Recent Progress in Computational Perturbative Quantum Field Theory

Z. Bern
UCLA

ICHEP 2002, Amsterdam

Outline

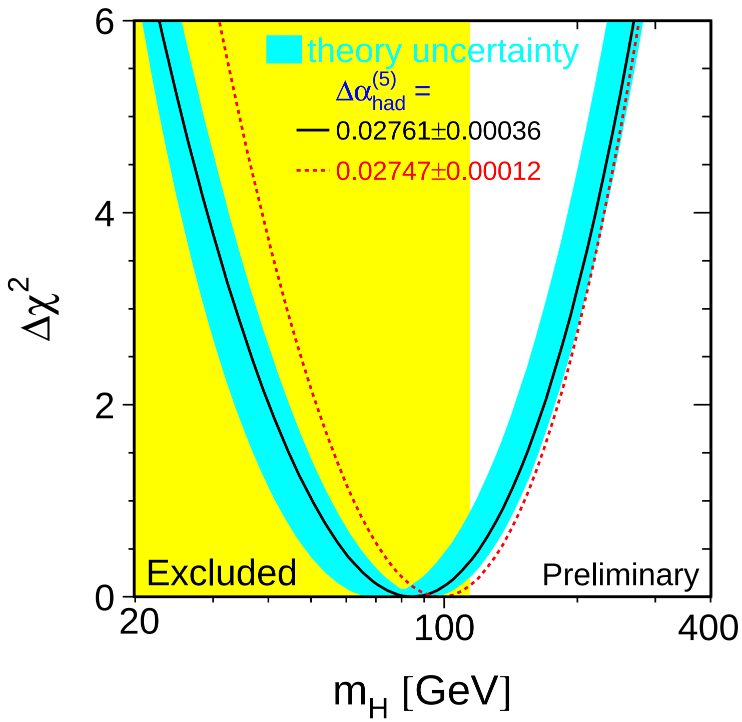
The last few years have seen remarkable progress in computing higher loop corrections to Standard Model processes.

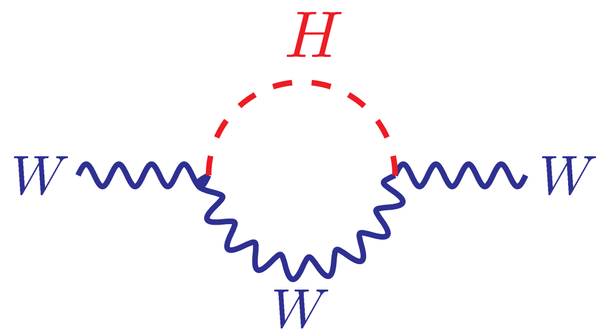
- Recent example demonstrating the crucial importance of challenging perturbative computations for experiments: Muon $g-2$
- Specific computational progress of the last few years:
  (a) Two-loop Feynman diagram calculations
  (b) Improved loop integration methods
- Applications:
  (a) Higgs signal and backgrounds at the LHC
  (b) Promise for future.

Precision Calculations

Precision calculations in the Standard Model have a long history. 1999 Nobel Prize awarded to ‘t Hooft and Veltman for providing the tools and theoretical foundation necessary for carrying out such computations.

LEPEWWG





$M_H < 195$ GeV

Famous example of the importance of precision calculations in quantum field theories

Importance of Higher Order Calculations

A spectacular recent example illustrating the importance of higher order perturbative calculations in quantum field theory is anomalous magnetic moment of the muon.

$$\vec{\mu} = g\,\frac{e}{2m}\,\vec{S}, \qquad\qquad a_\mu = \frac{g-2}{2}$$

Muon anomalous magnetic moment is approximately $(m_\mu/m_e)^2 \sim$ 40,000 times more sensitive to new high energy scale physics than the electron.

Experiment E821 at Brookhaven (PRL 2001, H.N. Brown et al.).

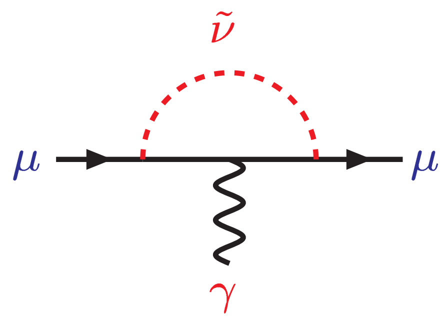
$$a_\mu^{\rm exp} = 116592023(151) \times 10^{-11}$$

Difference between experimental result and Standard Model prediction is in principle new physics

$$a_\mu^{\mathrm{exp}} - a_\mu^{\mathrm{SM}} = a_\mu^{\mathrm{new\ physics}}$$

Brookhaven experiment caused excitement because of a 2.6σ discrepancy between theory and experiment.

New physics? Hint of supersymmetry?

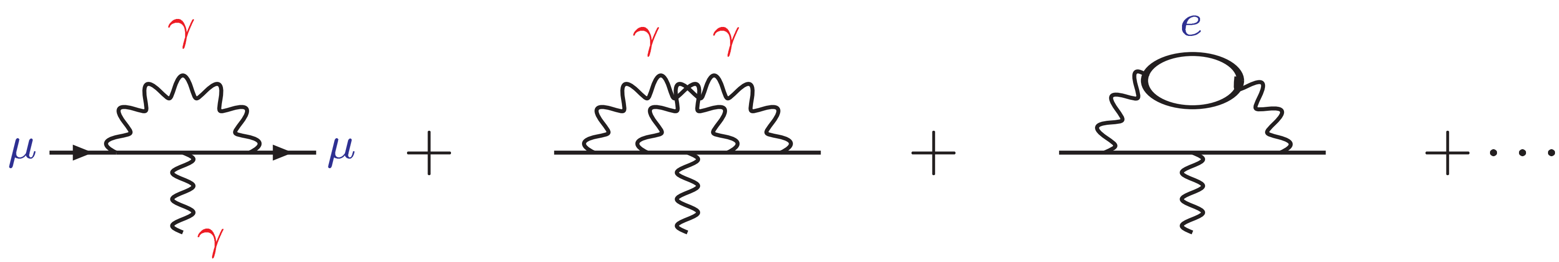


The origin of this discrepancy is now understood as due to glitches in the theoretical calculation.

$g-2$ theoretical calculation

Astonishingly intricate computations involving QED, electroweak theory and strong interactions.

QED:



Schwinger

Kinoshita *et. al.* $\longrightarrow$

Laporta and Remiddi

$$a_\mu^{\mathrm{QED}} = \frac{\alpha}{2\pi} + 0.765857376(27)\left(\frac{\alpha}{\pi}\right)^2 + 24.05050898(44)\left(\frac{\alpha}{\pi}\right)^3$$

$$+ 126.07(41)\left(\frac{\alpha}{\pi}\right)^4 + 930(170)\left(\frac{\alpha}{\pi}\right)^5 + \cdots$$

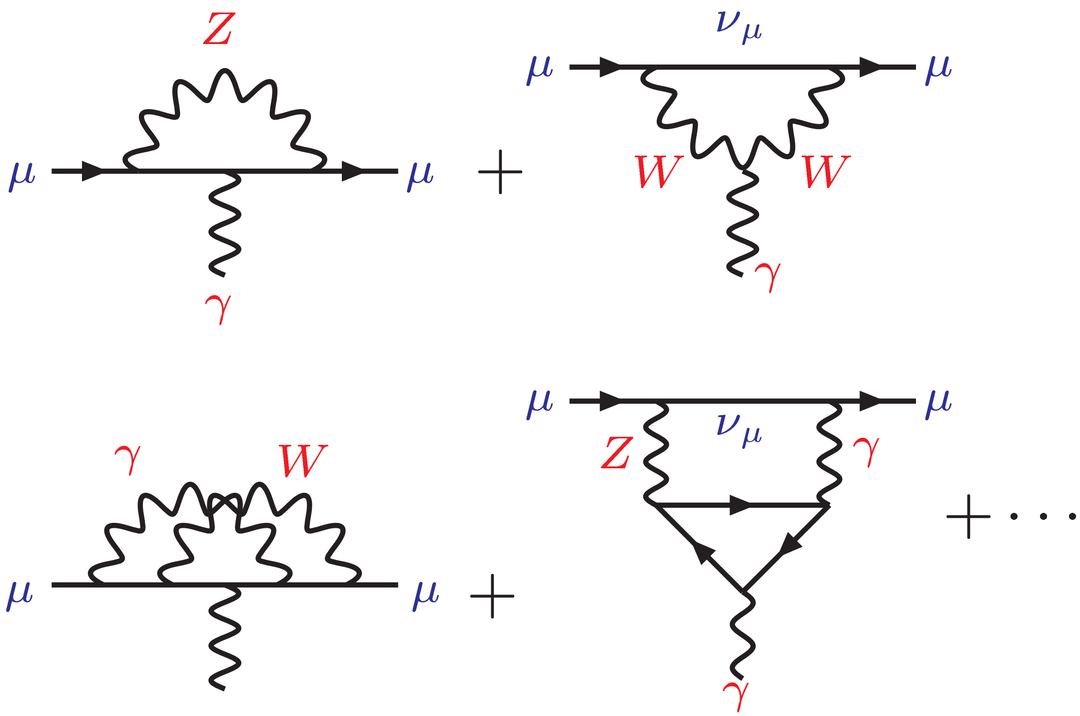
891 diagrams

Approximate

Milstein and Yelkhovsky; Karshenboim

$$a_\mu^{\mathrm{QED}} = 116584705.9(2.9) \times 10^{-11}$$

Electroweak



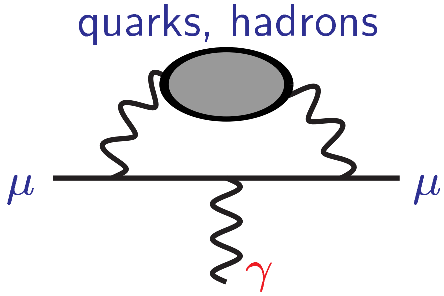
Kukhto, Kuraev, Schiller Silagadze
Czarnecki, Krause, Marciano
Peris, Perrottet, de Rafael
Degrassi, Giudice

1650 two-loop electroweak diagrams! (~ 200 important)

The two-loop EW contribution is surprisingly large (23% shift)

Final result: $a_\mu^{\rm EW} = 152(4) \times 10^{-11}$

Hadronic Loop Corrections



No first principles calculation yet (Lattice?)

At low energy the QCD coupling becomes strong and perturbation theory no longer is valid.

Use experiments to determine the vacuum polarization due to hadrons.

From optical theorem: $\mathrm{Im}\,\Pi(s) \sim \sigma(s)_{e^+e^- \to \mathrm{hadrons}}$

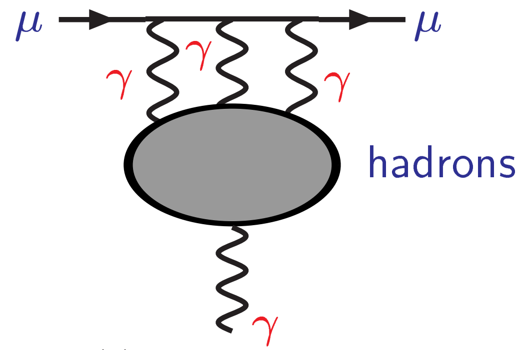
Then use analytic properties (dispersion relations) to obtain the vacuum polarization and then the anomalous magnetic moment.

For $\mathcal{O}(\alpha^2)$: $a_\mu^{\mathrm{Had}}(\mathrm{vac\ pol}) = 6924(62) \times 10^{-11}$ Davier and Hocker
Alemany, Davier and Hocker

For $\mathcal{O}(\alpha^3)$: $a_\mu^{\mathrm{Had}}(\mathrm{vac\ pol}) = -100(6) \times 10^{-11}$ Krause

Above numbers are from the time when 2001 experimental result announced. They have changed somewhat because of new data and theoretical analysis.

Hadronic Light by Light



Brief History:

1985: Kinoshita, Nizić, Okamoto $\to + 49(5) \times 10^{-11}$ (sign changed)

1995: Hayakawa, Kinoshita, Sanda $\to - 52(18) \times 10^{-11}$

1995: Bijnens, Pallante, Prades $\to - 92(32) \times 10^{-11}$

1995: Hayakawa, Kinoshita $\to - 79(15) \times 10^{-11}$

June 2001: Bartos *et al.* $\to - (54 \sim 82) \times 10^{-11}$

Nov 2001: Knecht and Nyffeler $\to + 83(12) \times 10^{-11}$

Other groups verify the new sign: Knecht, Nyffeler, Perrottet, De Rafael; Bardeen; Blokland, Czarnecki and Melnikov.

Dec 2001: Hayakawa and Kinoshita find the sign error: Problem could be reduced to an interpretation of the metric used in FORM (Vermaseren).

Bottom Line for Anomalous Moment of Muon

Net Effect: After sign correction, disagreement between Standard Model prediction and experiment fell to $1.6\,\sigma$.

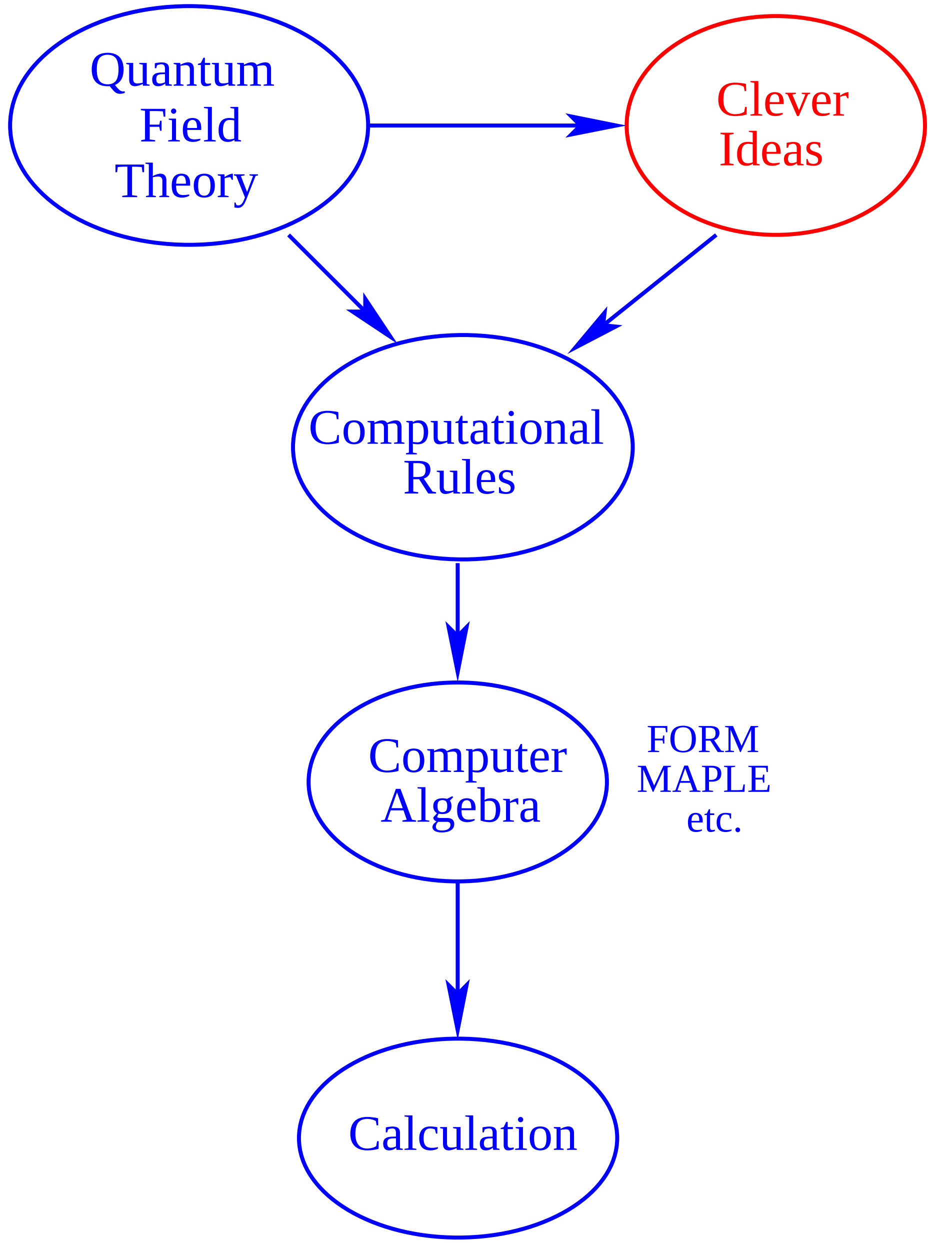
Since the publication of the experimental results, the hadronic corrections have been updated, shifting the error and central value slightly. Latest status described in a parallel session by Teubner.

Stay tuned for improved experimental numbers.
(See talk tomorrow by Yannis Semertzidis)

In any case, this is a great triumph for both the Brookhaven experiment and the theoretical calculations: 6-7 digits of precision.

The anomalous magnetic moment of the muon provides a spectacular recent example of the essential role of these extremely challenging theoretical calculations.

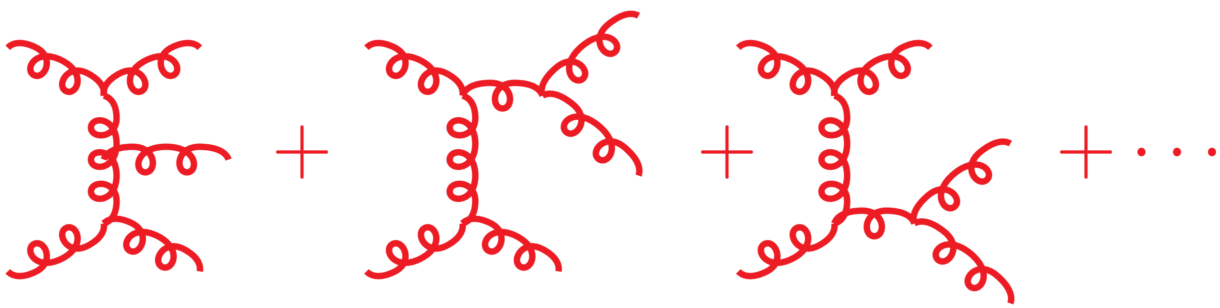
What enters complicated calculations



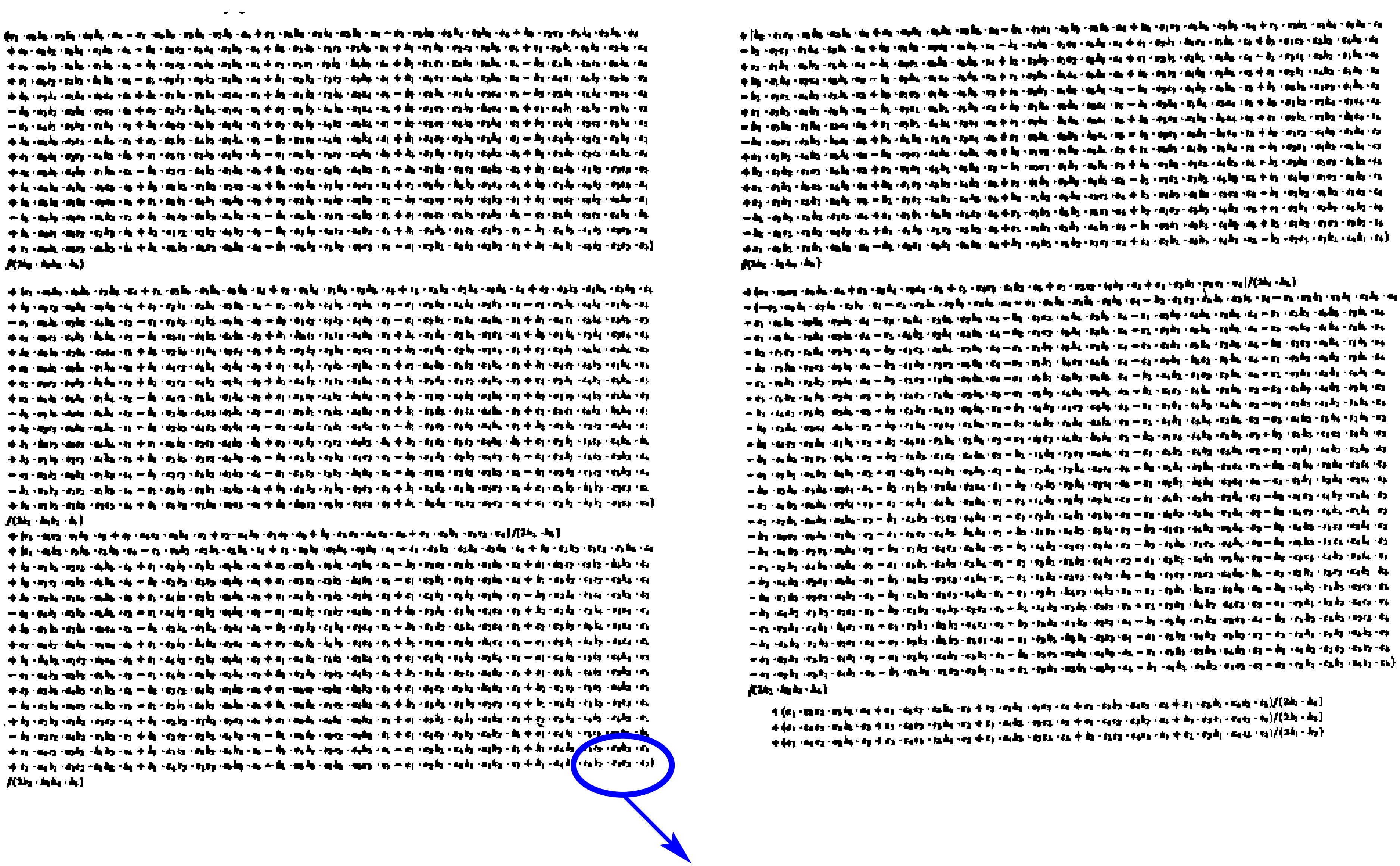
Examples of Clever Ideas

Consider the five-gluon tree-level amplitude of QCD. Enters in calculation of three-jet production at hadron colliders.

Described by following Feynman diagrams:



Result of a brute force calculation:



$$k_1 \cdot k_4 \, \varepsilon_2 \cdot k_1 \, \varepsilon_1 \cdot \varepsilon_3 \, \varepsilon_4 \cdot \varepsilon_5$$

Helicity

Berends, Kleiss, De Causmaecker,
Gastmans, Troost, Wu (1981)
Kleiss and Stirling (1985)
Gunion and Kunszt (1985)
Xu, Zhang and Chang, (1987)

etc.

Instead use helicity or circular polarization,

$$\varepsilon_\mu^\pm = (0, 1, \pm i, 0)$$

we obtain

$$s_{ij} \equiv (k_i + k_j)^2$$

$$A_5(1^\pm, 2^+, 3^+, 4^+, 5^+) = 0,$$

$$A_5(1^-, 2^-, 3^+, 4^+, 5^+) = \left(\frac{s_{12}^3}{s_{23}s_{34}s_{45}s_{51}} \right)^{1/2}$$

Precisely the same physical information as on the previous page.

There are actually much better ways to do things by representing the circular polarizations in terms of spinors. ('Chinese Magic')

Illustrates the importance of understanding the underlying physics.

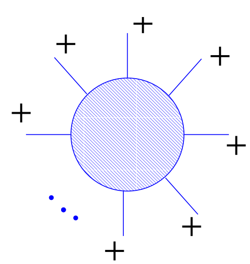
Supersymmetry as a computational tool

Cancellations between bosons and fermions quantum loops.

1. If done appropriately, calculations in susy theories generally far easier than in quantum chromodynamics. Susy calculations can serve as toys.

2. A QCD calculation is a cousin of a susy calculation: quarks $\to$ gluinos.

Grisaru, Pendleton and van Nieuwenhuizen (1977)
Parke and Taylor, (1985)
Z. Kunszt (1986)

Susy identity 

$= 0$

Applied to state-of-the-art two-loop computations.

Binoth, Glover, Marquard, van der Bij (2002); Bern, De Freitas, Dixon, Wong (2002)

Check on 4-loop QCD β-function computed by van Ritbergen, Vermaseren and Larin (1998).

Jack, Jones, North (1997)

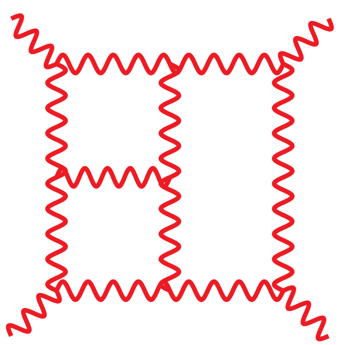
Example of Growth of Difficulties

Some problems cannot possibly be solved by brute force alone.

The often repeated statement that quantum mechanics and general relativity are incompatible arises from the bad ultra-violet divergence properties of (super) gravity theories.

Strong arguments for this based on power counting. Confirmed by two-loop calculation in pure Einstein gravity. Sagnotti and Goroff (1986).

But there is no proof for supergravity: Requires a 3-loop computation.



$\sim 10^{21}$ terms in diagram

At 10^9 terms/sec

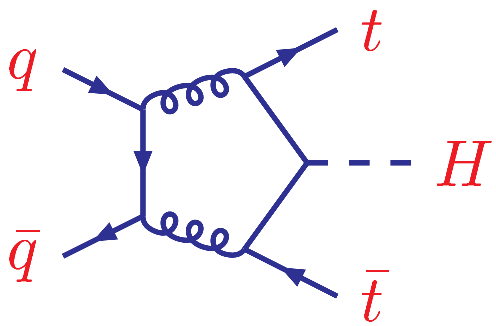
$\sim 20,000$ years to complete 1 diagram

Trick: map gravity $\rightarrow$ (gauge theory)^2 Kawai, Lewellen and Tye ('85) Bern, Dixon, Dunbar, Perelstein, Rozowsky ('98)

Impossible problem $\rightarrow$ extremely difficult problem (not solved).

Status of One Loop Computations

At one-loop the state of the art is five external legs, $e.g.$ $pp \to \bar{t}tH$.



Described by D. Wackeroth in a parallel session.

Reina, Dawson and Wackeroth (2001)
Beenakker, Dittmaier, Kramer, Plumper, Spira (2001)

At 6 points complete answers only for very special theories: $N = 4$ supersymmetric Yang-Mills and the Yukawa Model.

Bern, Dixon, Dunbar and Kosower (1994)
Binoth, Guillet, Heinrich and Schubert (2001)

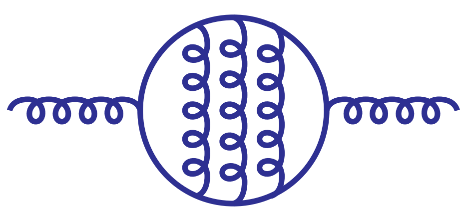
Status of Higher Loop Computations

Some examples of well known impressive higher loop computations:

- $g-2$, 4 loops, Kinoshita and friends.

- $R = \sigma(e^+e^- \to \text{hadrons})/\sigma(e^+e^- \to \mu^+\mu^-)$, $O(\alpha_s^3)$ Gorishny, Kataev and Larin, etc

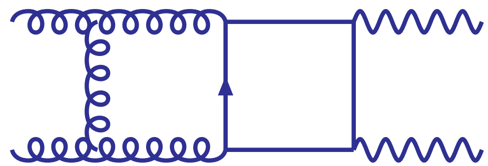
- 4-Loop QCD β function van Ritbergen, Vermaseren and Larin, $\sim 50,000$ Feynman diagrams!



These are all in the class of zero or one kinematic variable.

Major Advance of Past Two Years

Computations involving more than 1 kinematic variable is a new art < 2 years old. This is what we focus on here.



Key to Progress

In the past few years the field of high loop computations has gotten a tremendous boost due to the influx of energetic bright young people.

Babis Anastasiou, Andrzej Czarnecki, Daniel de Florian, Thomas Gehrmann, Massimiliano Grazzini, Robert Harlander, Sven Heinemeyer, Bill Kilgore, Kirill Melnikov, Sven Moch, Zoltan Nagy, Carlo Oleari, Matthias Steinhauser, M.E. Tejeda-Yeomans, Peter Uwer, Doreen Wackeroth, Georg Weiglein, Stefan Weinzierl, and many others

The difficulties at NNLO

Every step in the construction of a physical cross-section involving two-loop amplitudes has major difficulties.

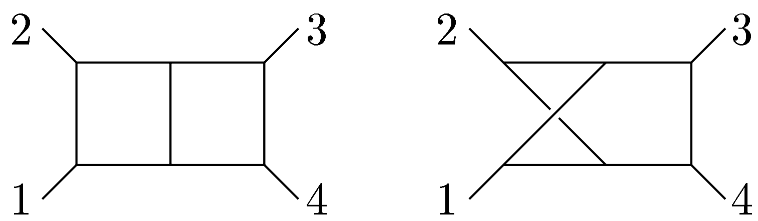
1. Loop integrals.
2. Scattering Amplitudes.
3. Infrared divergences and phase space integrals.
4. NNLO parton evolution (needed for Tevatron or LHC) via DGLAP equation.
5. Numerical programs for making quantitative predictions for experiments.

Remarkable progress in the past two years.

Loop Integrals

A crucial ingredient has been the breakthrough in obtaining two-loop massless double-box integrals.

Evaluation of massless scalar double box integrals:

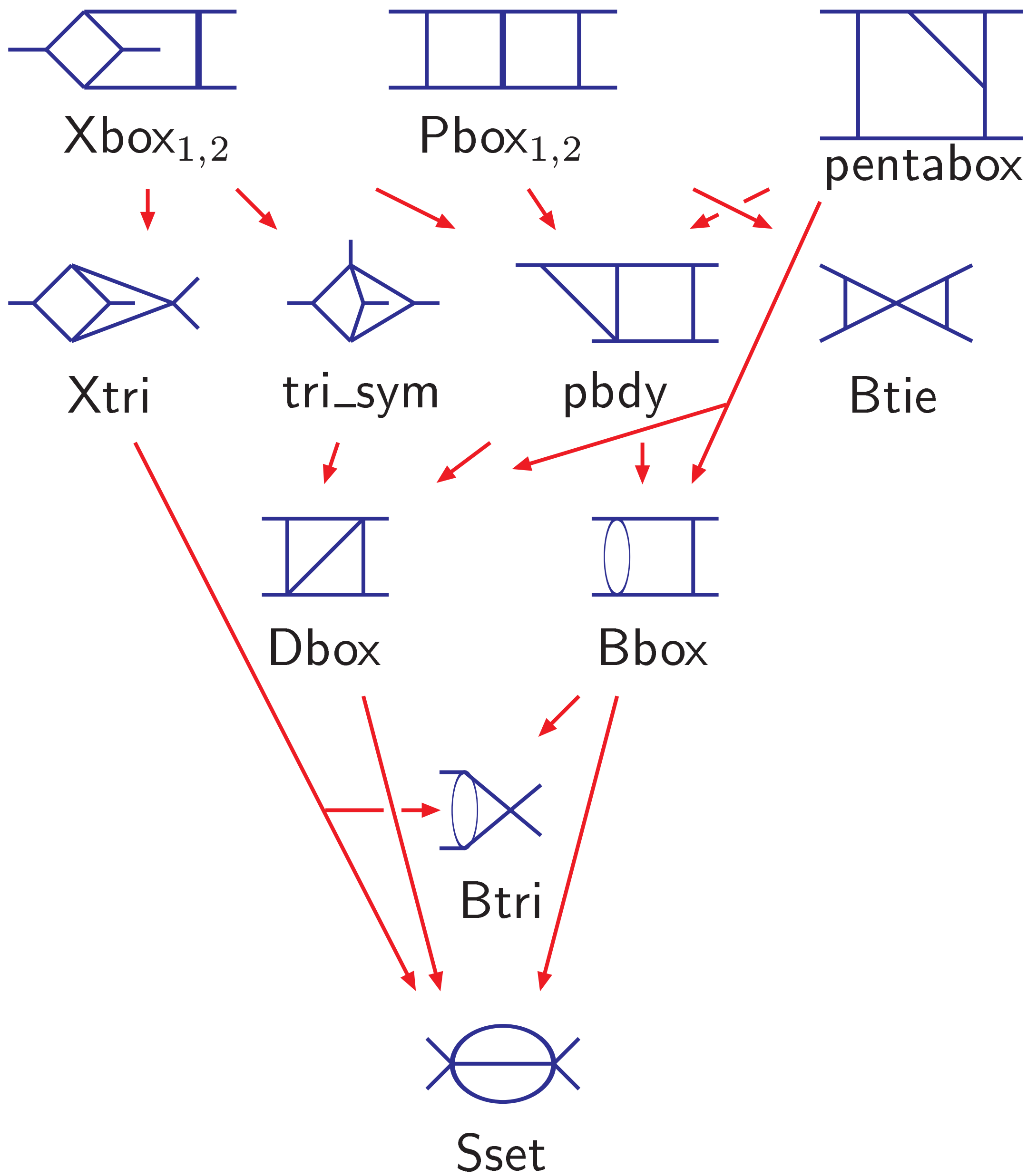


$$\int \frac{d^Dp}{(2\pi)^D}\frac{d^Dq}{(2\pi)^D}\frac{1}{p^2\,q^2\,(p+q)^2(p-k_1)^2\,(p-k_1-k_2)^2\,(q-k_4)^2\,(q-k_3-k_4)^2}$$

Another crucial ingredient: algorithms for evaluating integrals with powers of loop momenta in the numerator.

Answers in terms of standard polylogarithms.
IR divergent and a complicated analytic structure.

Two-loop integral inheritance chart



Some All-Massless Integrals References

$\text{Pbox}_{1,2}$, Bbox, Dbox: Smirnov, hep-ph/9905323; Smirnov & Veretin, hep-ph/9907385

Xbox_1, tri_sym: Tausk, hep-ph/9909506

$\text{Xbox}_{1,2}$, Xtri: Anastasiou et al., hep-ph/0003261

pentabox, Dbox: Anastasiou, Glover, Oleari, hep-ph/9912251

Bbox: Anastasiou, Glover, Oleari, hep-ph/9907523

Basic identities:

- Integration by parts — Tkachov, PLB (1981); Chetyrkin & Tkachov, NPB (1981)
- Lorentz invariance — Gehrmann & Remiddi, hep-ph/9912329

Now, even more powerful techniques available.

Laporta (2000); Moch, Uwer, Weinzierl (2002)

Integrals with Multiple Mass Scales

Tkachov, Smirnov, Chetyrkin; Beneke and Smirnov

In the past few years there has also been quite a bit of progress in dealing with integrals with multiple scales via the 'Strategy of Regions':

Series expansion in various kinematic limits making use of dimensional regularization $(D = 4 - 2\epsilon)$. Boundaries between regions can be ignored greatly simplifying the calculations.

The key progress is that now there is a universal method for dealing with this, instead of a case-by-case analysis depending on boundaries.

This has been applied to various cases such as atomic physics, heavy quark production and decay, and large electroweak logarithms.

Chetyrkin and Steinhauser; Blokland, Czarnecki and Melnikov; Kniehl, Penin, Smirnov, Steinhauser
Chetyrkin, Kühn and Steinhauser; Kühn, Penin and Smirnov; Czarnecki, Krause and Marciano;
Hoang and Teubner; Melnikov and Yelkhovsky; Beneke, Signer and Smirnov *etc.*

Example from parallel session: Electroweak large logs discussed in talk by Kühn obtained using this technique.

Numerical Integration Methods

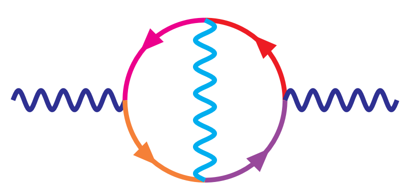
Two numerical approaches for loop integrals were discussed in parallel sessions:

Using a numerical approach for performing the integrations, Dave Soper described a technique which has so far reproduced the old result of Ellis, Ross and Terrano (1980) for NLO $e^+e^- \to 3$ jets. But it is not yet clear if it will be practical for state-of-the art calculations.

Another numerical approach was discussed by Passarino This approach is targeted for two-loop electroweak processes. Masses make analytic approaches ever more complicated.

Ghinculov and Yao (2000)
Passarino and Ucciritti (2002)

This has been applied to massive two-loop two- and three-point diagrams:



New Two-loop Amplitudes

After 4-point integration breakthrough many new two-loop calculations:

- Two-loop Bhabha scattering in QED, $e^+e^- \to e^+e^-$.

  Bern, Dixon and Ghinculov (2000)

- All two-loop $2 \to 2$ QCD processes.

  Anastasiou, Glover, Oleari and Tejeda-Yeomans (2001)

- $\gamma\gamma \to \gamma\gamma$

  Bern, Dixon, De Freitas, A. Ghinculov and H.L. Wong (2001)

- $gg \to \gamma\gamma$. (Background to Higgs decay.)

  Bern, De Freitas, Dixon (2001)

- $\bar{q}q \to \gamma\gamma$, $\bar{q}q \to g\gamma$, $e^+e^- \to \gamma\gamma$

  Anastasiou, Glover and Tejeda-Yeomans (2002)

- $e^+e^- \to 3$ partons

  Garland, Gehrmann, Glover, Koukoutsakis and Remiddi (2002)
  Moch, Uwer, Weinzierl (2002)

- DIS 2 jet and $pp \to W, Z + 1$ jet

  Gehrmann and Remiddi (2002)

Universal Two-loop Infrared Singularities

In a beautiful paper, Stefano Catani (1998) specified essentially the complete IR divergences of any two-loop QCD (and QED) process.

All IR divergences must cancel from a physical result:

real emission singularities $+$ wizardry $\longrightarrow$ two-loop IR divergences.

Catani's IR divergence formula is extremely useful because:

- Substantial fraction of the answer for a two-loop amplitude is known prior to starting calculation.
- Provides a very stringent check on any calculation.
- Provides a way for organizing amplitudes.

Parton Distribution Functions

For initial state protons one also needs parton distributions evolved using NNLO QCD.

There has been a large amount of work on this. So far only approximate solution.

van Neerven and Zijlstra (1993); Catani and Hautman(1994)
Larin, Nogueira, Retey, van Ritbergen, Vermaseren (1997)

van Neerven and Vogt (2000); Retey and Vermaseren (2001)

An important related milestone: NNLO quark and gluon distributions inside photons.

Moch, Vermaseren, and Vogt (2001)

Starting to be implemented in global fits (MRST) for pdfs.

Martin, Roberts, Stirling, Thorne (2002)

This is very important for true NNLO calculations.

Physical Predictions

Everything still needs to be put together in a numerical program for producing physical cross-sections.

In general, this is non-trivial, because of the IR divergences. At NLO general solutions to this problem exist. Giele, Glover and Kosower (1993)

Frixione, Kunszt and Signer (1995); Catani and Seymour (1996)

Done at NNLO in special cases:

- Drell-Yan, W or Z production Hamberg, van Neerven and Matsuura (1991)

- Inclusive Higgs production at hadron colliders.
  Catani, de Florian and Grazzini (2001); Harlander and Kilgore (2002); Anastasiou and Melnikov (2002)

This suggests that it can be done more generally.

This is key technical problem that still needs to be solved.

Recent Concrete Applications

Two very recent examples of phenomenological studies based on the above two-loop progress:

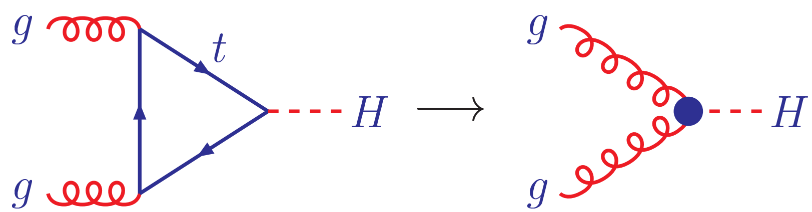
- Inclusive Higgs production at NNLO (July 2002).
- Improved understanding of the QCD background to Higgs production at the LHC for $M_H < 140$ GeV. (June 2002).

This is just the beginning!

These examples, bypass the technical problem with dealing with IR divergent phase space at NNLO.

Inclusive Higgs Production

In the talk by Frixione we heard about the important quest to obtain reliable predictions for Higgs production at Tevatron and LHC.

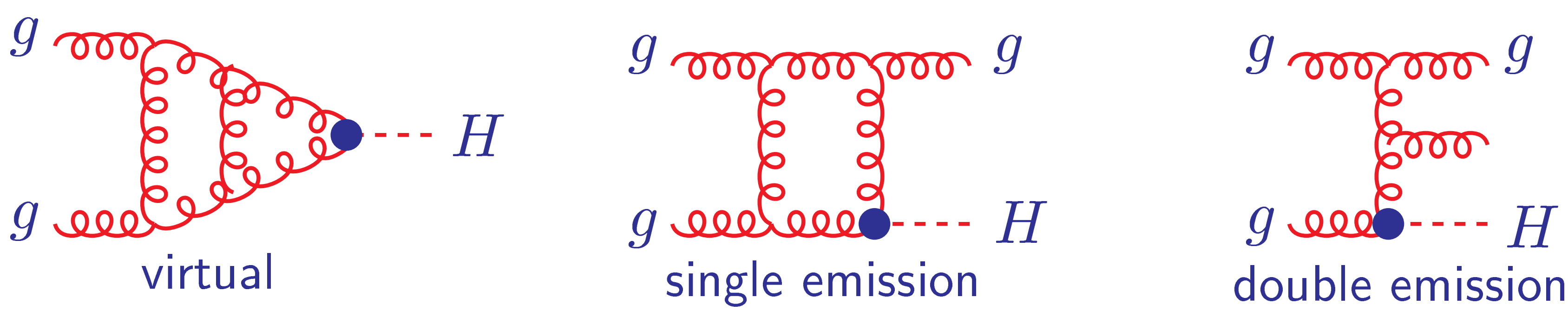


Vainshtein et al., 1979

Loop corrections for the effective vertex computed through $\mathcal{O}(\alpha_s^4)$

Chetyrkin et al., 1997

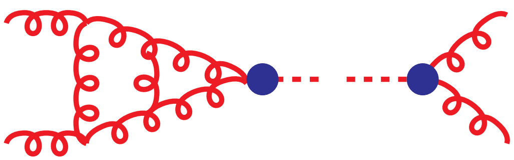
NNLO sampling of diagrams:



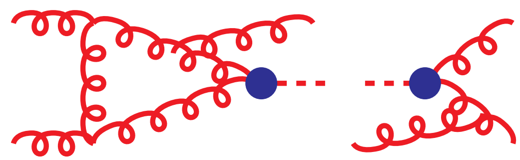
The difficult part of the calculation is dealing with the IR singular parts of the double real emission contributions. This was worked out by Harlander and Kilgore (PRL 2002) using a series expansion of the answer.

Application of New Technology

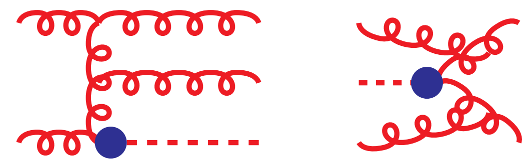
A useful application of the new integration technology has been to provide a straightforward way to obtain the exact phase space integration over the double real emission.



148 terms



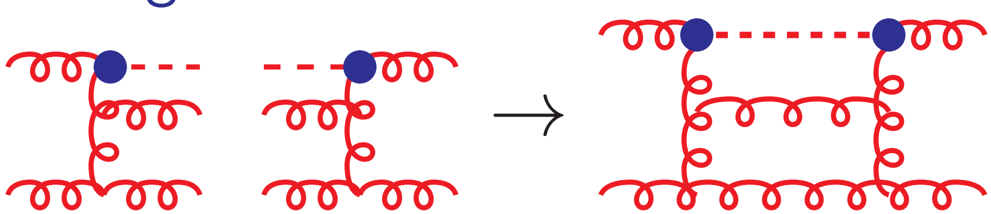
635 terms



594 terms

Use the optical theorem (unitarity) to promote the phase-space integral to the imaginary part of a forward scattering amplitude computable via Feynman diagrams:

Anastasiou and Melnikov (hep-ph/0207004)

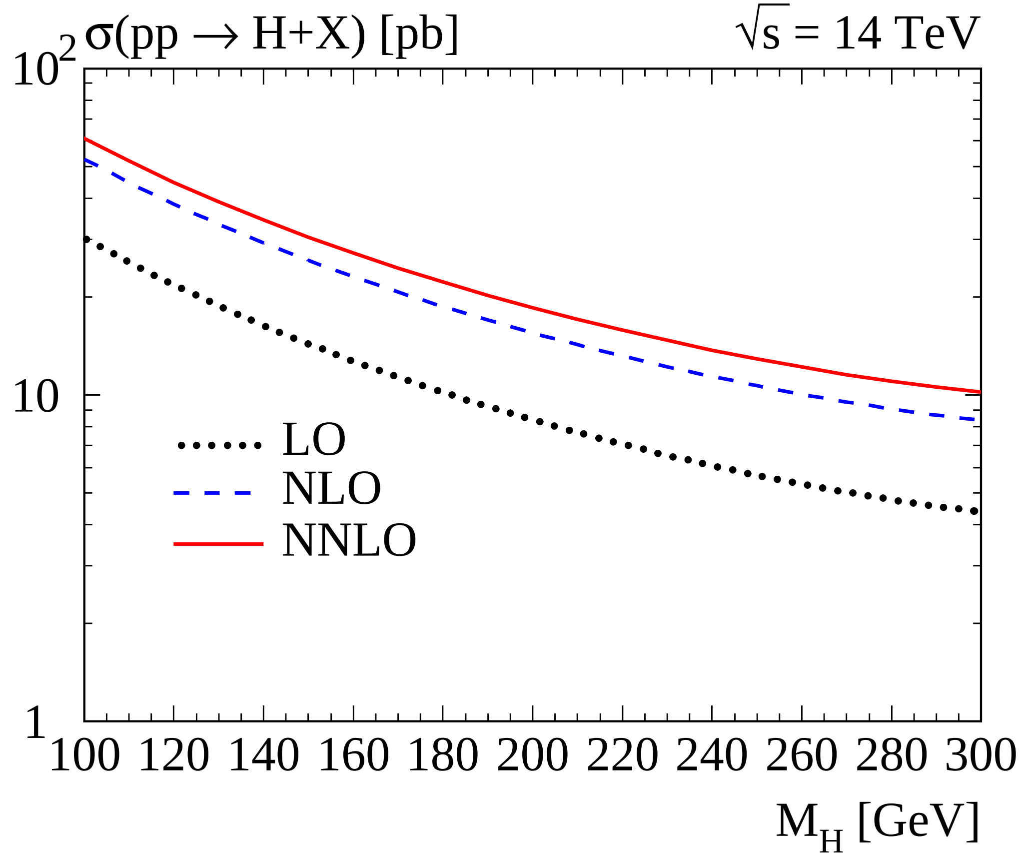


Use automated computer program (QGRAF) to produce Feynman diagrams. Then apply the previous integration technology.

So far this trick works only for totally inclusive cross-sections...

NNLO Inclusive Higgs Production at LHC

Harlander and Kilgore (PRL 2002)
Anastasiou and Melnikov (hep-ph/0207004)



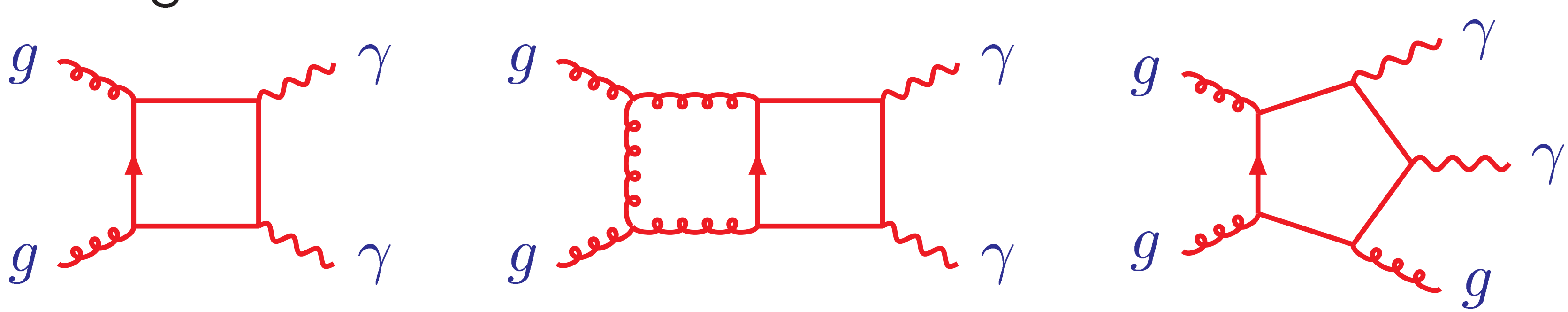
Fact that the NNLO value is close to to the NLO value suggests perturbation theory is under control. Result is also close to earlier approximate calculations of Catani, de Florian and Grazzini. Further improvements are coming.

LHC Higgs Search: background to $H \to \gamma\gamma$

For a low mass Higgs ($M_H < 140$ GeV) the preferred search mode is via the rare decay $H \to \gamma\gamma$.

Leading and next-to-leading order QCD subprocesses for $pp \to \gamma\gamma X$ form an irreducible background:

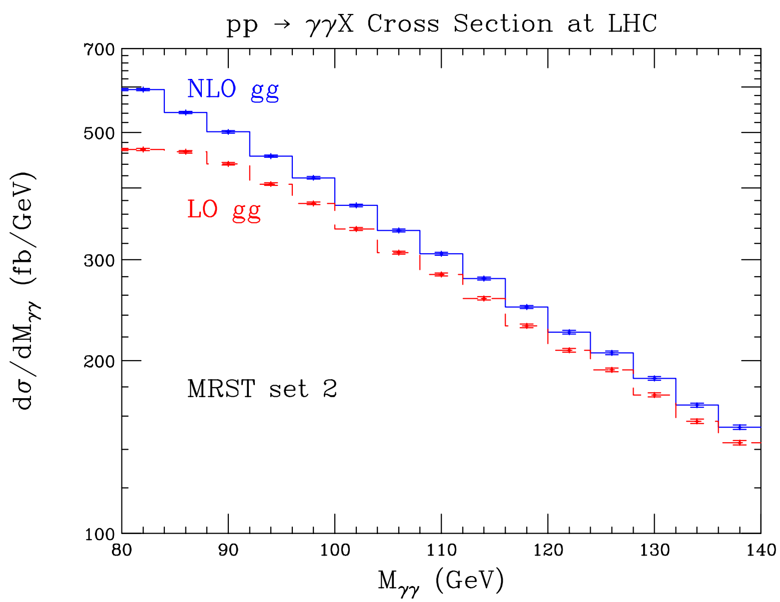
LHC is a glue factory (for low mass Higgs), hence gluon fusion is important background:



NLO corrections to gluon fusion are sizable. Two-loop amplitude obtained via the new technology.

Phenomenological Implications

To obtain a physical cross-section the two-loop $gg \to \gamma\gamma$ amplitude must be combined with the one-loop real emission diagrams as well as contributions with initial state quarks.



Binoth, Guillet, Pilon, Werlen (2000)
Bern, De Freitas, Dixon (2001)
Bern, Dixon and Schmidt (2002)

K-factor smaller than previous estimates.

2 years required to pull the Higgs signal out of the above background if $m_H < 140$ GeV.

Can we improve the situation using quantitative theoretical predictions to find appropriate kinematic variables and cuts?

Promise for Future

Other examples of what is on the horizon:

- $e^+e^- \to 3$ jets at NNLO. 'Precision' QCD at future linear collider *e.g.* 1% measurement of α_s.

  Garland, Gehrmann, Glover, Koukoutsakis and Remiddi (2002)

- NNLO DIS + 2 jets.

  Gehrmann and Remiddi (2002)

- A complete NNLO 2 jet program for the Tevatron and LHC.

  Anastasiou, Glover, Oleari and Tejeda-Yeomans (2001)

- NNLO parton distribution functions.

  van Neerven *et al.*; Vermaseren *et al.*

For this promise to be realized, the serious technical issue of NNLO IR divergent phase space integration needs to be resolved first.

The same technology should be applicable to electroweak processes once more integral types are worked out.

Summary

1. Challenging perturbative calculations are crucial for comparing theory to experiment. Example of muon anomalous magnetic moment.

2. Recent developments leading to new two-loop calculations. More will be forthcoming.

3. New technology already applied to produce physics results for inclusive Higgs production and to QCD background for Higgs production.

4. Potential for future: Unprecedented precision in high energy QCD and electroweak radiative corrections when more than a single kinematic invariant present.

The last few years have seen rapid development in our ability to calculate quantities of interest in perturbative quantum field theory. We can be optimistic that this rapid pace of progress will continue.