problems

\mathcal{P}_1 : Cannot simulate the emissions of hard partons

\mathcal{P}_2 : Cannot go beyond LO in the computation of the rates

Problems \mathcal{P}_1 and \mathcal{P}_2 are going to be acute at TeV-scale colliders: multi-jet channels are standard discovery tools, and huge backgrounds call for a precise estimate of the rates.

The solutions:

\mathcal{S}_1 : The improved MC is capable of simulating the emission of n_E extra hard partons

\mathcal{S}_2 : The improved MC knows how to compute the total rate to N^k LO accuracy

Implicit is the notion of Born \equiv LO level, as the process(es) with the smallest number n_B of final-state partons which contributes to a given reaction (usually, but not necessarily, a $2 \rightarrow 2$ process).

Implementation of \mathcal{S}_2 (multiplication by the K-factor) is not in the spirit of event generators (it's inclusive). And, it doesn't make sense for jets. Thus:

$\mathcal{S}_1 \implies$ Matrix Element corrections

$\mathcal{S}_1 \oplus \mathcal{S}_2 \implies$ MC@N^kLO, with $n_E = k$

A simple way to understand MC@NLO

A system S moves along a line between 0 and 1. It can radiate “photons”, whose energy we denote with x . S can undergo several further emissions; on the other hand, one photon cannot branch

$$\left(\frac{d\sigma}{dx}\right)_B = B\delta(x) \quad \longleftrightarrow \quad \begin{array}{c} \bullet \\ x=0 \end{array} \text{-----} \begin{array}{c} \\ x=1 \end{array}$$

$$\left(\frac{d\sigma}{dx}\right)_V = \alpha_S \left(\frac{B}{2\epsilon} + V\right) \delta(x) \quad \longleftrightarrow \quad \begin{array}{c} \star \\ \bullet \\ x=0 \end{array} \text{-----} \begin{array}{c} \\ x=1 \end{array}$$

$$\left(\frac{d\sigma}{dx}\right)_R = \alpha_S \frac{R(x)}{x} \quad \longleftrightarrow \quad \begin{array}{c} \text{~~~~~} \\ \bullet \\ x=0 \end{array} \text{-----} \begin{array}{c} \\ x=1 \end{array}$$

It doesn't need to be QCD, but must behave the same. Thus:

$$\lim_{x \rightarrow 0} R(x) = B$$

This condition guarantees that $V+R$ is finite. One can therefore proceed with standard techniques. B&V kinematics: $(S, 0)$; R kinematics: (S, x)

NLO \oplus MC \longrightarrow MC@NLO?

A: In standard MC's $(S, 0)$ happens to be the initial condition for the shower

B: At the NLO, kinematical configurations (S, x) and $(S, 0)$ are generated, and used to fill the histogram bins

Try the same at the NLO as at the LO: $(S, 0)$ and (S, x) are MC initial conditions

$$\frac{d\sigma}{dO} = \int_0^1 dx \left[I_{MC}(O; S, x) \frac{\alpha_S R(x)}{x} + I_{MC}(O; S, 0) \left(B + \alpha_S V - \frac{\alpha_S B}{x} \right) \right]$$

It doesn't work:

- Cancellations between (S, x) and $(S, 0)$ contributions occur **after the shower**: hopeless from the practical point of view (unweighting impossible)
- $d\sigma/dO - (d\sigma/dO)_{NLO} = \mathcal{O}(\alpha_S)$. In words: **double counting**

The problem is a fundamental one: **KLN cancellation** is achieved in standard MC's through **unitarity**, and embedded in Sudakovs. This is no longer possible: IR singularities **do appear in hard ME's**

MC@NLO: slicing

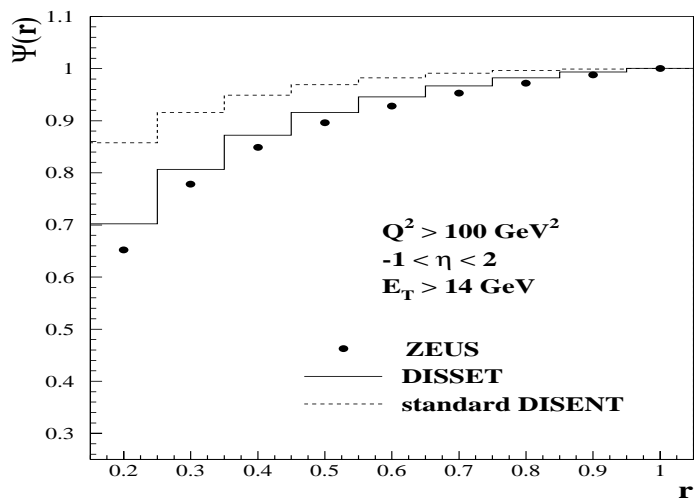
Exploit a proposal by Baer&Reno to get rid of the soft/unresolved configurations:

$$B + \alpha_S (B \log \delta_0 + V) = 0 \quad \Longrightarrow \quad \delta_0 = \exp \left[- (B + \alpha_S V) / \alpha_S B \right]$$

Another parameter $\delta_{PS} > \delta_0$ separates the shower region from the hard region (Pötter, Schörner, Dobbs)

$$\frac{d\sigma}{dO} = \alpha_S \int_{\delta_{PS}}^1 dx I_{MC}(O; S, x) \frac{R(x)}{x} + \alpha_S \int_{\delta_0}^{\delta_{PS}} dx I_{MC}(O; S, 0) \frac{R(x)}{x}$$

- + Only positive weights
- + Doesn't need to know details of MC implementation



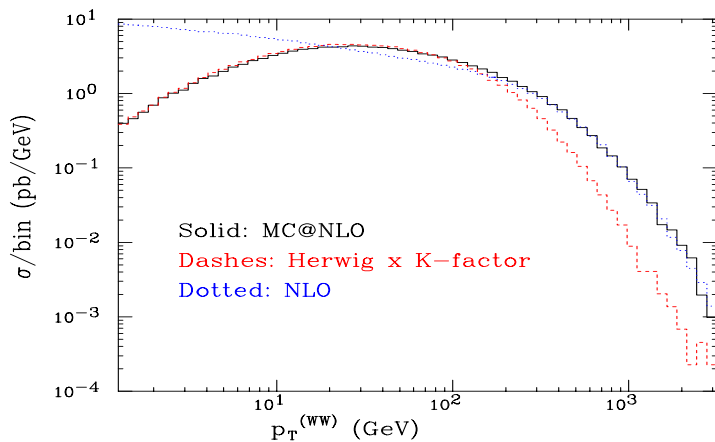
- Double counting for $x < \delta_{PS}$, and discontinuity at $x = \delta_{PS}$ imply dependence upon δ_{PS} , which is hidden by integration over Bjorken x 's
- Strictly speaking, the (perturbative) result is non-perturbative, since $\delta_0 \sim \exp(-1/\alpha_S)$

MC@NLO: modified subtraction

Get rid of the MC $\mathcal{O}(\alpha_s)$ contributions by an extra subtraction of the $\mathcal{O}(\alpha_s)$ term in the expansion of the Sudakov, $\alpha_s Q(x)/x$ (Webber & SF):

$$\frac{d\sigma}{dO} = \int_0^1 dx \left[I_{MC}(O; S, x) \frac{\alpha_s [R(x) - BQ(x)]}{x} + I_{MC}(O; S, 0) \left(B + \alpha_s V + \frac{\alpha_s B [Q(x) - 1]}{x} \right) \right]$$

- Negative weights ($\sim 10\%$) – can't be avoided completely
- The subtraction terms are MC-implementation dependent



- + There is **no unphysical parameter**. Soft and hard emissions are **smoothly** matched
- + NLO results **are recovered** upon expansion in α_s
- + The method can be applied to any process

Collins & al aim at implementing **NLL resummation**. The method is not fully defined in QCD so far (lacks gluon emission). Very recent work by **Kurihara**

ME corrections: a closer look

The approaches to this problem belong to two classes:

Class #1: Includes in the MC the computation of ME's with n_E as large as possible.

Thus, $n_B \rightarrow n'_B = n_B + n_E$. Processes with different n'_B 's are not related

- + The MC is not modified: ME computation provide it with the kinematics and the colour flow of the initial configuration
- The results depend on an unphysical parameter δ_{sep} , which must be introduced at the parton level to avoid divergences

Class #2: This improves on the results of Class #1: processes with different numbers of hard partons are consistently combined

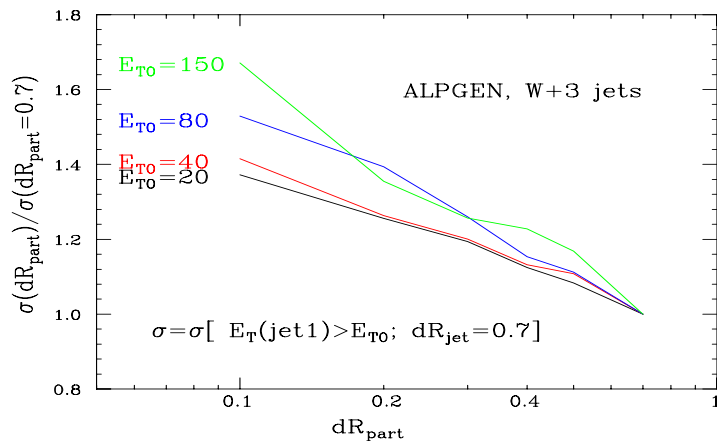
- + The dependence on the unphysical parameter is much reduced
- The MC showering mechanism has to be modified

ME corrections: class #1



This is a *very* active field. Many packages available ([AcerMC](#), [ALPGEN](#), [CompHEP](#), [Grace](#), [MadGraph2](#)), with $W/Z/nQ/nq/ng$ final states. This motivated the definition of a standard (*Les Houches accord*) for the middle box above. Problems solved:

- Efficient ME/phase space generation for n -parton final states. ALPGEN ([Mangano & al](#)) is the only one not based on diagrammatics, thus deals with larger n (thanks to Alpha, which is $\sim n!/3^n$ more efficient – [Caravaglios&Moretti](#))
- Information on colour flow passed on to the MC. Different generators should be equivalent up to $1/N_C^2$ terms



WARNING! Physical predictions may depend on the unphysical parameter δ_{sep}

The cross section is known at the 10–20% level

Still, a *very significant* improvement wrt standard MC's

Getting rid of δ_{sep} dependence: class #2

When $n_E = 1$, just reweight the MC cross section to match smoothly the ME result (Seymour, Sjöstrand)

In a new approach to e^+e^- , Catani, Krauss, Kuhn & Webber show that the problem cannot be solved at fixed n'_B , and with standard MC's. Extended to colour dipoles by Lönnblad; proposal for hadronic collisions by Krauss

- In the n -jet region, any observable is accurate to $\mathcal{O}(\alpha_S^{n-2})$, for any n
- In the n -jet region, large logs of the observable O are resummed according to

$$\sigma_n \sim \alpha_S^{n-2} \sum_k \left(a_k \alpha_S^k \log^{2k} O + b_k \alpha_S^k \log^{2k-1} O \right)$$

- The dependence upon δ_{sep} is:

$$\sigma_n \sim \alpha_S^{n-2} \left(\delta_{sep}^a + \sum_k c_k \alpha_S^k \log^{2k-2} \delta_{sep} \right)$$

When only $n \leq N$ ME's are available, results are accurate to $\mathcal{O}(\alpha_S^{N-1})$

The implementation in e^+e^-

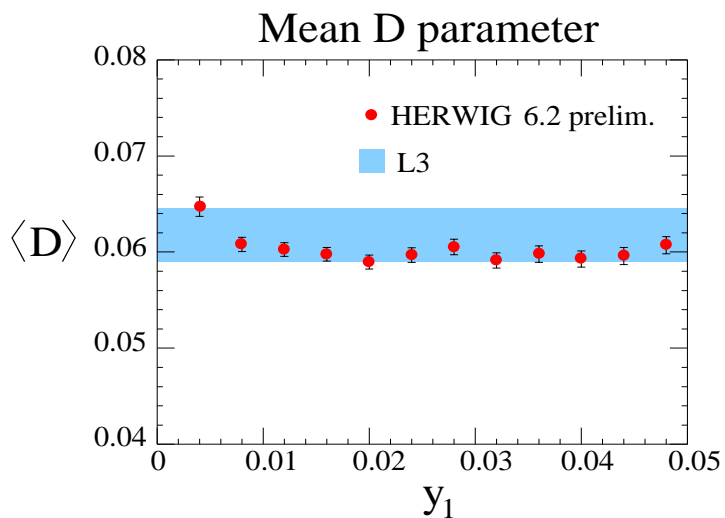
The procedure implies a modification of both the ME's **and** of the shower evolution.

After fixing δ_{sep} :

- Choose n according to the jet rates obtained with resolution δ_{sep} : $y_{ij} > \delta_{sep}$, with

$$y_{ij} = 2 \min\{E_i^2, E_j^2\}(1 - \cos \theta_{ij})/Q^2$$

- Generate an n -parton kinematical configuration according to ME, and **reweight** it by the probability of no **further branchings** (a combination of Sudakovs)
- After successful unweighting, use the n -parton kinematics as initial condition for the shower, **vetoing** all branchings such that $y_{ij} > \delta_{sep}$



n -parton contributions may have LL ($\alpha_S^k \log^{2k} \delta_{sep}$) dependence, which reduces to NNLL when all n are included

A **practical** problem for the extension to hadronic physics is the computation of all the total rates, in order to distribute events in n

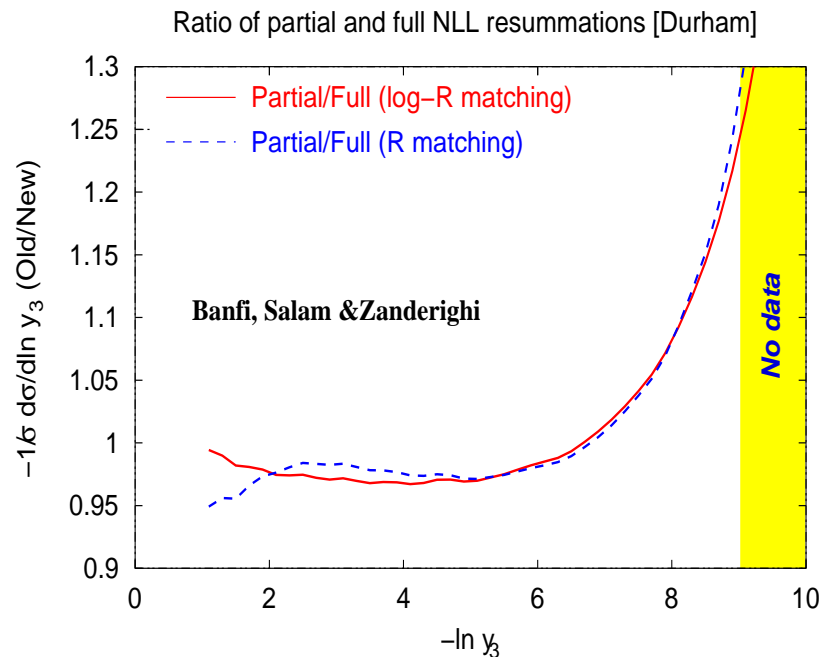
Numerical resummations

Beyond-LL results are difficult to obtain analytically because of multiple-emission (\sim recoil) effects. Can use MC's, which however lack terms in any N^k LL towers

A new approach (Banfi, Salam & Zanderighi): given the observable \mathcal{T} , define a “simple” observable \mathcal{T}_s which has the same LL structure as \mathcal{T} , but is trivial to exponentiate:

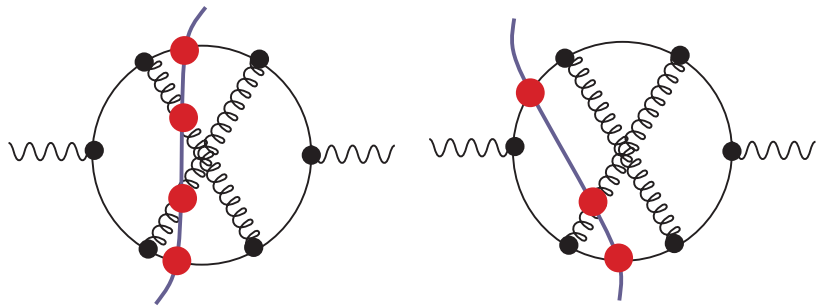
$$\frac{d\sigma}{d\log \mathcal{T}} = \frac{d\sigma_s}{d\log \mathcal{T}_s} \mathcal{F}(\mathcal{T}, \mathcal{T}_s)$$

\mathcal{F} includes all the recoil effects, and is computed numerically



- Cancellation of NNLL and beyond requires either arbitrary precision numerics, or extra analytic work
- Applies to a (rather broad) class of observable (thrust in 3-jet region, Jade jets *not* included)
- Flexible: new results in e^+e^- obtained (T_M, O, y_3), DIS almost completed, work for hadronic collisions is under way

Numerical NLO computations



Consider $e^+e^- \rightarrow 3 \text{ jets}$:

$$\sigma(\mathcal{O}(\alpha_s^2)) = \sum_G \sum_C \int d\vec{l}_1 d\vec{l}_2 d\vec{l}_3 \mathcal{G}(G, C; \vec{l}_i)$$

Standard procedure: If C includes a loop, integrate over it analytically; if not, use slicing/subtraction to extract divergences. Sum over cuts and obtain finite results

Numerical procedure (Soper): move \sum_C under the integral sign, exploit analyticity to deform the integral contour, and perform the integral by MC methods

Results for $e^+e^- \rightarrow 3 \text{ jets}$ (the only ones available) in agreement with standard results

- Extra work required for more partons (contour deformation), or new UV-divergent graphs. IR divergences problem solved for any number of partons
- Extension to NNLO has the same problems as analytical+numerical procedure
- Results in Coulomb gauge available (**new!**), as is desirable to interface with shower (**Krämer&Soper**). Without any cutoffs, only weighted events produced

Conclusions

It is unfortunately impossible to squeeze the enormous amount of theoretical and experimental work into a short talk. I wish I could have mentioned:

- A lot of new NLO computations
- Small- x physics (impact factors to NLO)
- Tests of DGLAP evolution
- ... and much more

It is reassuring, and shouldn't be taken for granted, that

- 1 There is no compelling evidence of a serious problem in QCD
- 2 There has been great progress recently

Although a lot of work remains to be done, we are on the right track. Stay tuned: other interesting results will appear soon

By the way: you will not find SUSY if you don't understand QCD