String theory and mathematical quantum field theory

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⇒ apologize for lack of references.

Four themes:

- Realistic (standard model like) models from string theory.

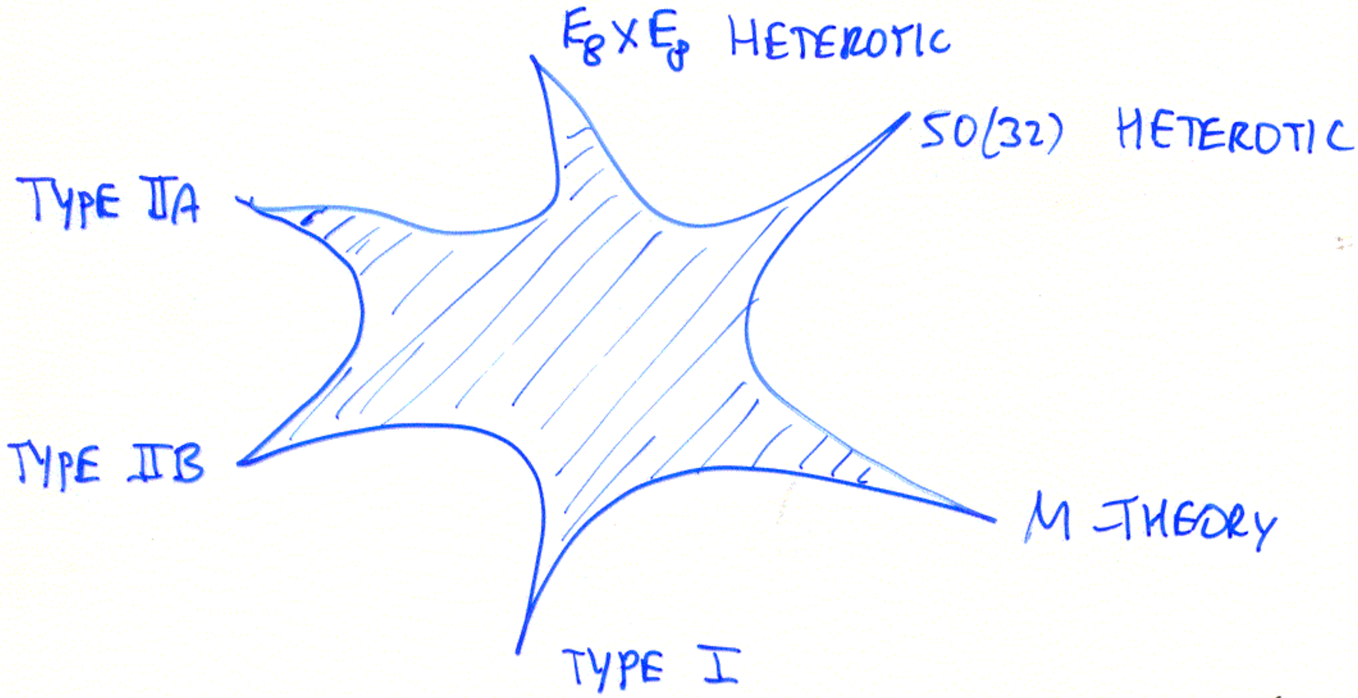
- Geometric engineering and theories with fluxes.

- The gauge theory - gravity correspondence.

- Time dependent backgrounds and string theory.

Realistic (standard model like) models from string theory

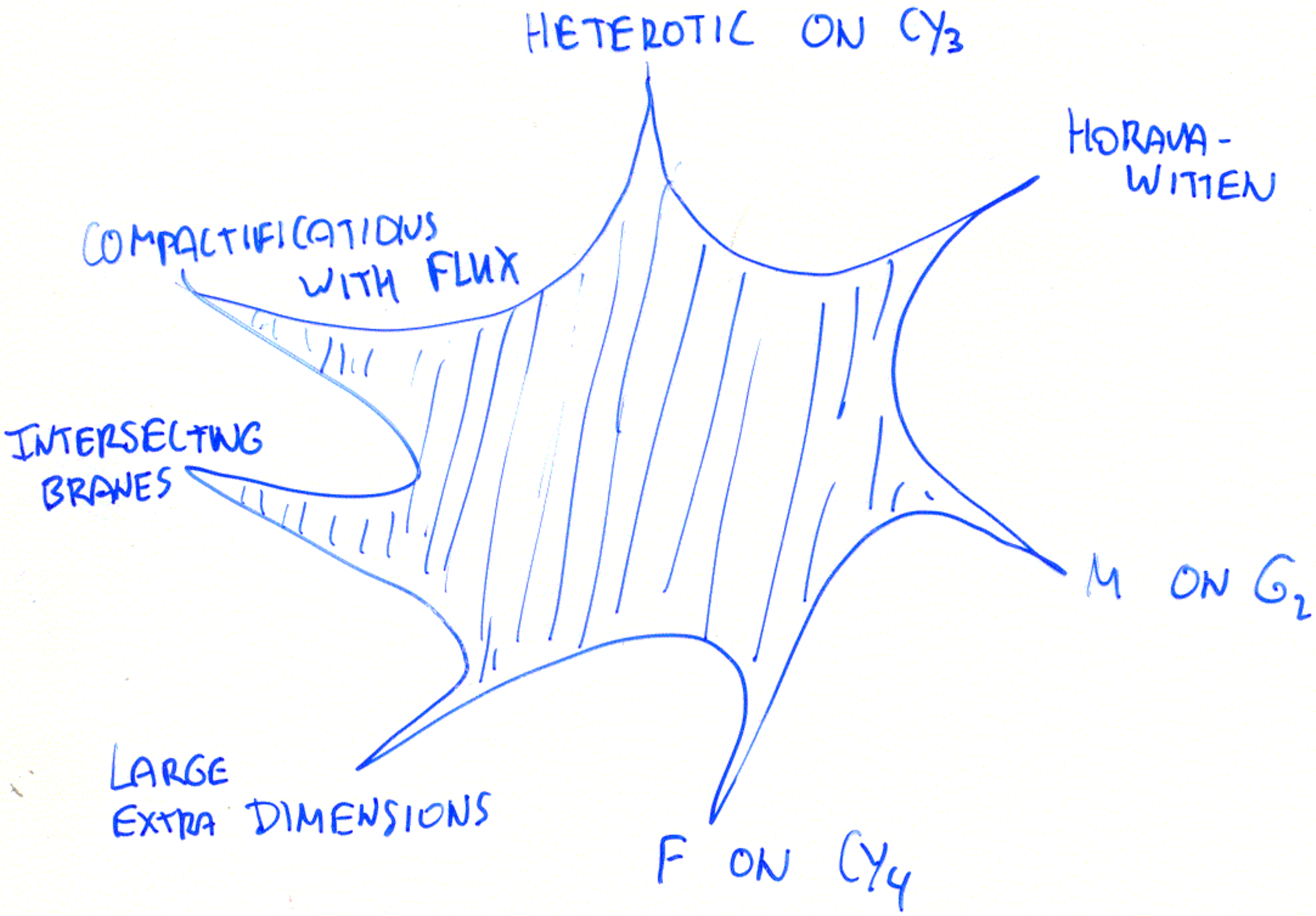
Before the 'second string revolution' in 1995, there were several different string theories. As we understand it now, these are all weakly coupled limits of a single theory. (Not clear what happens with less supersymmetry though).



Example: eleven dimensional M-theory (which isn't really a string theory) compactified on a circle of radius R. For large R, get weakly coupled M-theory. For small R, get weakly coupled IIA string theory.

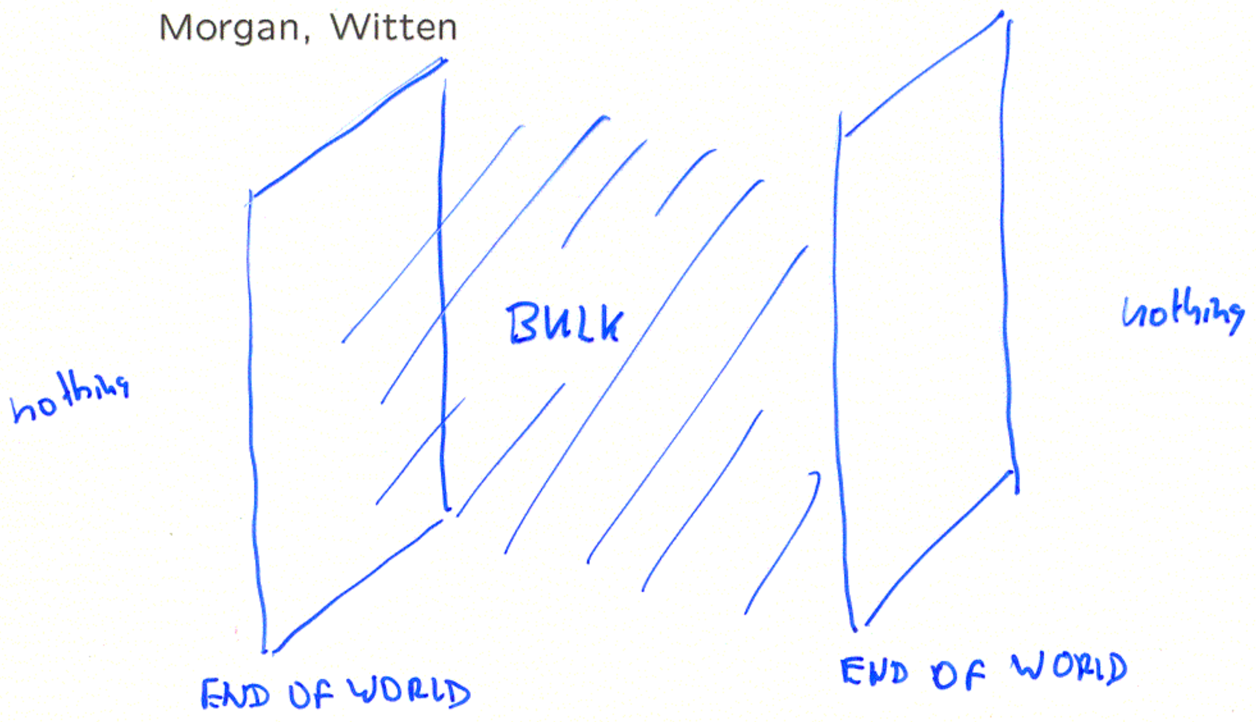
The traditional way to obtain realistic models from string theory was to start with the (10d) $E_8 \times E_8$ heterotic string, compactified on a (6d) Calabi-Yau manifold. This results in a theory with $N = 1$ supersymmetry in four dimensions.

Many other constructions have emerged over the past few years. These are part of a much more intricate duality web.



Horava-Wittten

Describes a strongly coupled version of the heterotic string compactified on a Calabi-Yau space, via M-theory on an interval times the Calabi-Yau space. The endpoints of the interval support 'end of the world' branes that carry gauge degrees of freedom. The phenomenology is comparable to that of the heterotic string, but $M_{GUT} = M_{Pl}$ can be achieved as well. Donagi, Lukas, Ovrut, Stelle, Waldram; Friedman, Morgan, Witten

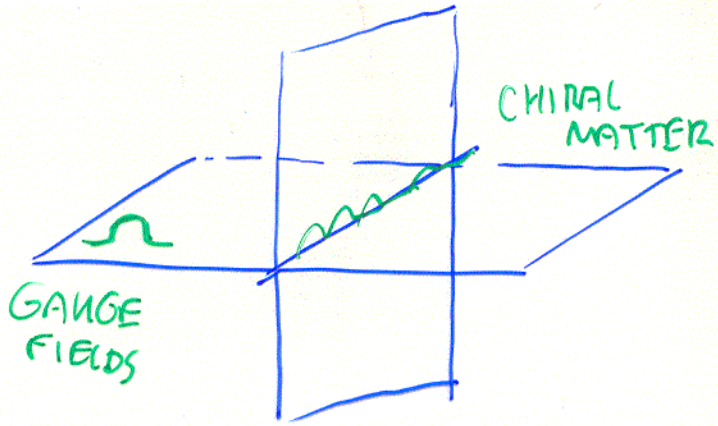


$\underline{M \text{ on } G_2}$

M-theory compactified on seven-dimensional manifolds of G_2 holonomy. Leaves $N = 1$ in four dimensions unbroken. In some sense, these are seven-dimensional version of Calabi-Yau spaces. The geometry of these spaces is rather poorly understood, only good examples are non-compact. Chiral fermions can only arise from singularities. Acharya, Atiyah, Witten

$\underline{F \text{ on } CY_4}$

F-theory compactified on eight-dimensional Calab Yau spaces. Really describe strongly coupled IIB compactifications. Very few nice models are known.



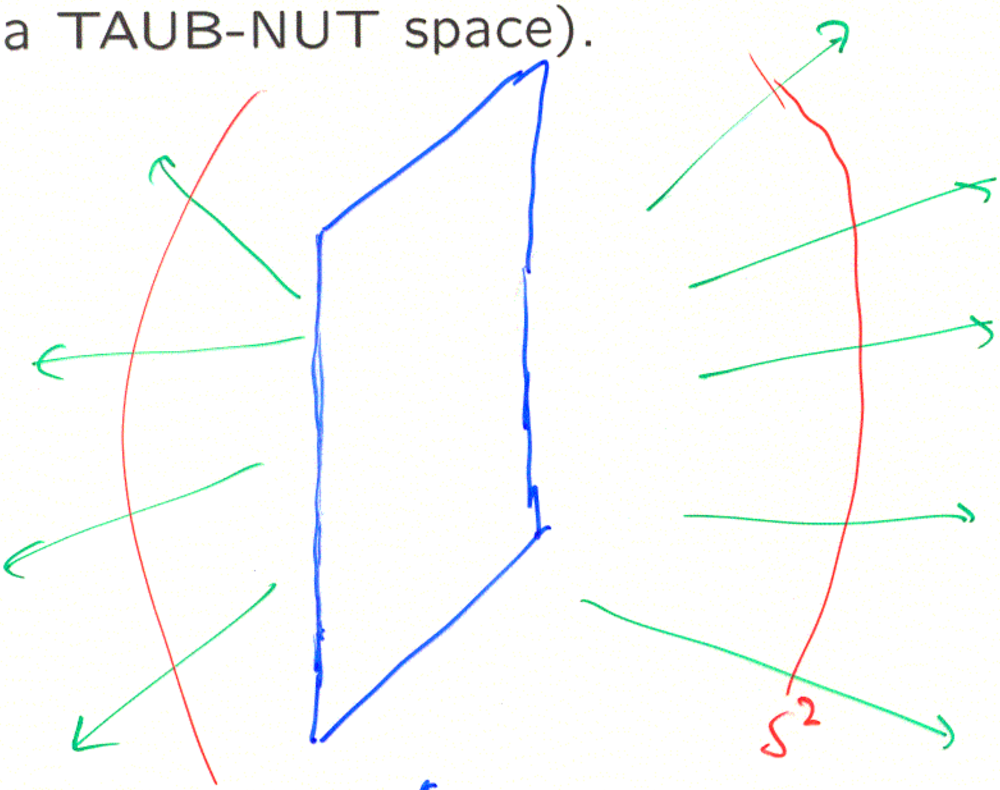
Intersecting branes

Start e.g. with IIA compactified on a six-torus. Choose sets of three circles in the six-torus and wrap D6-branes around them. Four of the seven dimensions of the D6-brane remain non-compact and support gauge bosons. Intersections of such branes can yield chiral matter. Chiral SM and MSSM spectra can be obtained. Relatively easy to control matter content. Stability of non-supersymmetric configurations is not clear. Proton is stable because baryon number is a gauge symmetry. Aldazabel, Cremades, Franco, Ibañez, Marchesano, Rabadan, Uranga; Blumenhagen, Görlich, Körs, Lüst, Ott; Cvetic, Shiu, Uranga; Bailin, Kraniotis, Love; Förste, Honecker, Schreyer; Kokorelis; Kataoka, Shimojo; Everett, Kane, King, Rigolin, Wang

Large extra dimensions

Similar to above, now with a large space transversal to the D-branes.

Duality between branes and geometry: M-theory on a circle yields type IIA theory. From the eleven dimensional metric we obtain a Kaluza-Klein gauge field. D6-branes are charged under this gauge field. If we integrate the field strength of the gauge field over a two sphere surrounding the D6 brane, we obtain a non-zero quantized answer. The lift of the D6 brane to M-theory becomes purely geometric (a TAUB-NUT space).



$$\int_{S^2} F \neq 0$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

Right now, the heterotic string compactified on a Calabi-Yau space is according to the majority still the most realistic string compactification, though this may change in the near future.

The main problem with the other approaches is to have a successful gauge coupling unification.

Virtually all models have additional degrees of freedom at low energies, beyond those of the (MS)SM.

Geometric engineering and theories with fluxes

Type II string theories compactified on a Calabi-Yau space give rise to a theory with $N = 2$ supersymmetry in four dimensions.

Geometric engineering refers to the process of cooking up a geometry that produces an a priori given $N = 2$ gauge theory in four dimensions. Many gauge theories can be geometrically engineered, especially those of 'quiver' or 'moose' type.

It is possible to break these $N = 2$ theories to $N = 1$ by either adding branes, or by adding fluxes. The duality between these two possibilities has been explored in great detail by Vafa and collaborators.

Fluxes refers to the following: type II string theory has several massless tensor fields, besides the graviton and dilaton. For example, type IIA has a gauge field and a three-form. Suppose that the Calabi-Yau manifold has a hole that looks like a sphere, then we can turn on a field strength for the gauge field so that there is non-zero flux through the two-sphere, exactly as if there were a charged particle sitting in the middle of the hole.

In principle, such fluxes will back-react on the geometry. The new geometry is not known, but can be constructed order by order in an expansion in one over the volume to some power.

Such type II string theories with fluxes have many interesting properties.

- They have a rich duality structure. The full geometry remains to be uncovered (very complicated math, see e.g. Douglas).

- The gauge kinetic terms and superpotential of the low-energy effective field theory can often be computed exactly using topological string theory. This has also yielded many previously unknown mathematical results. Witten; Bershadsky, Cecotti, Ooguri, Vafa; Antoniadis, Gava, Narain, Taylor; Ooguri, Vafa; many more recent

- Due to the presence of a superpotential, many of the moduli (corresponding to massless scalars in $d = 4$) are lifted, i.e. become massive. This is much easier to analyze than e.g. in heterotic strings on a Calabi-Yau manifold. Also, one naturally finds warped compactifications. Gukov, Vafa, Witten; Greene, Schalm, Shiu; Dasgupta, Rajesh, Sethi; Giddings, Kachru, Polchinski; Silverstein; Curio, Klemm, Lüst, Theisen; Mayr

- Provides topological versions of the gauge theory-gravity correspondence Gopakumar,Vafa; many more

One concrete recent application (Dijkgraaf, Vafa):

Take an $N = 1$ theory with a classical superpotential $W_{\text{classical}}$, and assume that W has a critical point where the gauge group is broken to a product of $U(N_i)$ factors. Introduce a gluino condensate field S_i for each of the factors.

The full Veneziano-Yankielowicz quantum superpotential $W[S_i]$ can then be computed by summing the planar diagrams of the matrix model with action $S = W_{\text{classical}}$. In many cases this can be proven using various string dualities and properties of topological strings. Field theory interpretation?

In a perhaps related development, recently Nekrassov managed to do explicitly all-order instanton calculations in $N = 2$ super Yang-Mills theory.

Gauge theory-gravity correspondence

The strong form of the AdS/CFT correspondence (Maldacena):

There is an exact equivalence between type IIB string theory on $AdS_5 \times S^5$ and $d = 4$ $N = 4$ supersymmetric Yang-Mills (SYM) theory.

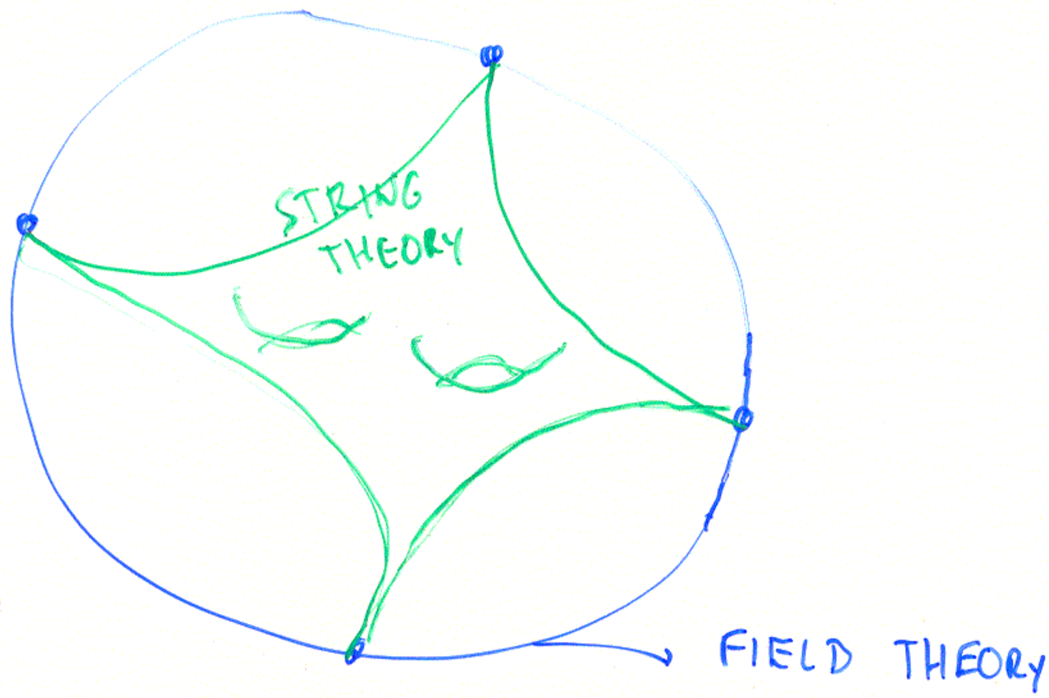
Hard to prove because we don't have a definition of non-perturbative type IIB string theory; even at string tree level we do not (yet) know how to solve the theory.

From this point of view, $N = 4$ SYM theory can be viewed as the definition of non-perturbative string theory on this particular background.

The weakest form of the AdS/CFT correspondence:

The large $g_{YM}^2 N$, large N limit of $\mathcal{N} = 4$ $d = 4$ SYM theory is equivalent to classical type IIB supergravity on $AdS_5 \times S^5$.

This statement has been very well tested by now.



Related to two deep ideas.

1. Large N gauge theory is a string theory ('t Hooft)

2. Holography: The Bekenstein-Hawking entropy of black holes $S = A/4G_N$ suggests that quantum gravity in d dimensions can be equivalent to a local quantum field theory in $d-1$ dimensions.

Gauge theory is a good description at small $g_{YM}^2 N$, small g_{YM}^2. String theory is good for large $g_{YM}^2 N$, small g_{YM}^2. Thus we can hope to

Use string theory to learn about gauge theory.
Use gauge theory to learn about string theory.

Difficult, unsolved problem: reconstruct local gravitational physics from field theory, and explain how the local gravitational description breaks down ($\rightarrow$ black hole information paradox).

The hard scattering of glueballs at high-energy in confining gauge theories can be understood using AdS/CFT correspondence. Although string scattering is soft, the warping of the geometry compensates for this. (Polchinski, Strassler)

Actually, strong gravity processes, including black hole formation, play an important role in the string description of such processes, and may saturate the Froissart bound $\sigma \sim \log^2 E$. (Giddings)

One can also study other aspects of the parton model and Deep Inelastic Scattering (Polchinski, Susskind).

Towards a QCD string?

An interesting $N = 1$ theory can be made by starting with a conifold singularity

$$zw - uv = 0$$

and putting N D3 branes and M wrapped D5 branes at the singularity.

The resulting gauge theory has gauge group $SU(N) \times SU(N + M)$, two chiral superfields A_i in the $(N, \overline{N + M})$ representation, and two chiral superfields B_i in the $(\overline{N}, N + M)$. In addition, there is a superpotential

$$W \sim \epsilon^{ij} \epsilon^{kl} \mathrm{tr}(A_i B_k A_j B_l)$$

The field theory has running gauge couplings, and once a coupling gets strong the theory admits a Seiberg-dual description. This, in turn has running couplings, and the process continues. The process continues indefinitely in the UV, and continues in the IR until e.g. a gauge group $SU(M)$ is reached.

This "duality cascade" is nicely reproduced in the dual description, where N and M become non-trivial functions of the radial coordinate z

In addition, these supergravity solutions are non-singular, and exhibit:

- confinement

- glueballs and baryons with a mass scale that emerges through dimensional transmutation

- gluino condensates that break the $\mathbf{Z}_{2M}$ chiral symmetry to $\mathbf{Z}_2$

- domain walls separating different vacua

Is this a candidate for a QCD string in the infrared of the gauge theory? Not really, because there are additional degrees of freedom at the scale Λ_{QCD}. This is a generic problem in trying to find weakly coupled string theory descriptions of gauge theories.

To decouple the extra degrees of freedom, we need to make the curvature of the space-time large, but the string coupling remains small. Therefore, all we need to do is to solve a strongly-coupled sigma model. Due to the presence of RR fluxes, the structure of these sigma models is not very well understood, but there has been progress recently (Berkovits and others) so who knows....

Other ways to see string effects in gauge theories? Look at operators with a large scaling dimension. E.g. baryons with $\Delta \sim N$ correspond to wrapped branes (Witten and other).

Other example: operators like

$$\mathrm{Tr}(\Phi D_{\mu_1} \ldots D_{\mu_S} \Phi)$$

with large S. These correspond to folded rotating closed strings. Both AdS and perturbative gauge theory calculations show that $\Delta - S \sim \log S$. Gubser, Klebanov, Polyakov; Gonzalez-Arrroyo, Lopez

String effects is gauge theories also emerge in another recent development: pp waves (Berenstein, Maldacena, Nastase, hep-th/0202021, 127 citations).

Idea: take a scaling limit of the AdS/CFT duality. In this "Penrose limit" the AdS space reduces to a pp-wave background of string theory which are exactly solvable sigma models. In particular, we know the free string spectrum.

From the field theory point of view, this is a large N limit, where $\Delta + J$ scales as $N^{1/2}$, but $\Delta - J$ remains of order one. (Here J is the generator of some distinguished global $U(1)$ symmetry).

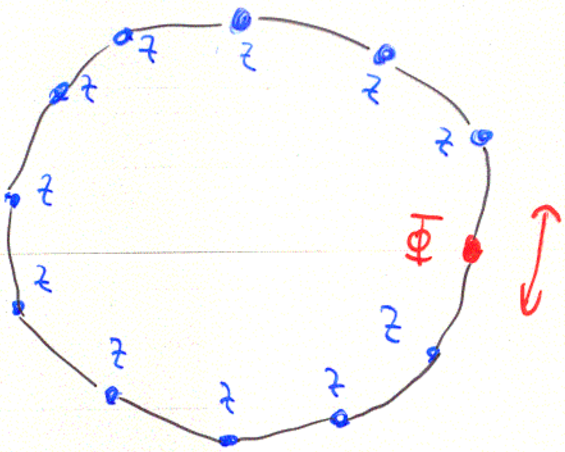
Amazingly, it seems one can recover the complete string spectrum from the gauge theory.

ground state: tr(Z^J)

string states $\sim \sum_k e^{2\pi i k/J} \mathrm{tr}(Z^i \Phi Z^{J-i})$

Realizes the "string bit" model.

Holography for flat space? Recover string interactions or even string field theory from Yang-Mills theory?



Time dependent backgrounds and string theory

As is well-known, string theorists pay a lot of attention to experimental data.

Therefore, it does not come as a surprise that the observation of a positive cosmological constant from supernovae observations has generated a lot of interest in time-dependent backgrounds.

Many cosmological scenario's involving branes have been proposed. Though this is great stuff for newspapers, a concrete embedding of these models in strings theory is often difficult, since string theory does not allow any freedom in the choice of brane tensions and/or interbrane interactions. Also, they often contain singularities that are hard to study.

String theory can deal with singularities of orbifold type, i.e. when space-time is of the form M/G, with a discrete group G.

Are there any orbifolds that provide a good example of a cosmological singularity? Many examples have been studied. Horowitz, Steif; Kounnas, Lüst; Khoury, Ovrut, Seiberg, Steinhardt, Turok; Balasubramanian, Hassan, Keski-Vakkuri, Naqvi; Nekrassov; Simon; Cornalba, Costa; Liu, Moore, Seiberg; Craps, Kutasov, Rajesh; Elitzur, Giveon, Kutasov, Rabinovici

In the simplest case we can take an orbifold of flat space by a discrete subgroup of the Lorentz group $SO(d,1)$, or the Poincaré group. To some extent, the physics of these theories can be extracted from that of the ambient (flat) space, although the lack of time translation, Wick rotation ($+i\epsilon$ prescription??), a Hamiltonian that is bounded from below etc make the interpretation rather confusing.

In addition, if the group is not finite, it has been argued that these backgrounds are generically unstable. Lawrence; Liu, Moore, Seiberg; Fabinger, McGreevy; Horowitz, Polchinski. This happens roughly because a single particle in the orbifold corresponds to infinitely many particles in the covering space, and these generically form a black hole. To evade this, we need a large number of non-compact transversal directions.

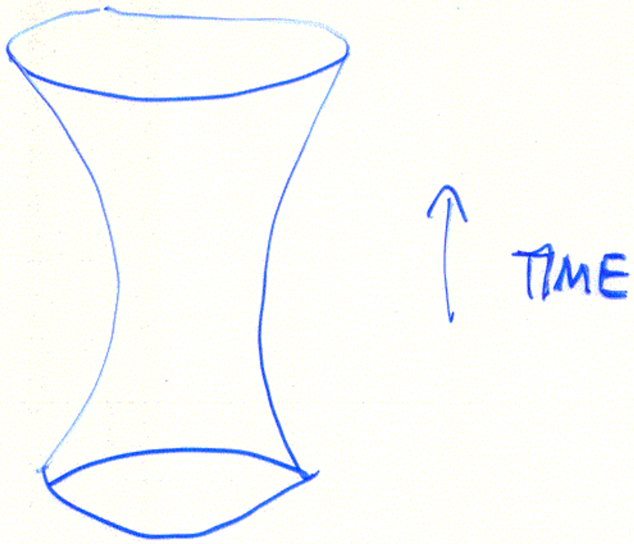
Another approach: S-branes. Since AdS arises from Dp-branes with one time and p space directions, perhaps we can obtain more realistic solutions from branes with zero time and $p+1$ space directions, so-called S-branes Gutperle, Strominger

No stable S-branes known at present.

Or perhaps we need to change the rules of string theory in a more drastic way.

Particle creation in string theory can be formulated in terms of non-local string theories. Aharony, Berkooz, Silverstein; Aharony, Fabinger, Horowitz, Silverstein

Or perhaps, we need to make sense of string theories with imaginary fluxes. Hull; Balasubramanian, JdB, Minic



What about holography for de Sitter space? De Sitter space seems to be dual to some peculiar conformal field theories. Strominger; Bousso, Maloney, Strominger; Spradlin, Volovich; Balasubramanian, JdB, Minic.

Leads to an interesting picture that the transition from the inflationary phase to the present phase of the universe could be described by an inverse renormalization group flow in such peculiar conformal field theories. Strominger; Balasubramanian, JdB, Minic.

Is the tachyon a candidate for the inflaton?

In many string theory setups, the tachyon is under a reasonable amount of control, especially in unstable branes and brane-anti brane pairs Sen and many others.

We can in principle try to find time dependent solutions of the string equations of motion including the tachyon that describe the tachyon rolling down the potential. Some solutions have been found by Sen. They generate a gas of excited string states after the tachyon has tolled down the potential. This can be captured by an effective field theory for the tachyon,

$$S \sim \int e^{-aT} \sqrt{1 + (\partial_\mu T)^2}$$

It is hard to get realistic models in this way, since there is no naturally small parameter in the theory and slow-roll conditions will typically not be satisfied.

If we are entering a de Sitter phase right now, one may wonder whether there is a solution of string theory that involves de Sitter space, or more generally any solution with $\Lambda > 0$.

This is remarkably difficult to achieve. Solutions with $\Lambda < 0$ like AdS are easily obtained by turning on fluxes.

Smooth solutions of supergravity involving de Sitter space and some compact internal space do not exist according to no go theorems (de Wit,Smit,Hari Dass; Maldacena, Nunez).

Ways around: higher derivative terms, negative tension objects.

Can we see signatures of string theory in cosmology?

String theory will affect the power spectrum of curvature fluctuations by terms of order $(H/M)^2$, where M is the scale of new physics (e.g. the string mass), and H the Hubble constant. This assumes a standard choice of vacuum. Non standard choices of vacua can enhance this to (H/M). In any case, it will hard to disentangle such effects from the data; probably we will also need the spectrum of tensor fluctuations.

Still, it would be great if the answer to this question would be yes.