A New Generation of CTEQ Parton Distribution Functions with Uncertainty Analysis

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CTEQ6

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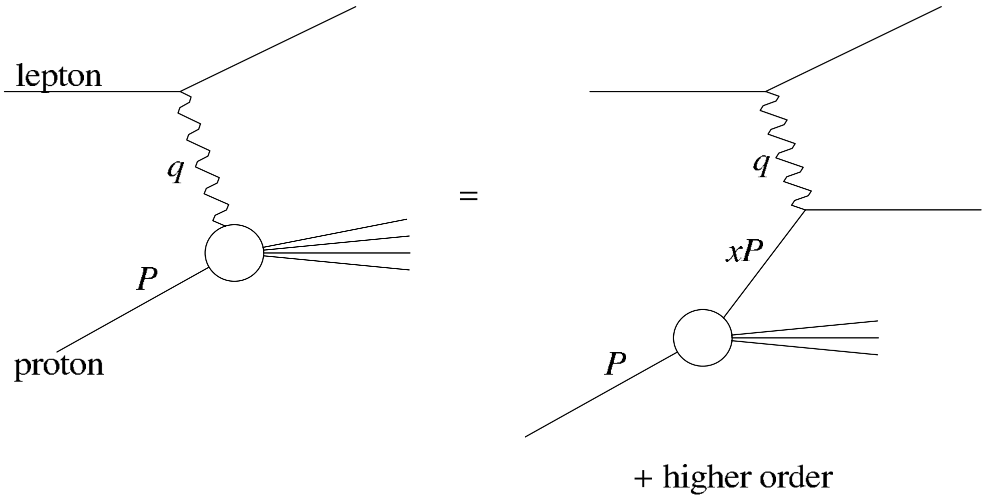
Parton Distribution Functions

For any short-distance process,

e.g., *ep* DIS,



the factorization theorem of QCD

(*schematically*)

$$\sigma(Q) = \int \quad f_i(x, Q)\ \hat{\sigma}_i(x, Q)dx$$

relates experimental $\sigma(Q)$ and perturbatively calculated $\hat{\sigma}_i(x,Q)$.

Global analysis

- Use data from many processes to determine the universal PDF's.

- Parametrize the $f_i(x,Q_0)$ at Q_0=1.3 GeV, with 20 fitting parameters

$\{a_1, a_2, \ldots, a_n\}$.

What is new in the CTEQ6 analysis?

- New Data
  - H1 and ZEUS : deep inelastic ep and $\bar{e}p$ scattering
  - DØ : $p\bar{p} \rightarrow jet$ cross section, as a function of η and E_T.
- New methods of analysis
  - For systematic errors … The published systematic errors are included in the fitting procedure.
  - For uncertainties … Methods are available to evaluate the uncertainties of the PDF's and their predictions.

Data used in the CTEQ6 global analysis

Data for which detailed systematic errors have been published and used in the fit:

process	data set	χ^2_e/N_e
DIS μp	BCDMS p	$378/339$
DIS μd	BCDMS d	$280/251$
DIS ep	H1a	$99/104$
DIS ep	H1b	$129/126$
DIS ep	ZEUS	$263/229$
DIS μp	NMC F2p	$305/201$
DIS μd	NMC F2d/p	$112/123$
$p\bar{p} \to$ jet	DØ jet	$69/90$
$p\bar{p} \to$ jet	CDF jet	$49/33$

Other data used in the global analysis but without systematics:

process	data set	χ^2_e/N_e
DIS ν Fe	CCFR	$150/156$
DY pp	E605	$95/119$
DY pd/pp	E866	$6/15$
$p\bar{p} \to W$	CDF W	$10/11$

Overall $\chi^2/N = 1954/1811$

χ^2 minimization

The simplest fitting method is to define

$$\chi_0^2 = \sum_{i=1}^{N} \frac{(D_i - T_i)^2}{\sigma_i^2} \qquad \begin{cases} D_i = & \text{data point} \\ T_i = & \text{theory value} \\ \sigma_i = & \text{“expt. error”} \end{cases}$$

and minimize χ_0^2 with respect to the PDF model parameters $\{a_\lambda\}$. However, the systematic errors imply

$$D_i = T_i(a) + \alpha_i r_{\text{stat},i} + \sum_{k=1}^{K} r_k \beta_{ki}$$

where α_i = uncorrelated error on D_i

$\beta_{ki} = k^{\text{th}}$ systematic error on D_i

(numbers published by the expt.)

($r_{\text{stat},i}$ and r_k are random variables with standard deviation 1.)

To take into account the systematic errors† we define

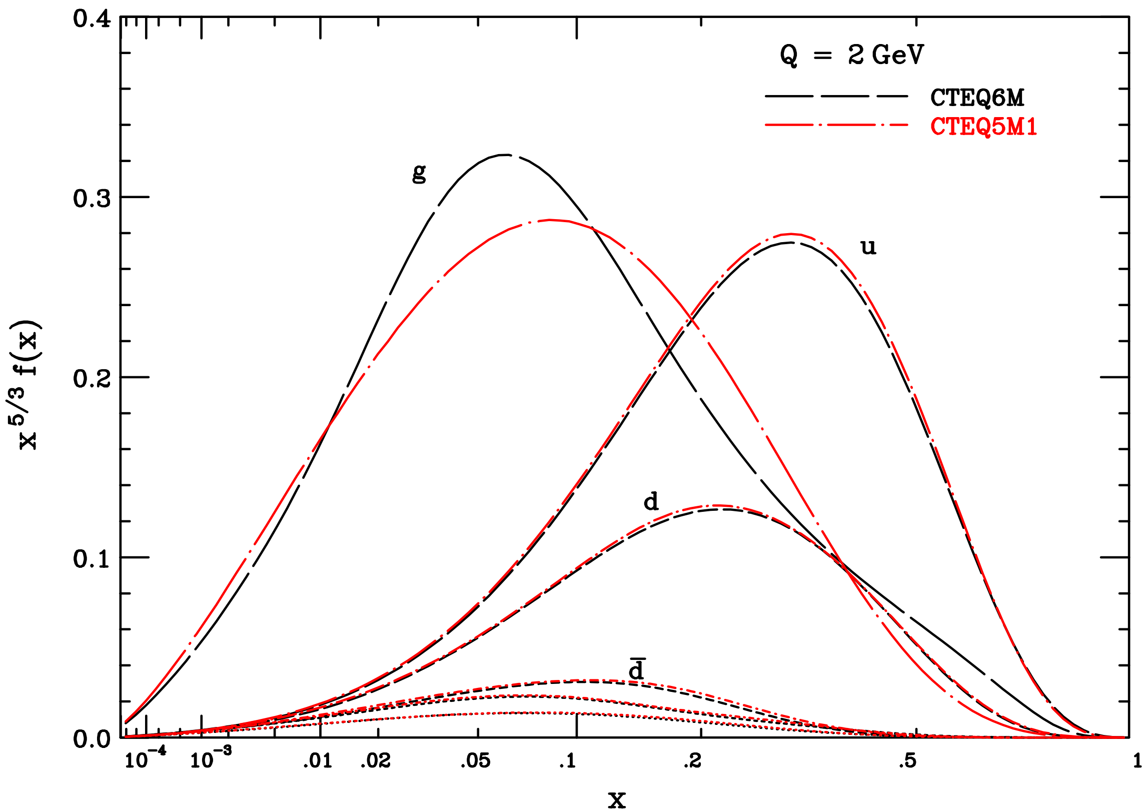
$$\chi'^2(a_\lambda, r_k) = \sum_{i=1}^{N} \frac{(D_i - \sum_k r_k \beta_{ki} - T_i)^2}{\alpha_i^2} + \sum_k r_k^2,$$

and minimize with respect to both $\{r_k\}$ (≡ the systematic shifts) and $\{a_\lambda\}$ (≡ the PDF model parameters).

Because we use a quadratic penalty term r_k^2, the minimization with respect to $\{r_k\}$ can be done analytically (for arbitrary $\{a_\lambda\}$). Then the minimization w. r. t. $\{a_\lambda\}$ is done numerically.

† In CTEQ6 we symmetrize the systematic errors.

Comparison of CTEQ6 and CTEQ5

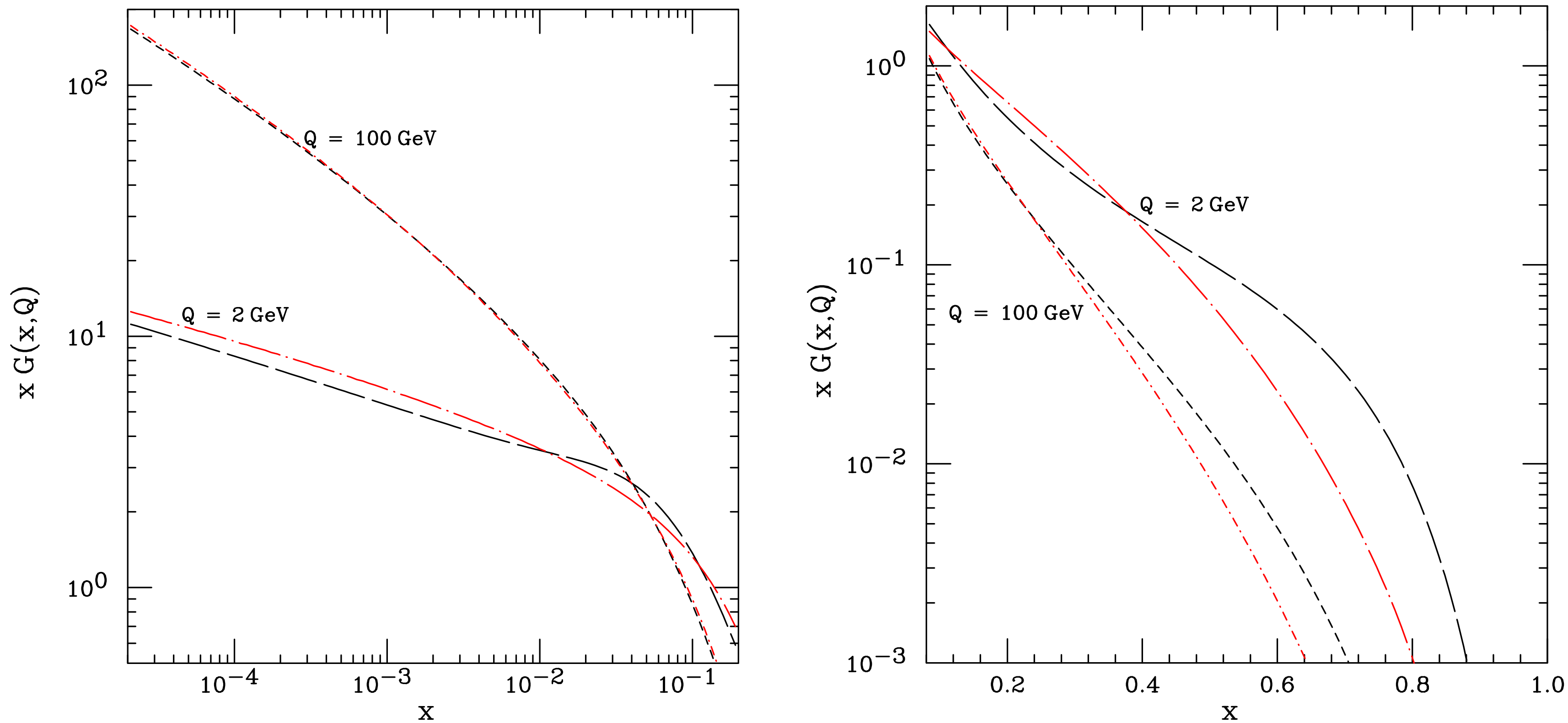


CTEQ6M and CTEQ5M1 PDF's at Q=2 GeV.

(unlabeled:
$\bar{u}$ and $s = \bar{s}$;
MS-bar scheme)

- ► The quark distributions have not changed much.
- ► The gluon is noticeably different.

The Gluon Distribution



CTEQ6M and CTEQ5M1 gluon distributions at Q = 2 and 100 GeV. (a) Small-x region; (b) large-x region.

► ***Note the hard gluon distribution in CTEQ6M.***

Comparison to Data

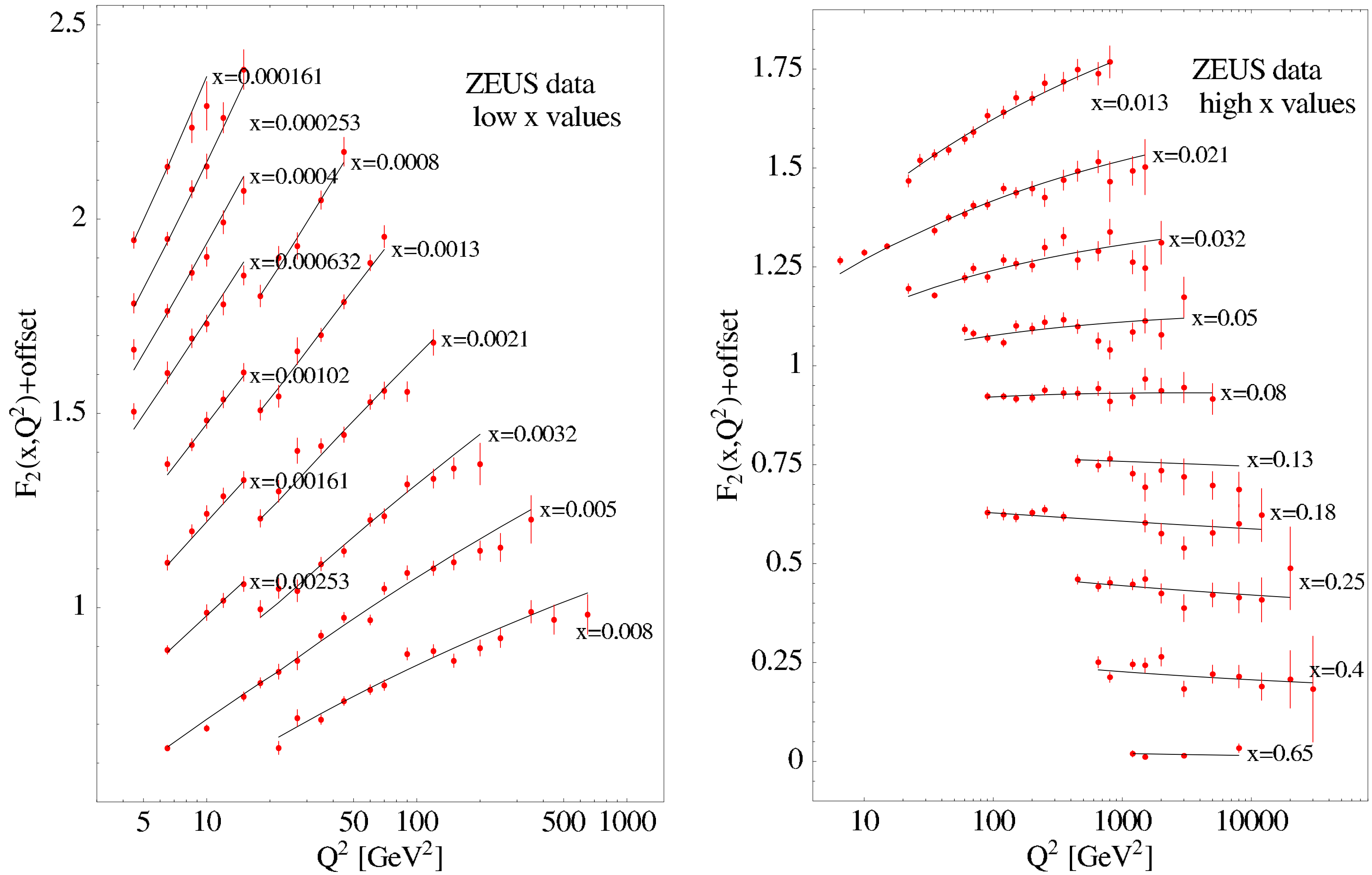
Data sets with published correlated systematic errors—

data set	N_e	χ_e^2/N_e
BCDMS p	339	1.114
BCDMS d	251	1.114
H1a	104	0.948
H1b	126	1.024
ZEUS	229	1.147
NMC F2p	201	1.517
NMC F2d/p	123	0.909
DØ jet	90	0.766
CDF jet	33	1.472

There is good qualitative agreement between theory and data,

but detailed studies are necessary to assess the uncertainties.

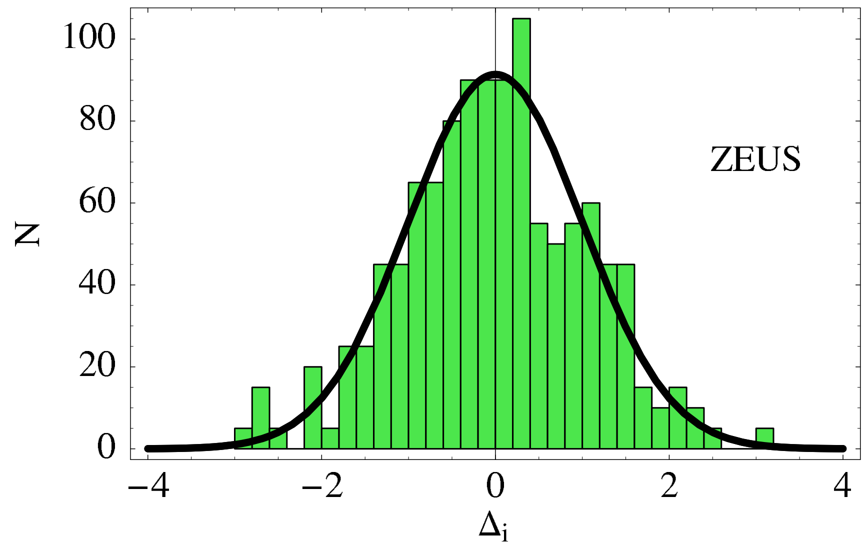
Consider ZEUS F_2 measurements as an example …



CTEQ6M model and ZEUS data in separate x bins. The data points include the optimal shifts for systematic errors. The error bars are statistical errors only.

ZEUS collaboration: S. Chekanov *et al*, Eur Phys J, **C21** (2001) 443.

The "pull" distribution



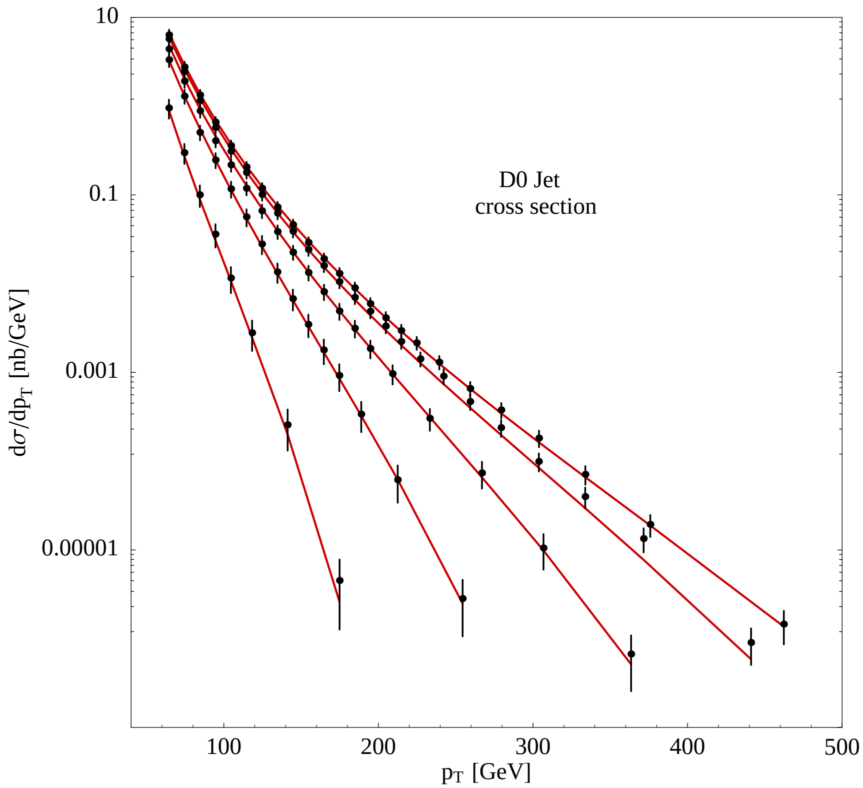
Histogram of residuals for the ZEUS data

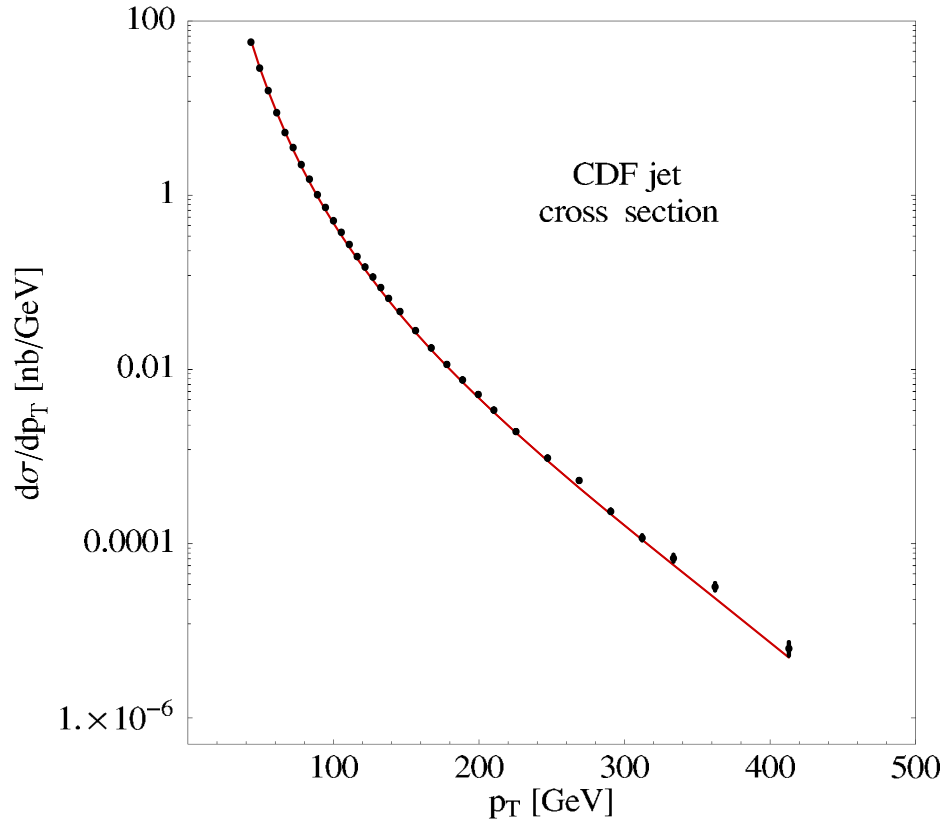
$$\Delta_i = (D_i^{\text{shifted}} - T_i)/\alpha_i \quad \text{where} \quad D_i^{\text{shifted}} = D_i - \sum_{k=1}^{K} \hat{r}_k \beta_{ki}.$$

The curve is a Gaussian of width 1.

The optimal systematic shifts $\hat{r}_k$ are all of order 1, consistent with expectations.

Tevatron $p\bar{p} \rightarrow jet$ inclusive cross section





The CTEQ6M fit to the inclusive jet data.

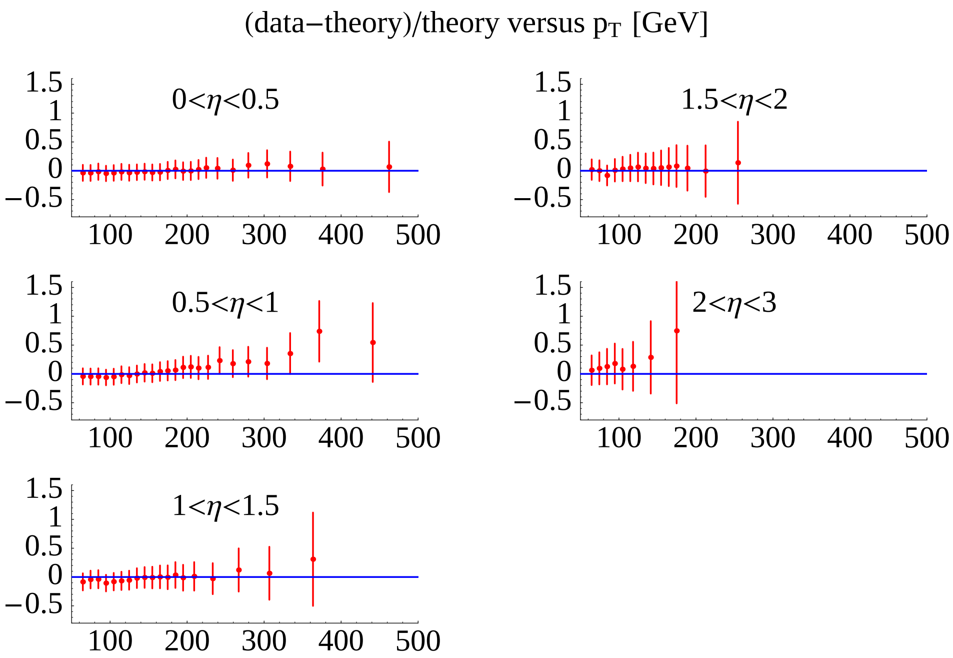
(a) DØ data for 5 rapidity bins (0.0 0.5 1.0 1.5 2.0 3.0);

(b) CDF data for central rapidity ($0.1 < |\eta| < 0.7$).

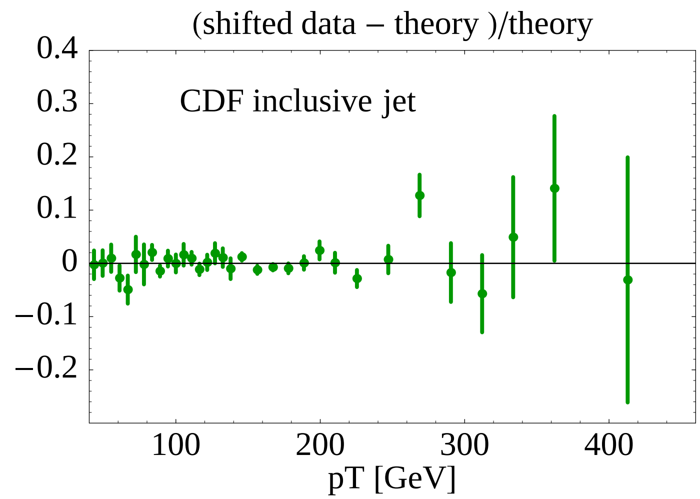
DØ Collaboration: B. Abbott *et al* ; CDF Collaboration: T. Affolder *et al.*

Closer comparison (data - theory / theory) between CTEQ6M and the jet cross section

DØ jet cross section



CDF jet cross section



The Tevatron inclusive jet cross section implies a hard gluon distribution, i.e., *g(x,Q)* is large at large *x*.

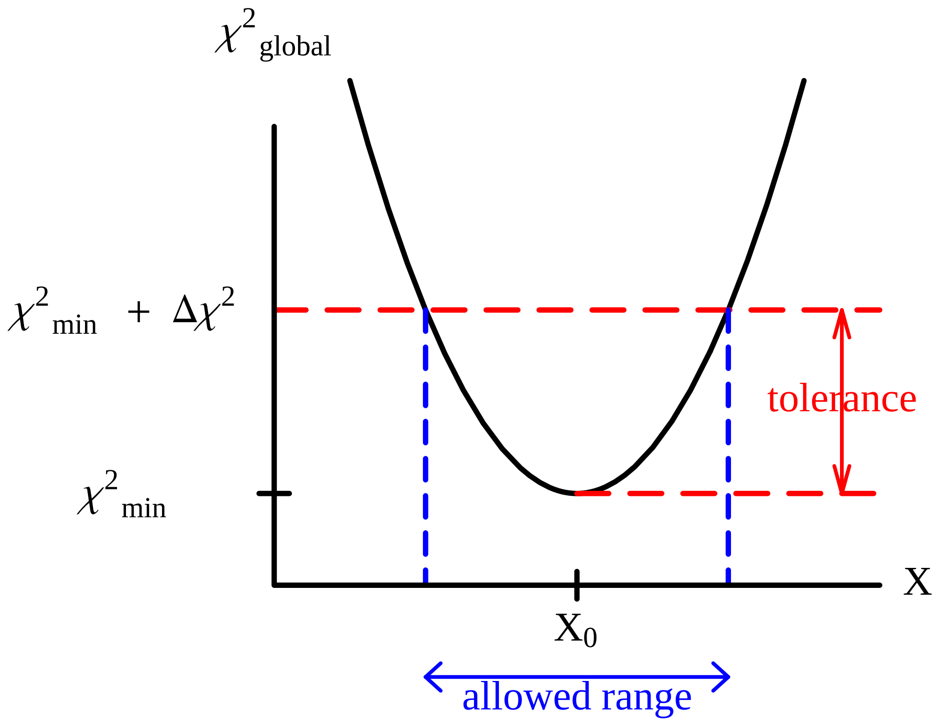
(Recall CTEQ4HJ and CTEQ5HJ.)

Quantitative Uncertainties of PDF's

Computational tools

- Lagrange Multiplier Method (constrained fitting)
- Hessian Matrix Method (a *complete set* of allowed variations using the eigenvector basis)

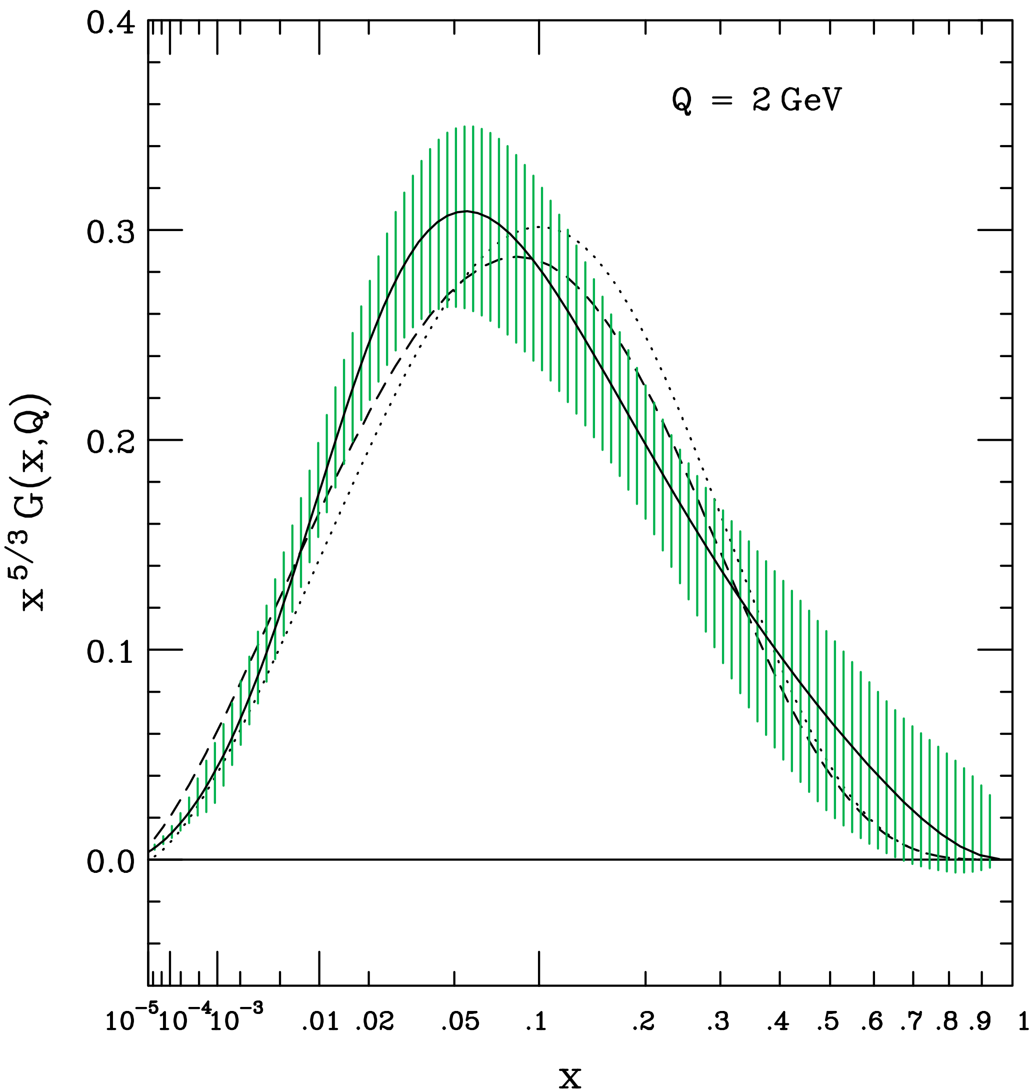
Plot χ^2_{global} versus an observable X.



The question of ***tolerance***

We conclude that a large tolerance ($\Delta\chi^2 \sim 100$ for 1800 data points) is realistic.

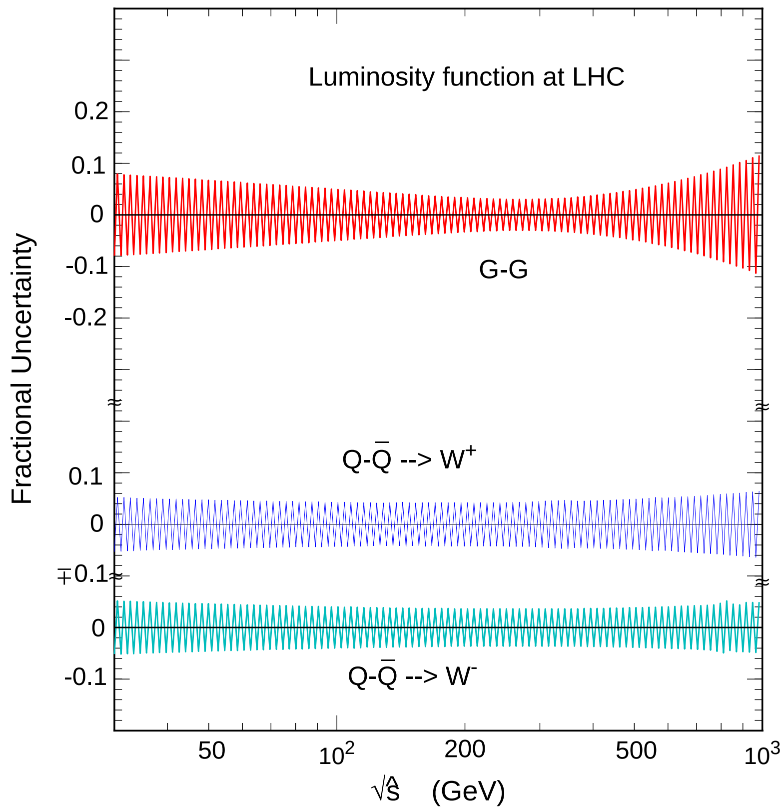
Uncertainty band for the gluon distribution function (at Q = 2 GeV).

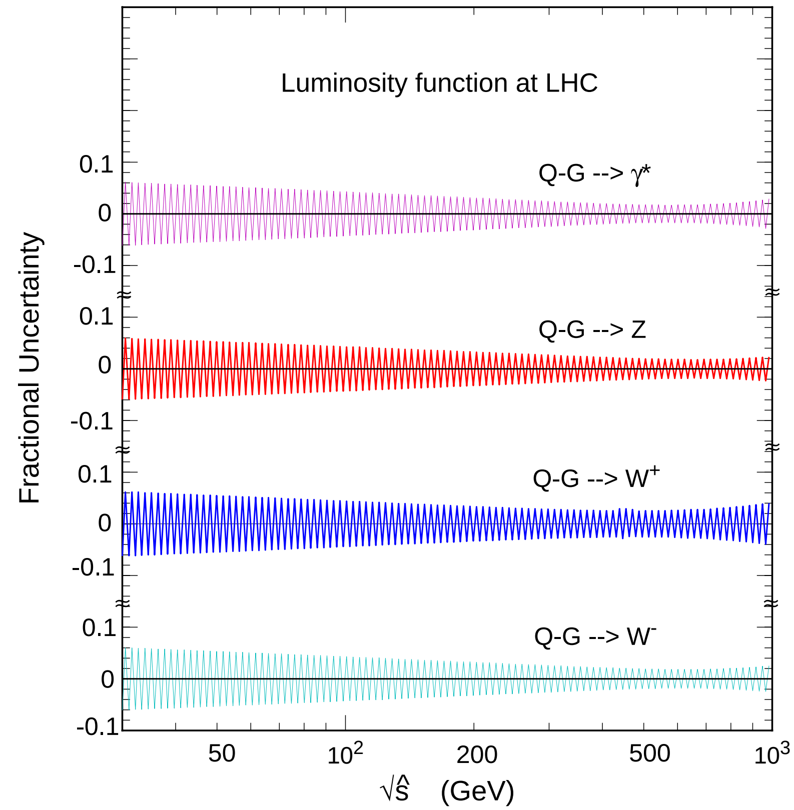


curves ≡
solid : CTEQ6M
dashed : CTEQ5M1
dotted : MRST 2001

Uncertainty band ≡ envelope of allowed variations

Uncertainties of LHC parton-parton luminosities

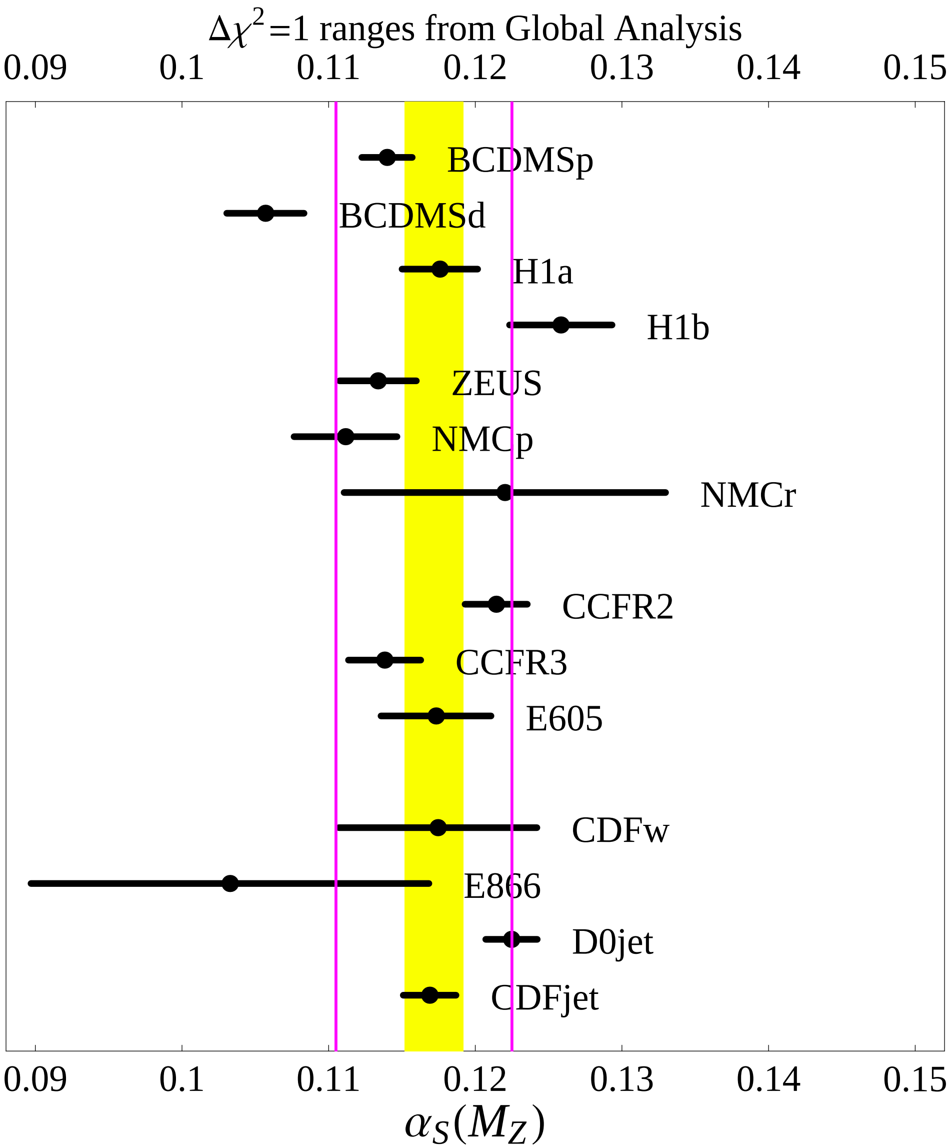




$$\mathcal{L}(\hat{s}) = \sum_{i,j} C_{ij} \int f_i(x_1) f_j(x_2) \delta(\hat{s} - x_1 x_2 s) dx_1 dx_2$$

provides simple estimates of PDF uncertainties at the LHC.

Example $\alpha_S(M_Z)$ from the CTEQ6 global analysis



Each data set gives a best value of α_S (from min. χ^2) and an "allowed range" of α_S (from $\Delta\chi^2 \leq 1$).

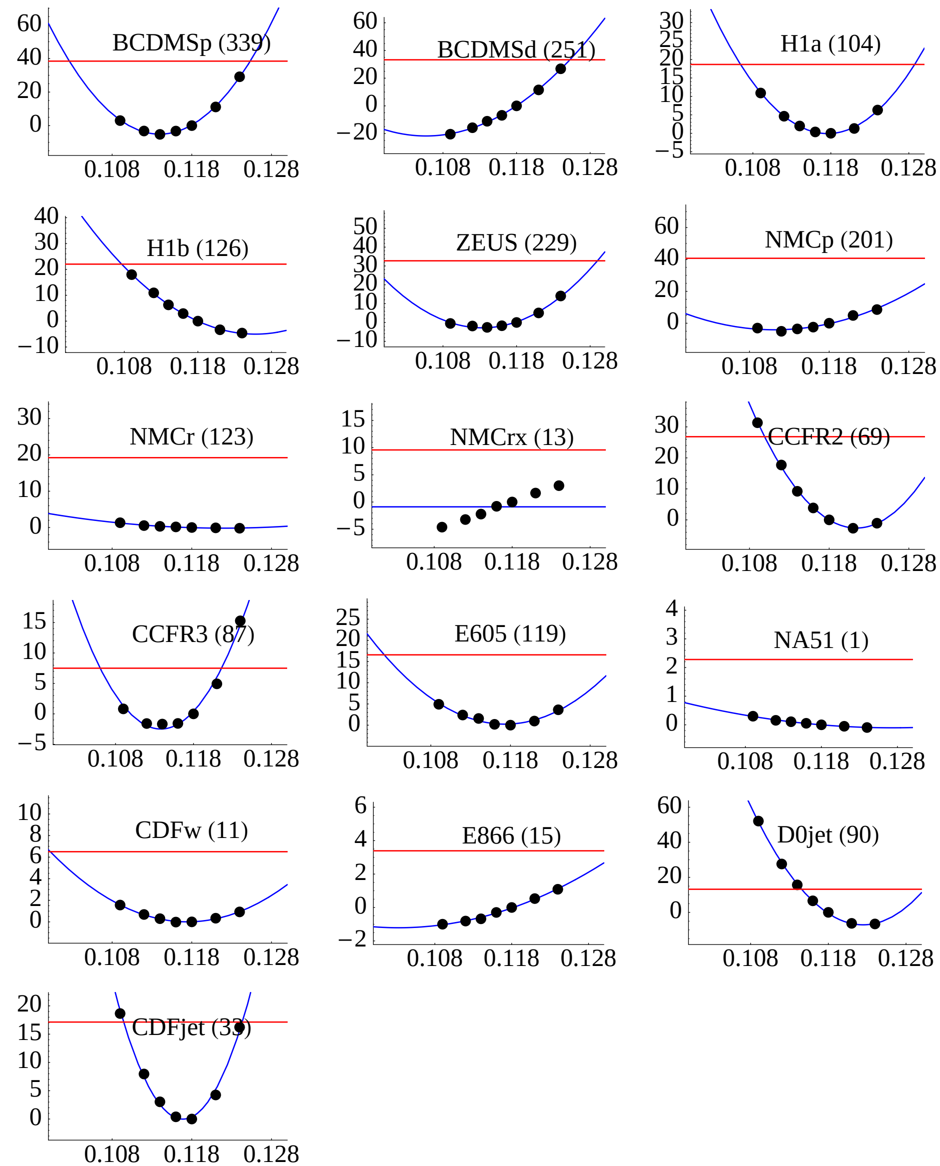
Particle Data Group (shaded strip) is 0.117 ± 0.002.

The fluctuations are larger than expected for normal statistics. The vertical lines have $\Delta\chi^2_{global} = 100$,

$$\alpha_S(M_Z) = 0.1165 \pm 0.0065$$

Determination of α_S from the CTEQ global fit …

Plot χ^2 versus α_S for the individual data sets.



Conclusions

► The CTEQ6 parton distributions are available at www.cteq.org

CTEQ6M	MS-bar scheme
CTEQ6D	DIS scheme
CTEQ6L and L1	LO model
40 extreme sets	(eigenvector basis)

► … also available in the LHAPDF Interface at pdf.fnal.gov.

► We are still early in the study of PDF uncertainties—important for precision measurements at hadron colliders.