Measurement of the QCD β-Function using RGI

Klaus Hamacher

Bergische Universität Wuppertal & DELPHI Collaboration

Outline

- Data and Corrections
- Power Corrections and
  Renormalisation Group Invariant Perturbation Theory
- Comparing to Data (determine α_s and power terms)
- Measurement of the β-Function
- Summary

Data and Corrections

Talk is based on: DELPHI Coll., R. Reinhardt et al.,
A study of the energy evolution of event shape distributions and their means with the DELPHI detector at LEP

- Data
  - DELPHI data from LEP1 and LEP2 (E_{CM} =89 to 202 GeV)
  - DELPHI radiative Z events at E_{CM} =45, 66 and 76 GeV
  - low energy data from various experiments (PETRA, PEP, TRISTAN)

- Observables
  - mean values of Thrust, C-Param., Major, jet–broadenings; ($\int$EEC, $\int$JCEF)
  - means of jet–masses ($M^2_{h/s}/E^2_{vis}$) in alternative definitions ("E-scheme")

- Corrections
  - usual detector corrections
  - **apply correction for b-mass effects (Monte Carlo)**

Theoretical Predictions - Power Corrections

$\mathcal{O}(\alpha_s^2)$ perturbative expansion for shape means reads:

$$\langle f_{pert} \rangle = A \cdot \frac{\alpha_s(\mu)}{2\pi} + \left(A \cdot 2\pi b \ln \frac{\mu^2}{E_{cm}^2} + B \right) \left(\frac{\alpha_s(\mu)}{2\pi} \right)^2$$

A,B perturbative parameters given in $\overline{MS}$-scheme.
Non-perturbative hadronisation $\longrightarrow$ mean values modified by a term $\propto 1/E_{\mathrm{cm}}$:

$$\langle f \rangle = \langle f_{\mathrm{pert}} \rangle + c_f \mathcal{P}$$

Dokshitzer and Webber et al.

$$\mathcal{P} = \frac{4C_F}{\pi^2} \mathcal{M} \frac{\mu_I}{E_{\mathrm{cm}}} \left[\alpha_0(\mu_I) - \alpha_s(\mu) - \left(b \cdot \log \frac{\mu^2}{\mu_I^2} + \frac{K}{2\pi} + 2b \right) \alpha_s^2(\mu) \right]$$

$\alpha_0(\mu_I)$ is a universal non-perturbative parameter ($\alpha_0 = \langle \alpha_s(k) \rangle |_{k<\mu_I}$).
c_f f dependent perturbative parameter known for resummable f's.
Assess all observables $\langle f \rangle$ using simple power corrections:

$$\langle f \rangle = \langle f_{\mathrm{pert}} \rangle + \frac{C_1^{(f)}}{E_{\mathrm{cm}}}$$

Renormalisation Group Invariant (RGI) Perturbation Theory

Basic idea: use observable $R = \langle f \rangle / A$ as expansion parameter;
require R to fulfil RGE:

$$Q\frac{dR}{dQ} = -\frac{\beta_0}{2}R^2(1 + \rho_1 R + \rho_2 R^2 + \ldots) = \frac{\beta_0}{2}\rho(R) \quad .$$

β_0, $\rho_1 = \beta_1/2\beta_0$ are universal, ρ_i scheme invariant ($\longrightarrow$ name of the method).

$$\frac{\beta_0}{2}\ln\frac{Q}{\Lambda_R} = \frac{1}{R} - \rho_1 \ln\left(1 + \frac{1}{\rho_1 R}\right) + \underbrace{\int_0^R dx \left(\frac{1}{\rho(x)} + \frac{1}{x^2(1+\rho_1 x)}\right)}_{\text{vanishes in second order}}$$

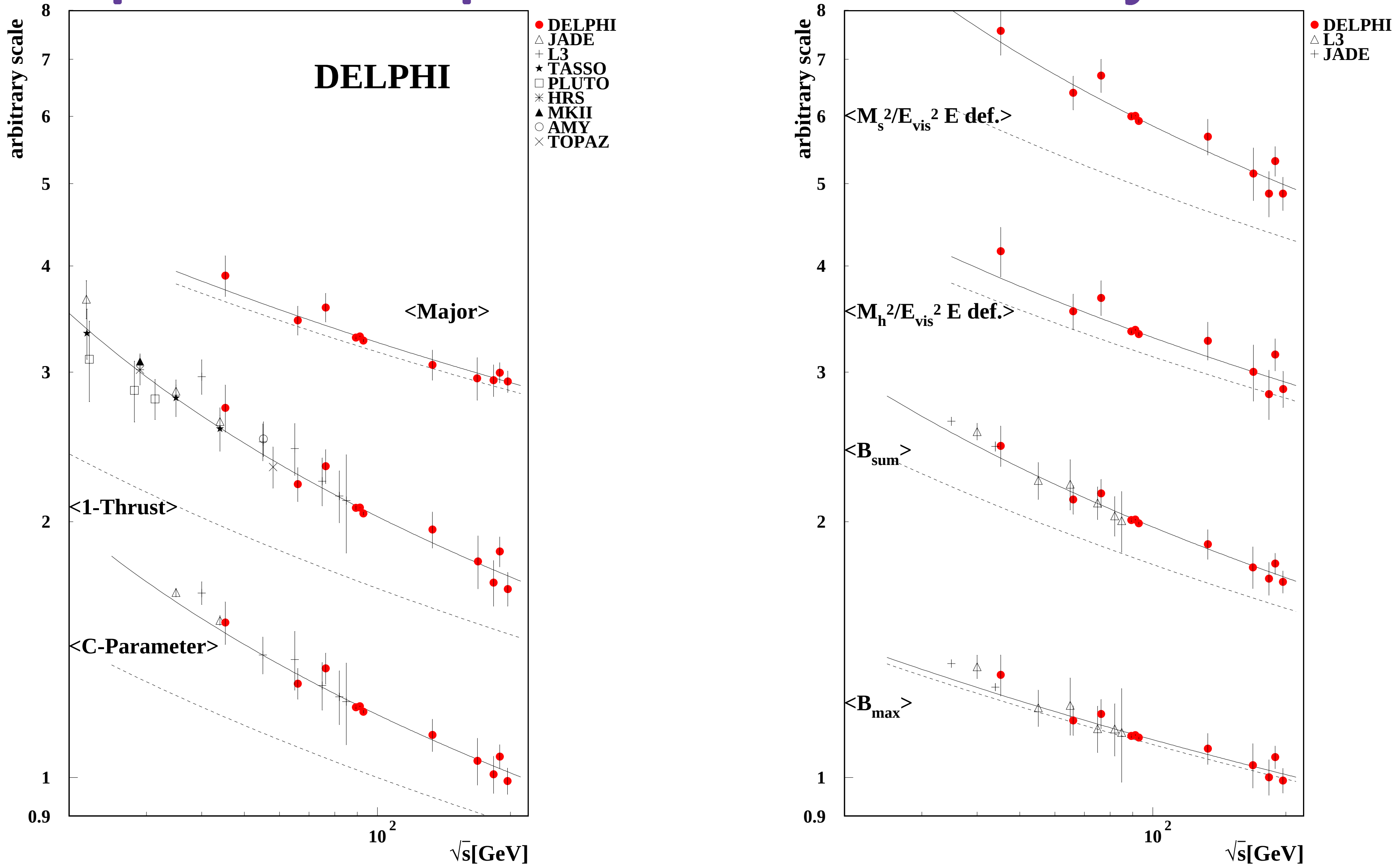
Simple RGI applies to observables depending on a single energy scale.
RGI resums UV log. terms. Numerically RGI=ECH.

Exact conversion to $\overline{MS}$ reference–scheme. Power corrections can be included.

$$\frac{\Lambda_R}{\Lambda_{\overline{MS}}} = e^{\frac{B}{A\beta_0}}\left(\frac{2\beta_1}{\beta_0^2}\right)^{-\frac{\beta_1}{\beta_0^2}}$$

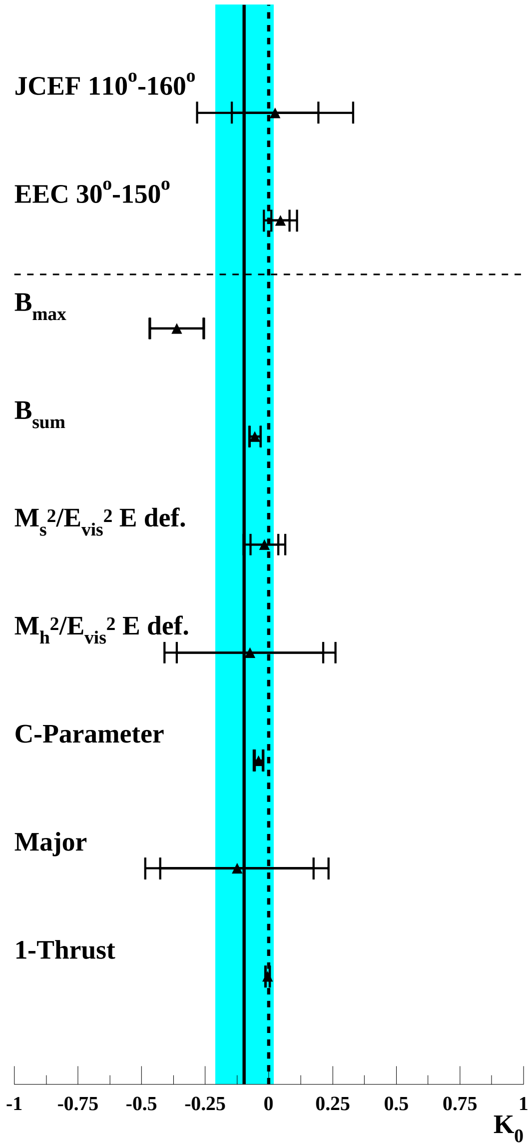
$$\rho(R) \to \rho(R) - \frac{2K_0}{\beta_0}R^{-\frac{\beta_1}{\beta_0^2}}e^{-\frac{2}{\beta_0 R}}$$

Description of Shape Observable Means by RGI



RGI (full line) – $\overline{MS}$ with same $\alpha_s(M_Z)$ (dotted) $=$ $\overline{MS}$ power correction.

No Significant Power Corrections Needed with RGI



Fitting RGI with power–corrections to a large set of observables:

$\longrightarrow$ Observe power terms $K_0 \sim 0$!

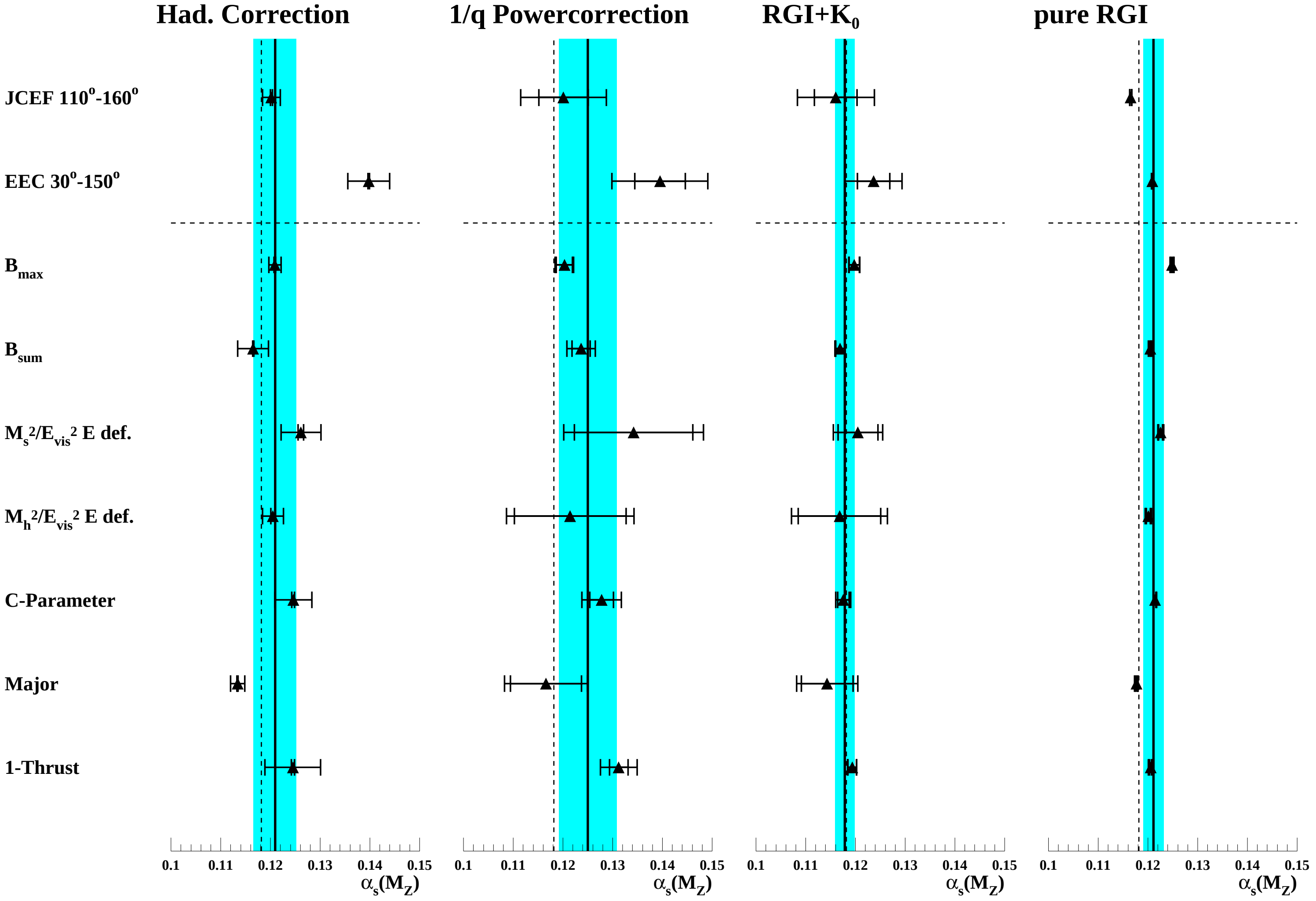
This should be viewed as a virtue of both: RGI and inclusiveness of mean values.

$\longrightarrow$ Power terms in $\overline{MS}$-analysis are due to missing higher order corrections.

Presence of genuine power suppressed terms for means unclear so far!

Possible contribution:
only $\sim 3\%$ (relative) at Z energies.

Compare α_s from Means Obtained with Various Methods



RGI – unique α_s scatter $\langle\alpha_s(M_Z)\rangle = 0.121 \pm 0.002$

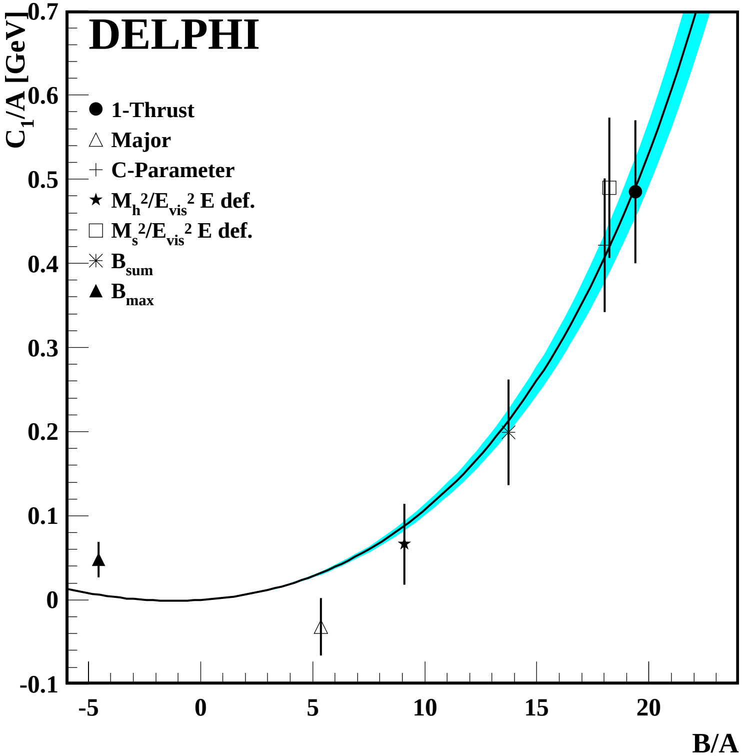
$\longrightarrow$ Data indicate better "convergence" of perturbative series for RGI.

“Predict” $\overline{MS}$ Power Terms using RGI

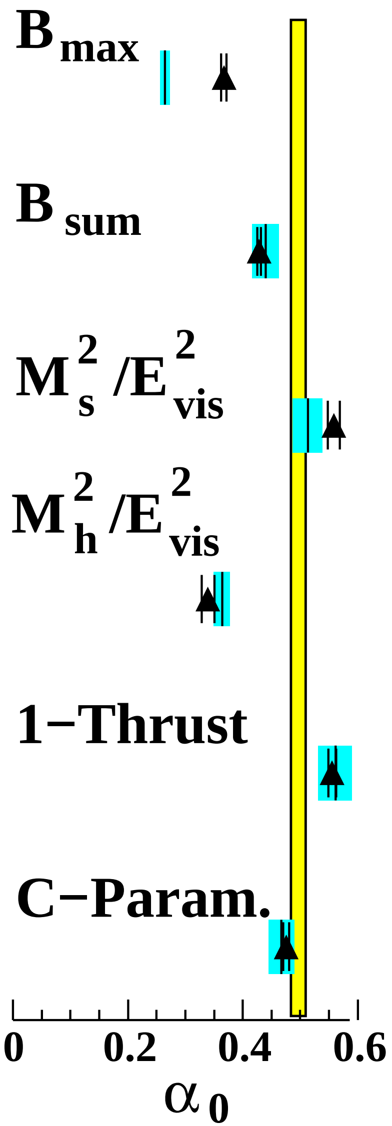
Set RGI = Power Model; solve for α_0 or C_1.

$$\langle R\rangle_{RGI} \cdot A = \langle f\rangle_{pert} + \langle f\rangle_{pow}$$

Plot: size of power corr. $\leftrightarrow$ size of 2^{nd} order term



$\Lambda_{\overline{MS}} = 220 MeV$



α_0 agrees better than any presumed universal value ~ 0.5. $\longrightarrow$

Measuring the β-Function

RGI imposes the RGE but does not fix the β-function!
QCD β-function can be directly measured from E evolution of event shape means.

Independent of α_s-measurements! Free of renormalisation scheme ambiguities!

$$\frac{\mathrm{d}R^{-1}}{\mathrm{d}\ln Q} = \frac{\beta(R)}{R^2} = \frac{\beta_0}{2} \cdot \left(1 + \frac{\beta_1}{2\beta_0} \cdot R + \dots\right)$$

$\beta(R) \propto$ slope of straight line fit to $\frac{1}{R}$ vs. $\ln E$.

Expectation:

$$\beta_0 = \frac{11C_A - 2n_f}{3} \quad \beta_1 = \frac{17C_A^2 - 5C_A n_f - 3C_F n_f}{3}$$

	β_0	β_1	
QCD:	$= 7.67$	$= 19.67$	$n_f = 5$
+ light gluinos:	$=\sim 5.7$		$n_f = 8$

β-Function from Energy Evolution of Event Shape Means

From DELPHI measurements of $\langle 1 - T \rangle$
(apply small correction for β_1-term):

$$\beta_0 = 7.7 \pm 1.1 \pm 0.2 \pm 0.1_{b-mass}$$

Uncertainty due to power terms small!
Other observables: consistent.

Including low energy data:

$$\beta_0 = 7.86 \pm 0.32$$

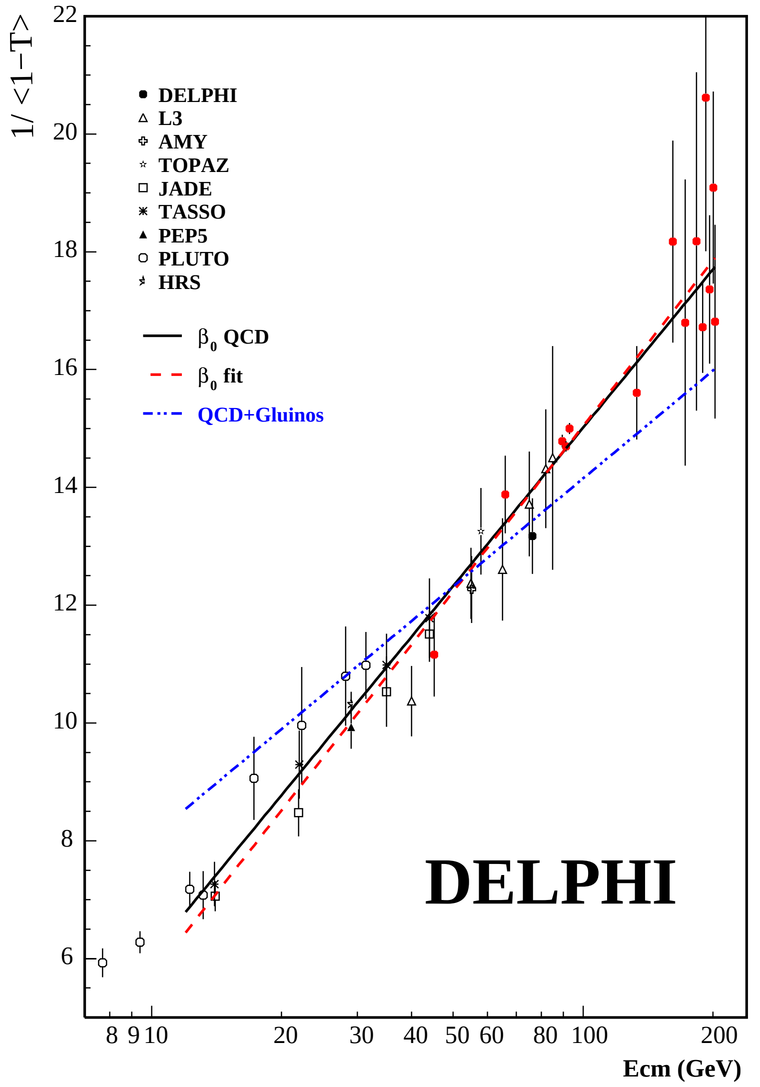
$$n_f = 4.75 \pm 0.44 \text{ (using QCD expression)}$$

Compare indirect measurement (via α_s):
LEP event shapes:

$$\beta_0 = 7.67 \pm 1.63$$

R_τ, F_2, F_3, R_Z, event shapes . . . world data:

$$\beta_0 = 7.76 \pm 0.44$$



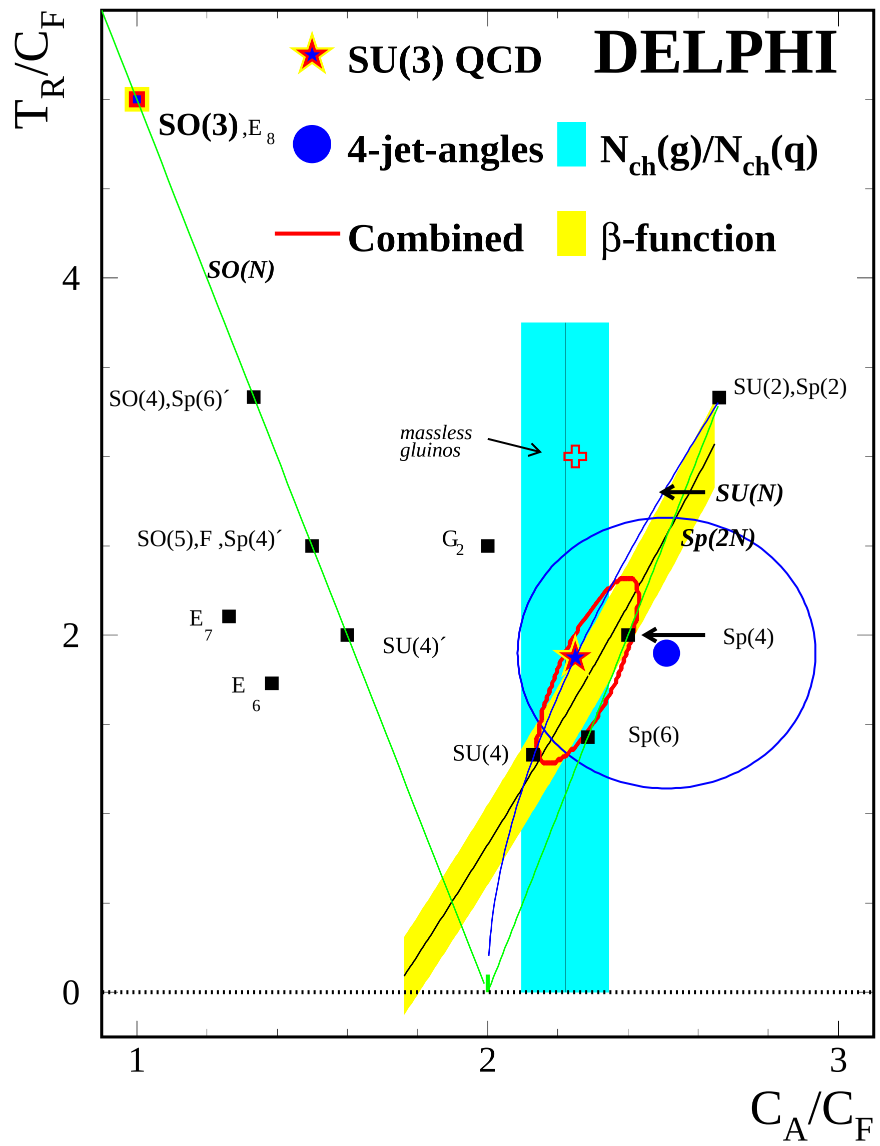
Consequences of β-Function Measurement

Measurement strongly restricts gauge group of the strong interaction.

Compare to DELPHI colour factor measurements from:

- 4–jet angular distributions.
- Multiplicity of gluon and quark jets

And implies
model independent limits
on new physics e.g. light gluinos



Summary

- Data on event shape means are successfully described by RGI theory.
  Obtain consistent result for α_s; scatter $\Delta\alpha_s/\alpha_s \sim \mathcal{O}(\alpha_s^2)$
  No artificial "theory" error required.

- We need to understand success of RGI – test more observables!

- Power terms appearing in models ($\overline{MS}$) can be "predicted" by RGI theory !
  $\longrightarrow$ power terms predominantly due to missing higher order contributions.

- Experimentally more (low energy) data (CLEO ...) is required to clarify role of "genuine" power terms.

- Obtain basic and clean measurement of the β–function of strong interaction.

  $\beta_0 = 7.86 \pm 0.32$ $\qquad$ $n_f = 4.75 \pm 0.44$

  Consistent with QCD expectation.

Mass Effects for Event Shapes

Hadron masses influence shape observables.
Two types of observables:

- e.g. Thrust: depending on particle momenta; mass–dependence via energy conservation.
- e.g. Jet Masses: direct mass–dependence; avoid by choosing E– or p–scheme:

$$p = (\vec{p}, E) \longrightarrow (\vec{p}, |\vec{p}|) \qquad (p\text{-scheme})$$

$$p = (\vec{p}, E) \longrightarrow (\hat{p}E, E) \qquad (E\text{-scheme})$$

Furthermore:

- Transverse momentum from B–decays.

Estimate effects using Monte Carlo models;
b-correction: calculate b+udsc/udsc;

Change to E-scheme, b-corr. for shape means $\Longrightarrow$