YFS Monte Carlo Approach to QCD Soft Gluon Exponentiation

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Outline:

- **Introduction**
- **Review of YFS Theory: An Abelian Gauge Theory Example**
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- **YFS Exponentiated QCD Corrections to $t\bar{t}$ Production at High Energies**
- **Conclusions**

Papers by S. Jadach and B.F.L. Ward, S. Jadach, *et al.*, M. Phys. Lett. A **14** (1999) 491, hep-ph/0205062; *ibid.* **12** (1997) 2425.

Motivation

- FNAL/RHIC $t\bar{t}$ PRODUCTION; POLARIZED pp PROCESSES; $b\bar{b}$ PRODUCTION; J/Ψ PRODUCTION: SOFT n(G) EFFECTS ALREADY NEEDED

  $\Delta m_t = 5.1$ GeV with SOFT n(G) UNCERTAINTY $\sim$ 2-3 GeV, ..., ETC.

- FOR THE LHC/TESLA/LC, THE REQUIREMENTS WILL BE EVEN MORE DEMANDING AND OUR QCD SOFT n(G) MC EXPONENTIATION RESULTS WILL BE AN IMPORTANT PART OF THE NECESSARY THEORY – YFS EXPONENTIATED $\mathcal{O}(\alpha_s^2)L$, ON AN EVENT-BY-EVENT BASIS

- CROSS CHECK OF LITERATURE:

  1. PHASE SPACE – CATANI, CATANI-SEYMOUR, ALL INITIAL PARTONS MASSLESS
  2. RESUMMATION – CATANI ET AL., BERGER ET AL., ...
  3. NO-GO THEOREMS

PRELIMINARIES

- WE USE THE GPS CONVENTIONS OF JWW FOR SPINORS.
- PHOTON-GLUON POLARIZATION VECTORS FOLLOW THEREFROM:

$$(\epsilon^{\mu}_{\sigma}(\beta))^* = \frac{\bar{u}_{\sigma}(k)\gamma^{\mu} u_{\sigma}(\beta)}{\sqrt{2}\ \bar{u}_{-\sigma}(k) u_{\sigma}(\beta)}, \quad (\epsilon^{\mu}_{\sigma}(\zeta))^* = \frac{\bar{u}_{\sigma}(k)\gamma^{\mu} u_{\sigma}(\zeta)}{\sqrt{2}\ \bar{u}_{-\sigma}(k) u_{\sigma}(\zeta)}, \tag{1}$$

- REPRESENTATIVE PROCESS IN FIG. 1

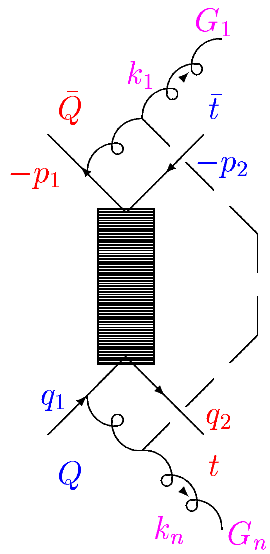


Figure 1: The process $\bar{Q}Q \to \bar{t} + t + n(G)$. The four–momenta are indicated in the standard manner: q_1 is the four–momentum of the incoming Q, q_2 is the four–momentum of the outgoing t, etc., and $Q = u, d, s, c, b, G$.

Review of YFS Theory: An Abelian Gauge Theory Example

QED CASE – S. Jadach et al., YFS2, YFS3, BHLUMI, BHWIDE, KORALZ, KKMC, YFSWW3, YFSZZ, KoralW

For $e^+(p_1)e^-(q_1) \to \bar{f}(p_2)f(q_2) + n(\gamma)(k_1, \cdot, k_n)$, renormalization group improved YFS theory (PRD36(1987)939) gives

$$d\sigma_{exp} = e^{2\alpha\, Re\, B + 2\alpha\, \tilde{B}} \sum_{n=0}^{\infty} \frac{1}{n!} \int \prod_{j=1}^{n} \frac{d^3 k_j}{k_j^0} \int \frac{d^4 y}{(2\pi)^4} e^{iy(p_1+q_1-p_2-q_2-\sum_j k_j)+D}$$

$$\bar{\beta}_n(k_1, \ldots, k_n) \frac{d^3 p_2 d^3 q_2}{p_2^0 q_2^0}$$

where the YFS real infrared function $\tilde{B}$ and the virtual infrared function B are known and where we note the usual connections

$$2\alpha\, \tilde{B} = \int^{k \le K_{max}} \frac{d^3 k}{k_0} \tilde{S}(k)$$

$$D = \int d^3 k \frac{\tilde{S}(k)}{k^0} \left(e^{-iy\cdot k} - \theta(K_{max} - k) \right) \qquad (2)$$

for the standard YFS infrared emission factor

$$\tilde{S}(k) = \frac{\alpha}{4\pi^2}\left[Q_f Q_{(\bar{f})'}\left(\frac{p_1}{p_1\cdot k} - \frac{q_1}{q_1\cdot k}\right)^2 + (\ldots)\right] \qquad (3)$$

if Q_f is the electric charge of f in units of the positron charge. For example, the YFS hard photon residuals $\bar{\beta}_i$ in (1), $i = 0, 1, 2$, are given in S. Jadach *et al.*,CPC102(1997)229 for BHLUMI 4.04 $\Rightarrow$ YFS exponentiated exact $\mathcal{O}(\alpha)$ and LL $\mathcal{O}(\alpha^2)$ cross section for Bhabha scattering via a corresponding Monte Carlo realization of (1). Here, we will develop and apply the analogous theoretical paradigm to the prototypical QCD higher order radiative corrections problem for $Q\bar{Q} \to t\bar{t} + n(G)$.

Extension to non-Abelian Gauge Theories: Proof

We focus proto-typically for the moment on the subprocess $q_i + \bar{q}_i \rightarrow t\bar{t} + n(G)$ for definiteness.

In DeLaney *et al.*,PRD52(1995)108, PLB342(1995)239, we have analyzed how in the special case of Born level color exchange in all of the s, t, and u channels one extends the YFS theory's IR singularity analysis to QCD to all orders in α_s. Here, we will take a somewhat different approach:

- **YFS theory as a general re-arrangement of renormalized perturbation theory based IR behavior $\sim$ the renormalization group as a general property of renormalized perturbation theory based on its UV(ultra-violet) behavior.**

Our arguments are entirely general from the outset $\Rightarrow$
Our result applies to any renormalized perturbation theory.

Let the amplitude for the emission of n real gluons be represented by

$$\mathcal{M}^{(n)\alpha\bar{\alpha}}_{\gamma\bar{\gamma}} = \sum_{\ell} M^{(n)\alpha\bar{\alpha}}_{\gamma\bar{\gamma}\ell}, \tag{4}$$

$M^{(n)}_{\ell} \Leftrightarrow$ contribution to $\mathcal{M}^{(n)}$ from Feynman diagrams with ℓ virtual loops.

Define the virtual gluon IR emission factor $S_{QCD}(k)$ such that

$$\lim_{k\to 0} k^2 \left(\rho^{(n)\alpha\bar{\alpha}}_{\gamma\bar{\gamma}1}(k) - S_{QCD}(k)\rho^{(n)\alpha\bar{\alpha}}_{\gamma\bar{\gamma}0}\right)|_{\alpha\neq\bar{\alpha}\neq\gamma\neq\alpha} = 0, \tag{5}$$

where we have now introduced the Born level color exchange condition as $\alpha \neq \bar{\alpha} \neq \gamma \neq \alpha$ for definiteness.

From the analogy of the YFS analysis, we get the "YFS representation",

$$\mathcal{M}^{(n)} = exp(\alpha_s B_{QCD}) \sum_{j=0}^{\infty} \mathrm{m}^{(n)}_{j}, \tag{6}$$

where we have defined

$$\alpha_s(Q)B_{QCD} = \int \frac{d^4k}{(k^2-\lambda^2+i\epsilon)} S_{QCD}(k) \tag{7}$$

and

$$\mathrm{m}_j^{(n)} = \frac{1}{j!}\int \prod_{i=1}^{j} \frac{d^4k_i}{k_i^2}\beta_j(k_1,\cdots,k_j). \tag{8}$$

Note: we have only shown that $\beta_i(k_1,...,k_i)$ do not contain the virtual IR singularities in the product $S_{QCD}(k_1)\cdots S_{QCD}(k_i)\rho_0^{(n)0}$.

We treat analogously the real IR singularities associated with the $\mathcal{M}^{(n)} \Rightarrow$ we write the respective cross section using the standard methods as

$$d\hat{\sigma}^n = \frac{e^{2\alpha_s ReB_{QCD}}}{n!}\int \prod_{m=1}^{n} \frac{d^3k_m}{(k_m^2+\lambda^2)^{1/2}}\delta(P_1+Q_1-P_2-Q_2-\sum_{i=1}^{n}k_i^0)$$
$$\bar{\rho}^{(n)}(P_1,P_2,Q_1,Q_2,k_1,\cdots,k_n)\frac{d^3P_2d^3Q_2}{P_2^0Q_2^0}, \tag{9}$$

where we have defined

$$\bar{\rho}^{(n)}(P_1,P_2,Q_1,Q_2,k_1,\cdots,k_n) = \sum_{color,spin} \| \sum_{j=0}^{\infty} \mathrm{m}_j^{(n)} \|^2 \tag{10}$$

in the incoming $q\bar{q}$ cms system, we get, for the definition

$$\lim_{|\vec{k}|\to 0} \vec{k}^2 \left(\bar{\rho}^{(1)}(k) - \tilde{S}_{QCD}(k)\bar{\rho}^{(0)} \right) = 0, \tag{11}$$

where the real infrared function $\tilde{S}_{QCD}(k)$ is explicitly computed in DeLaney *et al.*, op. cit., upon applying the analogous YFS expansion in $\tilde{S}_{QCD}(k)$ and summing on n the "YFS-like" result

$$\begin{aligned} d\hat{\sigma}_{\rm exp} &= \sum_n d\hat{\sigma}^n \\ &= e^{\rm SUM_{IR}(QCD)} \sum_{n=0}^{\infty} \int \prod_{j=1}^{n} \frac{d^3 k_j}{k^0{}_j} \int \frac{d^4 y}{(2\pi)^4} e^{iy\cdot(P_1+P_2-Q_1-Q_2-\sum k_j)+D_{\rm QCD}} \\ &* \bar{\beta}_n(k_1,\ldots,k_n) \frac{d^3 P_2}{P_2^0} \frac{d^3 Q_2}{Q_2^0} \end{aligned} \tag{12}$$

with

$$SUM_{IR}(QCD) = 2\alpha_s ReB_{QCD} + 2\alpha_s \tilde{B}_{QCD}(K_{\rm max}),$$

$$2\alpha_s \tilde{B}_{QCD}(K_{\rm max}) = \int \frac{d^3k}{k^o} \tilde{S}_{\rm QCD}(k)\theta(K_{\rm max} - k),$$

$$D_{\rm QCD} = \int \frac{d^3k}{k} \tilde{S}_{\rm QCD}(k) \left[e^{-iy\cdot k} - \theta(K_{\rm max} - k)\right], \quad (13)$$

$$\frac{1}{2}\bar{\beta}_0 = d\sigma^{(1-{\rm loop})} - 2\alpha_s {\rm Re} B_{\rm QCD} d\sigma_B,$$

$$\frac{1}{2}\bar{\beta}_1 = d\sigma^{B1} - \tilde{S}_{\rm QCD}(k) d\sigma_B, \quad \ldots \quad (14)$$

where the $\bar{\beta}_n$ are the QCD hard gluon residuals; Here, for illustration, we note that the exact one-loop and single bremsstrahlung cross sections, $d\sigma^{(1-{\rm loop})}$, $d\sigma^{B1}$, respectively, may be taken from Nason *et al.*(NPB303 (1988) 607; *ibid.* B327 (1989) 49; *ibid.* B335 (1990) 260) and Beenakker *et al.* (

PRD40 (1989) 54; NPB351 (1991) 507). We stress two things about the right-hand side of (12) :

- It is independent of the dummy parameter K_{max}.
- Its realization in our new CEEX (S. Jadach *et al.*, PRD63(2001)113009, and references therein) format is possible.

What about the presence of infrared divergences in the $\bar{\beta}_n$ which were not removed into the $S_{QCD}, \tilde{S}_{QCD}$?

Infrared finiteness of the left-hand side of (12) and infrared finiteness of $SUM_{IR}(QCD) \Rightarrow$

$$d\bar{\hat{\sigma}}_{\rm exp} \equiv \exp[-{\rm SUM_{IR}(QCD)}]d\hat{\sigma}_{\rm exp}$$

must also be infrared finite to all orders in α_s.

$\Rightarrow$ Each order in α_s must make an infrared finite contribution to $d\bar{\hat{\sigma}}_{\rm exp}$.

Definition:

$$\bar{\beta}_n^{(\ell)} = \tilde{\bar{\beta}}_n^{(\ell)} + D\bar{\beta}_n^{(\ell)}$$

where $\tilde{\bar{\beta}}_n^{(\ell)}$ is now completely free of any infrared divergences and $D\bar{\beta}_n^{(\ell)}$ contains all left-over infrared divergences in $\bar{\beta}_n^{(\ell)}$ which are of non-Abelian origin and vanishes in the Abelian limit $f_{abc} \to 0$.

We define $D\bar{\beta}_n^{(\ell)}$ by a minimal subtraction of the respective IR divergences: only $1/\epsilon - C_E$ or $\ln\lambda^2$ occur.

$\Rightarrow$

$$\int dPhD\bar{\beta}_n^{(\ell)} \equiv \sum_{i=1}^{n+\ell} d_i^{n,\ell} \ln^i(\lambda^2)$$

with $d_i^{n,\ell}$ independent of λ for $\lambda \to 0$ and with $dPh \equiv$ n-gluon LIPS.

At $\mathcal{O}(\alpha_s^n(Q))$, the IR finiteness of the contribution to $d\bar{\hat{\sigma}}_{\rm exp} \Rightarrow$

$$d\bar{\hat{\sigma}}^{(n)}_{\rm exp} \equiv \int \sum_{\ell=0}^{n} \frac{1}{\ell!} \prod_{j=1}^{\ell} \int_{k_j^0 \geq K_{max}} \frac{d^3k_j}{k_j} \tilde{S}_{QCD}(k_j) \sum_{i=0}^{n-\ell} \frac{1}{i!} \prod_{j=\ell+1}^{\ell+i} \int \frac{d^3k_j}{k_j} \bar{\beta}_i^{(n-\ell-i)}(k_{\ell+1}, \ldots, k_{\ell+i}) \frac{d^3P_2}{P_2^0} \frac{d^3Q_2}{Q_2^0} \quad (15)$$

to be finite.

From this it follows that

$$D d\bar{\hat{\sigma}}_{\rm exp}^{(n)} \equiv \int \sum_{\ell=0}^{n} \frac{1}{\ell!} \prod_{j=1}^{\ell} \int_{k_j^0 \geq K_{max}} \frac{d^3 k_j}{k_j} \tilde{S}_{QCD}(k_j) \sum_{i=0}^{n-\ell} \frac{1}{i!} \prod_{j=\ell+1}^{\ell+i} \int \frac{d^3 k_j}{k_j} D\bar{\beta}_i^{(n-\ell-i)}(k_{\ell+1}, \ldots, k_{\ell+i}) \frac{d^3 P_2}{P_2^0} \frac{d^3 Q_2}{Q_2^0} \tag{16}$$

is finite. Since the integration region for the final particles is arbitrary, the independent powers of the IR regulator $\ln(\lambda^2)$ in this last equation must give vanishing contributions. This means that we can drop the $D\bar{\beta}_n^{(\ell)}$ from our result (12) because they do not make a net contribution to the final parton

cross section $\hat{\sigma}_{\rm exp}$. We thus finally arrive at the **new rigorous result**

$$d\hat{\sigma}_{\rm exp} = \sum_n d\hat{\sigma}^n$$
$$= e^{\rm SUM_{IR}(QCD)} \sum_{n=0}^{\infty} \int \prod_{j=1}^{n} \frac{d^3 k_j}{k_j} \int \frac{d^4 y}{(2\pi)^4} e^{iy\cdot(P_1+P_2-Q_1-Q_2-\sum k_j)+D_{\rm QCD}}$$
$$* \tilde{\bar{\beta}}_n(k_1,\ldots,k_n) \frac{d^3 P_2}{P_2^{\,0}} \frac{d^3 Q_2}{Q_2^{\,0}} \quad (17)$$

where now the hard gluon residuals $\tilde{\bar{\beta}}_n(k_1,\ldots,k_n)$ defined by

$$\tilde{\bar{\beta}}_n(k_1,\ldots,k_n = \sum_{\ell=0}^{\infty} \tilde{\bar{\beta}}_n^{(\ell)}(k_1,\ldots,k_n)$$

are free of all infrared divergences to all orders in $\alpha_s(Q)$. This is a basic result of this section. Note that the arguments in the earlier references (DeLaney *et al.* op. cit.) are not really sufficient to derive the respective analog of eq.(17); for, they did not really expose the compensation between the left over genuine non-Abelian IR virtual and real singularities between $\int dPh\bar{\beta}_n$ and $\int dPh\bar{\beta}_{n+1}$ respectively that really distinguishes QCD from QED, where no such compensation occurs.

We comment on (17) as follows:

- One may use the results of Gatheral (PLB133(1983) 90) to compute h.o. corr. to SUM_{IR} and one may correct SUM_{IR} to apply to a chain type organization of the cross section as well following Dokshitzer *et al.*, Collins, etc.: both of these efforts are in progress.
- One will be able to view (17) as a master resummation formula $\Rightarrow$ contact with all other approaches to QCD resummation by an appropriate choice of the new $\tilde{\bar{\beta}}_n(k_1, \ldots, k_n)$ and SUM_{IR}, e.g.,
$\Pi\tilde{S}_{QCD}(\tilde{\bar{\beta}}_0 + \tilde{\bar{\beta}}_1/\tilde{S}_{QCD} + \cdots) \Leftrightarrow (J^2\hat{\sigma}_2 + J^3\hat{\sigma}_3 + \cdots)$
- YFS resummed Quantum Gravity – YFS resum the propagators in the NON-ABELIAN gauge theory of QG:
$\Rightarrow$ from the YFS formula

$$iS'_F(p) = \frac{ie^{-\alpha B''_\gamma}}{S_F^{-1}(p) - \Sigma'{}_F(p)}, \tag{18}$$

We find for Quantum Gravity, proceeding as above, the analogue of

$$\alpha B''_\gamma = \int \frac{d^4\ell}{(2\pi)^4} \frac{-i\eta^{\mu\nu}}{(\ell^2 - \lambda^2 + i\epsilon)} \frac{-ie(2ik_\mu)}{(\ell^2 - 2\ell k + \Delta + i\epsilon)} \frac{-ie(2ik'_\nu)}{(\ell^2 - 2\ell k' + \Delta' + i\epsilon)}\Big|_{k=k'} \tag{19}$$

as $-B''_g(k)$ **with**

$$B''_g(k) = -2i\kappa^2 k^4 \frac{\int d^4\ell}{16\pi^4} \frac{1}{\ell^2 - \lambda^2 + i\epsilon} \frac{1}{(\ell^2 + 2\ell k + \Delta + i\epsilon)^2} \tag{20}$$

for $\Delta = k^2 - m^2 \Rightarrow$ **for a scalar field**

$$i\Delta'_F(k)|_{YFS-resummed} = \frac{ie^{B''_g(k)}}{(k^2 - m^2 - \Sigma'_s + i\epsilon)} \tag{21}$$

$\Rightarrow$

Expand theory with the 'improved Born' propagators

$$iP_{\alpha_1\cdots;\alpha_1'\cdots}\Delta_F'(k)|_{YFS-resummed,\Sigma_s'=0} = \frac{iP_{\alpha_1\cdots;\alpha_1'\cdots}e^{B_g''(k)}}{(k^2-m^2+i\epsilon)} \quad (22)$$

where in the DEEP UV we get

$$B_g''(k) = \frac{\kappa^2|k^2|}{8\pi^2}\ln\left(\frac{m^2}{m^2+|k^2|}\right), \quad (23)$$

$\Rightarrow$ **ALL PROPAGATORS FALL FASTER THAN ANY POWER OF** $|k^2| \Rightarrow$ **QG IS FINITE (SEE hep-ph/0204102)!**

YFS Exponentiated QCD Corrections to $t\bar{t}$ Production

at High Energies

We shall realize the result above as it is applied to the process in Fig. 1 at high energies by the Monte Carlo event generator methods of two of us (S.J. and B.F.L.W.). We follow but extend earlier work in DeLaney *et al.* (op. cit.). Similar results will hold for pp incoming states at the LHC and ,for $q\bar{q}$ production, at RHIC. Sample MC data will be illustrated. We refer to the respective MC event generator as ttp1.0. It is in the EEX format but a CEEX version is imminent. It will be available from the authors soon.

Improving and extending PRD52(1995)108 to $q + \bar{q} \to t\bar{t} + n(G)$, we identify

the hard gluon residuals $\tilde{\bar{\beta}}_{0,1}$ via

$$\frac{1}{2}\tilde{\bar{\beta}}_0 = \frac{d\sigma^{1-loop}_{q\bar{q}}}{d\Omega_t} - 2\alpha_s \Re B_{QCD}\frac{d\sigma^{Born}_{q\bar{q}}}{d\Omega_t},$$
$$\frac{1}{2}\tilde{\bar{\beta}}_1 = \frac{d\sigma^{B1}_{q\bar{q}}}{d\Omega_t k dk d\Omega_G} - \tilde{S}_{QCD}(k)\frac{d\sigma^{Born}_{q\bar{q}}}{d\Omega_t}, \tag{24}$$

where the 1-loop cross section for $q\bar{q}$ to annihilate to t$\bar{\text{t}}$, $d\sigma^{1-loop}_{q\bar{q}}$, is taken from the literature as noted and where the single bremsstrahlung cross section for $q\bar{q} \to t\bar{t} + G$, $d\sigma^{B1}_{q\bar{q}}$, is also taken from that literature.

The respective YFS functions $\tilde{S}_{QCD}, \tilde{B}_{QCD}, \Re B_{QCD}$ are then used in the Monte Carlo algorithm presented in S. Jadach *et al.*(YFS2, BHLUMI, etc.) to realize the result derived in the previous section for the case of the $q\bar{q} \to t\bar{t} + n(G)$ subprocess. For the $G + G \to t\bar{t} + n(G)$ subprocess, we proceed in complete analogy with the $q\bar{q}$ annihilation subprocess, with the appropriate substitution of cross sections and color factors.

In order to apply these parton level results to the desired hadron level cross

section $\sigma(p\bar{p} \to t\bar{t} + X)$, we use the standard formula

$$\sigma(p\bar{p} \to t\bar{t} + X) = \int \sum_{i,j} F_i(x_i)\bar{F}_j(x_j) d\hat{\sigma}'_{\mathrm{exp},ij} dx_i dx_j \qquad (25)$$

where $F_i(\bar{F}_j)$ is the structure function of parton $i(j)$ in p(p̄) and where $\hat{\sigma}'_{\mathrm{exp},ij}$ is the result derived above for the tt̄ production subprocess with the incoming parton-i,parton-j initial state when the DGLAP synthesization procedure presented in MPLA14(1999)491 by BW and SJ is applied to it to avoid over-counting resummation effects already included in the structure function DGLAP evolution.

We have realized (25) by MC methods by extending the MC realizations of the subprocesses to include the two-dimensional structure function distribution in a standard way (PLB292(1992)413).

$\Rightarrow$ MC event generator ttp1.0, with YFS-style exponentiated soft $n(G)$ effects in pp̄$\to$tt̄$+X$.

Today's illustrations: at FNAL energies, $\bar{\beta}_0$ level; $\bar{\beta}_1$ level in progress.

Results

SEMI-ANALYTICAL NORMALIZATION n(G) EFFECT

MPLA12(1997)2425

$$r^{nls}_{exp} = \exp\left\{\frac{\alpha_s}{\pi}[(2C_F - \frac{1}{2}C_A)\frac{\pi^2}{3} - \frac{1}{2}C_F]\right\}$$
$$= \begin{cases} 1.086, & \alpha_s = \alpha_s(\sqrt{s}), \\ 1.103, & \alpha_s = \alpha_s(2m_t), \\ 1.110, & \alpha_s = \alpha_s(m_t), \end{cases} \quad (26)$$

$\Rightarrow$

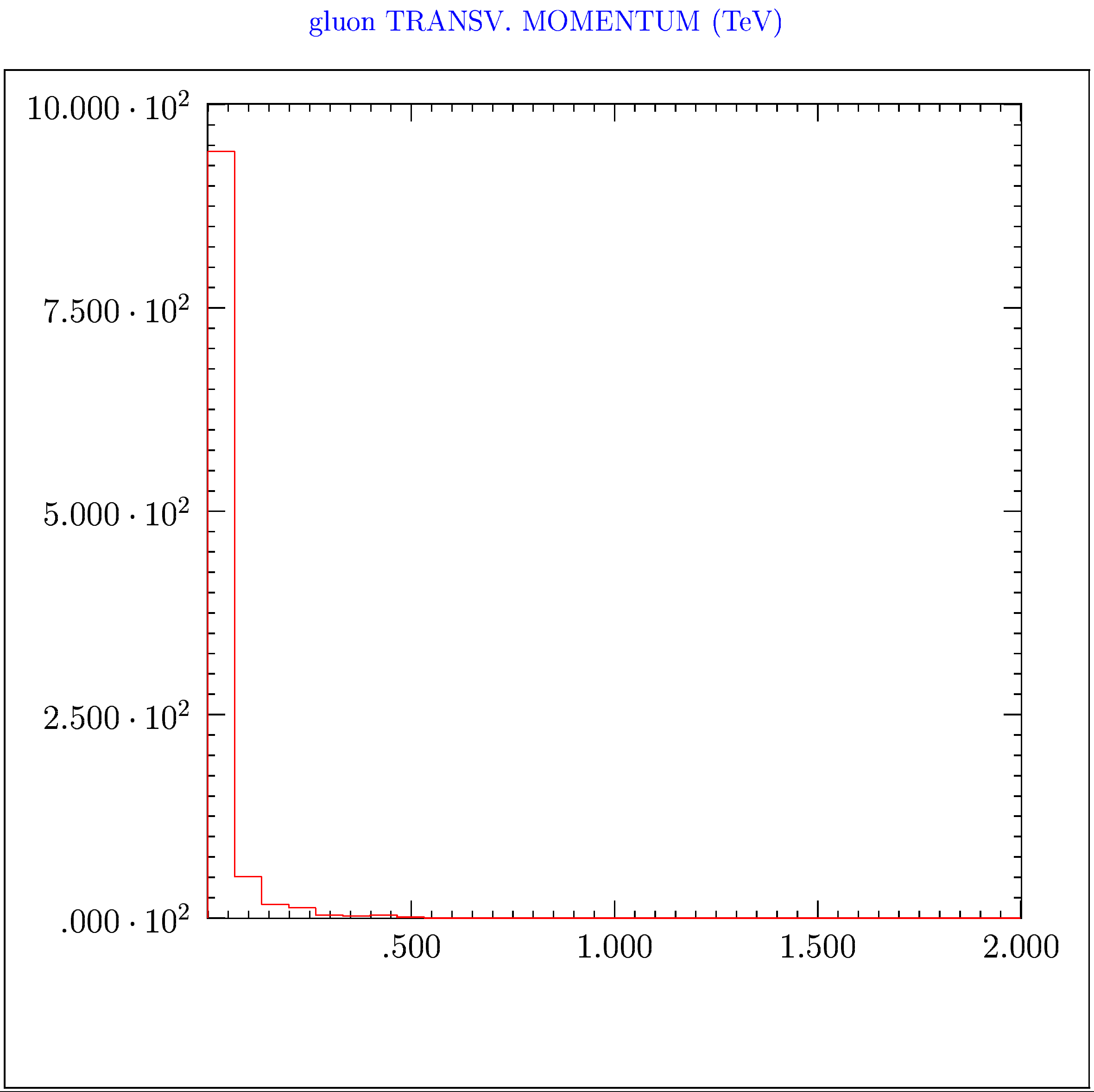
$$\delta\sigma(p\bar{p} \to t\bar{t})^{exp} = \int \sum_{i,j} D_i(x_i)\bar{D}_j(x_j)(r^{nls}_{exp} - 1 - \frac{\alpha_s}{\pi}[(2C_{ij} - \frac{1}{2}C_A)\frac{\pi^2}{3} - \frac{1}{2}C_{ij}])d\hat{\sigma}_B(x_i x_j s)dx_i dx_j, \quad (27)$$

where

$$C_{ij} = \begin{cases} C_F, & ij = q\bar{q},\ \bar{q}q, \\ C_A, & ij = GG. \end{cases} \tag{28}$$

$\Rightarrow (\mathcal{O}(\alpha_s^n), n \geq 2)$ contributes .006-.008 of the NLO cross section, in agreement with Catani *et al.*,**PLB378(1996)329.**

MC Data: Preliminary



Conclusions

YFS THEORY (EEX AND CEEX) EXTENDS TO NON-ABELIAN GAUGE THEORY FOR QCD

- FULL MC EVENT GENERATOR REALIZATION NEAR.
- SEMI-ANALYTICAL RESULTS AGREE WITH LITERATURE ON $t\bar{t}$ PRODUCTION
- PRELIMINARY MC DATA SHOW n(G) P_T SIGNIFICANT.
- A FIRM BASIS FOR THE COMPLETE $\mathcal{O}(\alpha_s^2)$ RESULTS NEEDED FOR THE FNAL/LHC/RHIC/TESLA/LC PHYSICS HAS BEEN DEMONSTRATED AND ALL THE LATTER IS IN PROGRESS.

FOR QG, APPLICATION OF OUR YFS-QCD METHODS $\Rightarrow$ FINITENESS WITH A PLANCK SCALE CUT-OFF.