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Multiplicities, Fluctuations and Correlations

Edward K. Sarkisyan-Grinbaum
CERN and Univ. Antwerpen

OUTLINE

- Introduction
- Multiplicity flavour dependence
  see also talk of M. Siebel
- Scaling of local fluctuations. Factorial moments
- Cumulants. Multiparticle correlations
  see also talk of Š. Todorova-Nová
- Summary and conclusions

Multiplicity in HEP

- Multiplicity, n, – a first measured variable in high-energy particle collisions
- Characterises event: trigger for specific processes, multiplicity distribution, from event to event multiplicity fluctuations
- Important model parameter: energy, flavour, particle species dependence
- Input for other variables' spectra (dn/dy, dn/dp_T etc.)
- Single-particle density $\rho_1(p_1)$
  - $\int \rho_1(p_1)\, dp_1 = \langle n \rangle$ – mean multiplicity
- Two-particle density $\rho_2(p_1, p_2)$
  - $\int \rho_2(p_1, p_2)\, dp_1\, dp_2 = \langle n\,(n-1) \rangle \equiv \langle n^{[2]} \rangle = f_2$ second order factorial moment
- 2-particle, 2nd order, correlation funct. $C_2(p_1, p_2)$
  - $C_2(p_1, p_2) = \rho_2(p_1, p_2) - \rho_1(p_1)\, \rho_1(p_2)$
- Multi-particle densities, factorial moments, correlation functions, cumulants – by analogy

I.M. Dremin, J.W. Gary, Phys. Reports 349 (2001) 301

Hadronisation of heavy & liqht quarks. I

DELPHI Collaboration, Abs. 232

OPAL Collaboration, Abs. 373

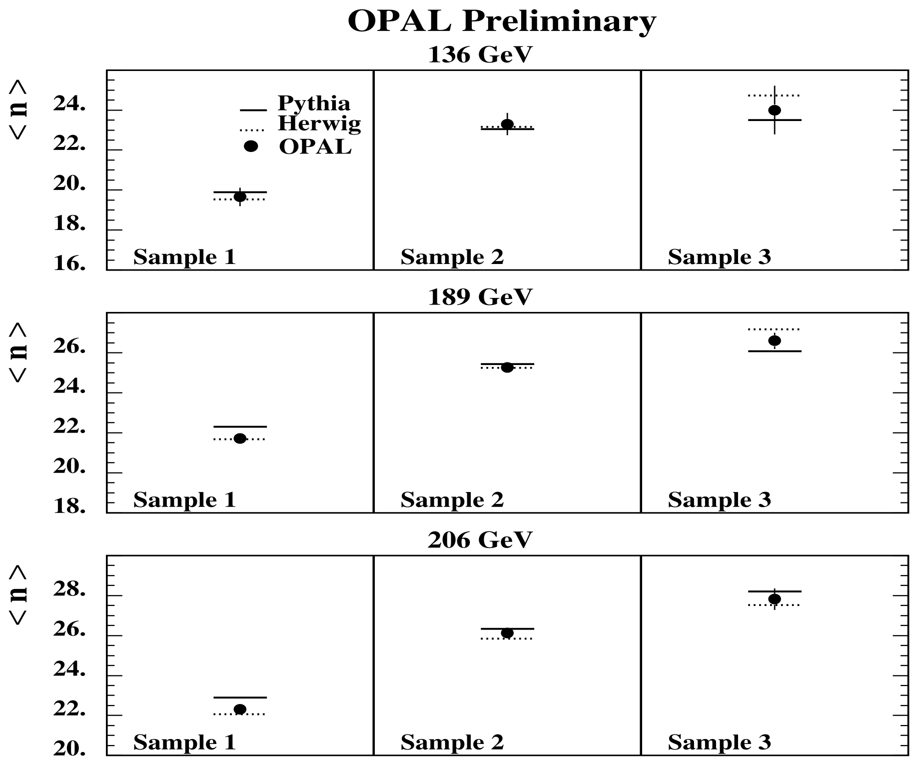
- **A basic test of QCD** is to search for

$$\delta_{hl} = \langle n_{h\bar{h}} \rangle - \langle n_{l\bar{l}} \rangle$$

  $l = \{u,\, d,\, s,\, (c)\}$, $h = \{b,\, (c)\}$
- The QCD coherence predicts $\delta_{hl} = \mathrm{const}(E_{\mathrm{cm}})$
- The flavour-independent (naive) hadronisation model: δ_{hl} decreases with E_{cm}
- **Experimental** studies with $h = b$, l=$\{u, d, s\}$ in $\mathrm{e}^+\mathrm{e}^- \to \mathrm{Z}^0/\gamma^* \to \mathrm{q\bar{q}}$ events
  - radiative $\mathrm{q\bar{q}}\gamma$ events reduction
  - multivariate b-tagging
  - $\langle n_{q\bar{q}} \rangle$ from b-, uds- and c-events (S=1,2,3),

$$\langle n^{(S)} \rangle = f_b^{(S)}\, C_b^{(S)}\, \langle n_{b\bar{b}} \rangle + f_l^{(S)}\, C_l^{(S)}\, \langle n_{l\bar{l}} \rangle + f_c^{(S)}\, C_c^{(S)}\, \langle n_{c\bar{c}} \rangle$$

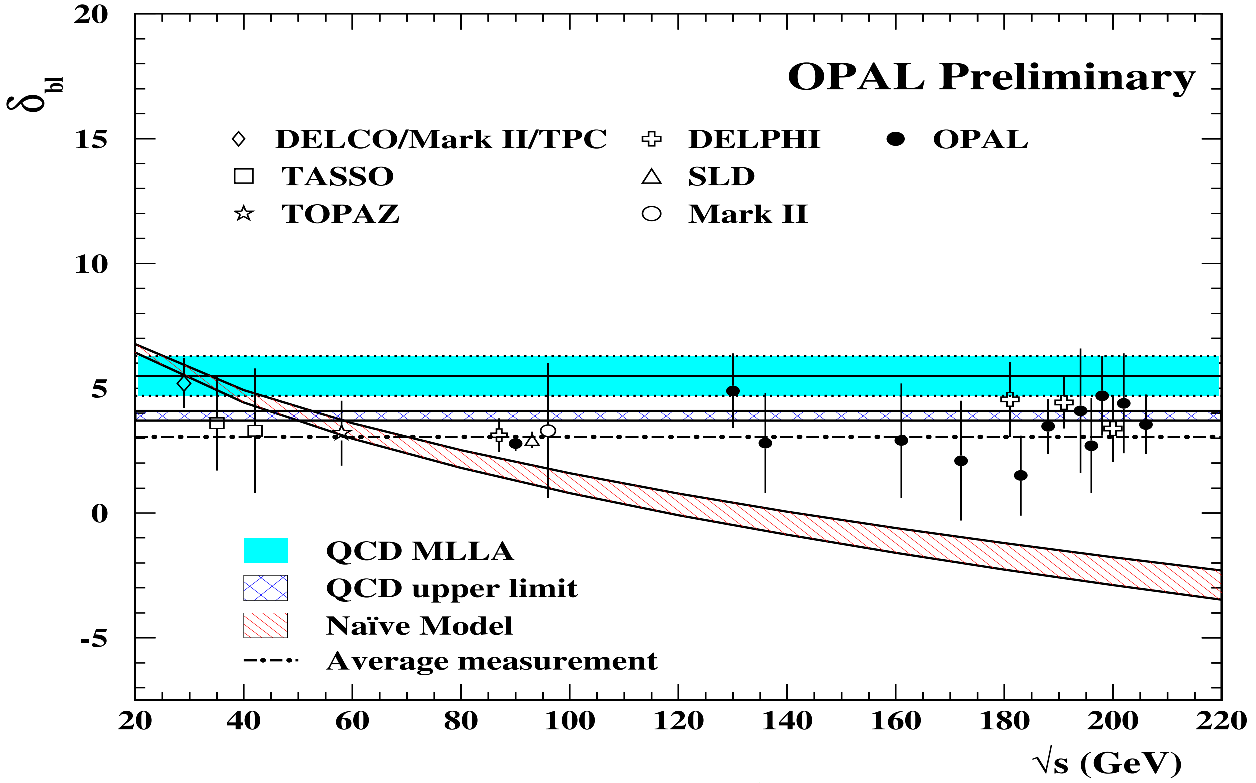
$f_q^{(S)}$ flavour fractions, $C_q^{(S)}$ correction factors from MC



OPAL Physics Note PN511 (2002)

Hadronisation of heavy & light quarks. II

OPAL Collaboration, Abs. 320, 373



OPAL PN511 (2002)

$e^+e^- \to Z^0/\gamma^* \to$hadrons at $\sqrt{s} = 130 - 206$ GeV

- **OPAL (preliminary)**

  $\delta_{bl} = 3.44 \pm 0.40(\mathrm{stat}) \pm 0.79(\mathrm{syst})$

  $\langle n_{b\bar{b}} \rangle \simeq 26 - 31,\ \langle n_{l\bar{l}} \rangle \simeq 21 - 28$

  - favoured by the QCD coherence calculations
  - inconsistent with flavour-independent model

- agrees with QCD flavour-independence of $\langle n_l \rangle$ in $e^+e^- \to Z^0 \to$hadrons: $\langle n_i \rangle : \langle n_j \rangle \sim 1$

  $\langle n_u \rangle = 17.77 \pm 0.51^{+0.86}_{-1.20}$

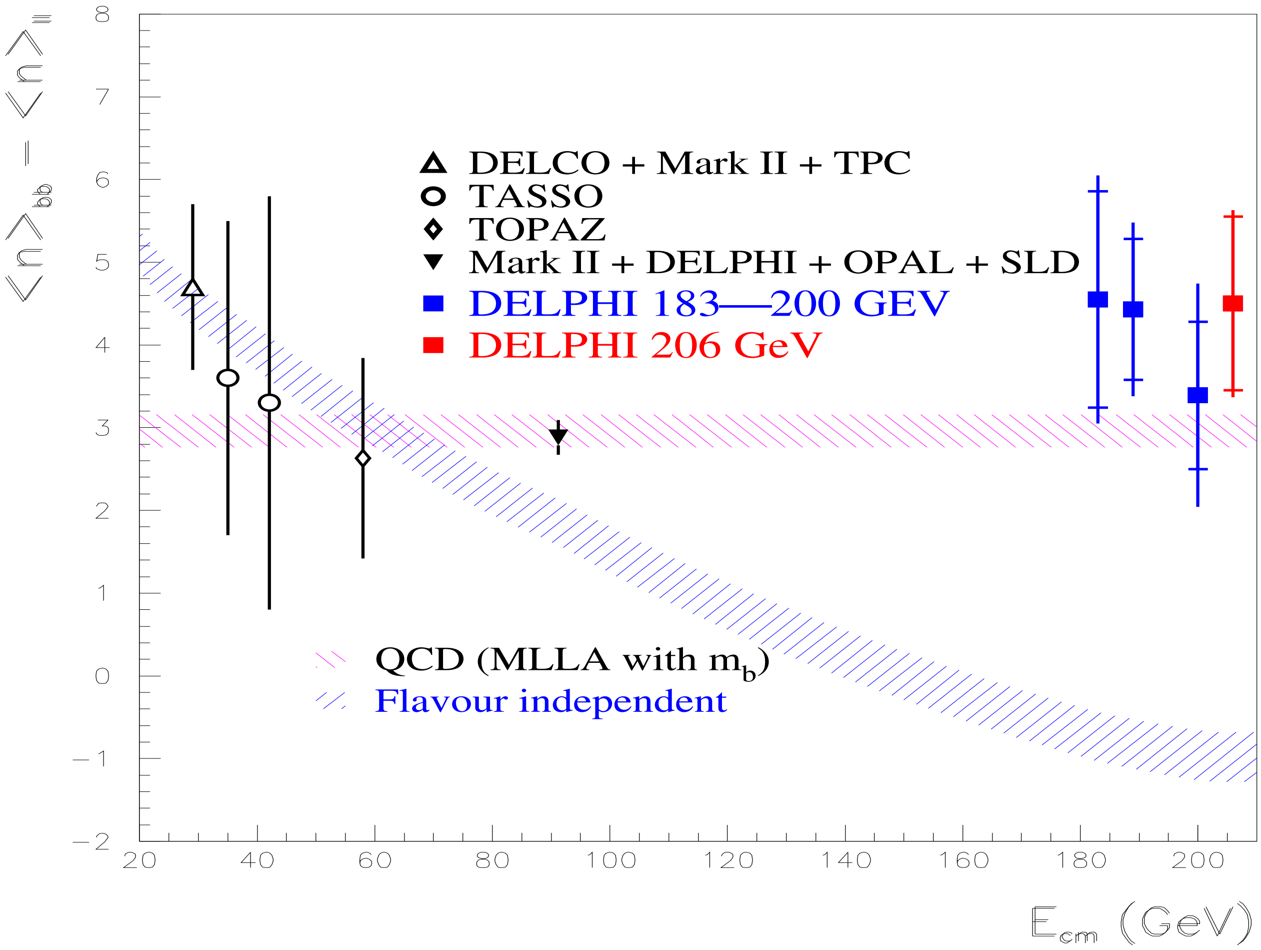
  $\langle n_d \rangle = 21.44 \pm 0.63^{+1.46}_{-1.17}$ $\quad \mathrm{corr}_{ud} = -0.90$

  $\langle n_s \rangle = 20.02 \pm 0.13^{+0.39}_{-0.37}$ $\quad \mathrm{corr}_{ds} = -0.42$

  OPAL, Eur. Phys. J. C19 (2002) 257

Hadronisation of heavy & light quarks. III

DELPHI Collaboration, Abs. 232



DELPHI 2002–052 CONF 586 (2002)

$e^+e^- \to Z^0/\gamma^* \to$hadrons at $\sqrt{s} = 206$ GeV

- **DELPHI (preliminary)**

$$\delta_{bl} = 4.50 \pm 1.05(\mathrm{stat}) \pm 0.52(\mathrm{syst})$$

$$\langle n_{b\bar{b}} \rangle = 28.72 \pm 0.70 \pm 0.32$$

$$\langle n_{l\bar{l}} \rangle = 24.22 \pm 1.09 \pm 0.61$$

- favoured by the QCD coherence calculations
- inconsistent with flavour-independent model

Fluctuations and factorial moments

- The **next step** to study observables beyond the global ones like one-particle spectra and averages
- To **overcome** difficulties of conventional method even in 1D for >3 particles
- Divide a phase space into **M cells of equal size**
- Calculate **cell-averaged normalized** factorial moments of order q:

$$\mathrm{F_q} = \frac{1}{\mathrm{M}} \sum_{\mathrm{m=1}}^{\mathrm{M}} \int_{\delta \mathrm{y}} \prod_{\mathrm{i}} \mathrm{dy_i} \frac{\rho_{\mathrm{q}}(\mathrm{y_1}, ..., \mathrm{y_2})}{[\int_{\delta \mathrm{y}} \mathrm{dy} \rho_1(\mathrm{y})]^{\mathrm{q}}}$$

- Factorial moments **provide dynamical** i.e. non-statistical (non-Poissonian) fluctuations
- Factorial moments **calculated** according to

$$\mathrm{F_q} = \frac{1}{\mathrm{M}} \sum_{\mathrm{m=1}}^{\mathrm{M}} \frac{\langle \mathrm{n_m} \, (\mathrm{n_m} - 1) \cdots (\mathrm{n_m} - \mathrm{q} + 1) \rangle}{\langle \mathrm{n_m} \rangle^{\mathrm{q}}}$$

- Self–similar **dynamical fluctuations** or intermittency scaling,

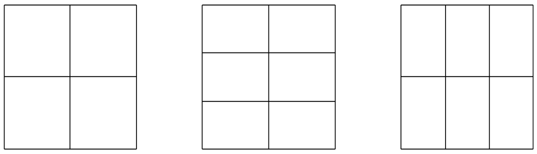
$$F_q \propto M^{\phi_q} \quad (M \to \infty)$$

$$0 < \phi_q \leqslant q - 1 \text{ is } \textbf{intermittency index} \ (q \geqslant 2)$$

- **Observed in all types of collisions**

E.A. De Wolf, I.M. Dremin, W. Kittel, Phys. Reports 270 (1996) 1

Self-affinity vs. self-similarity. I



(a) (b) (c)

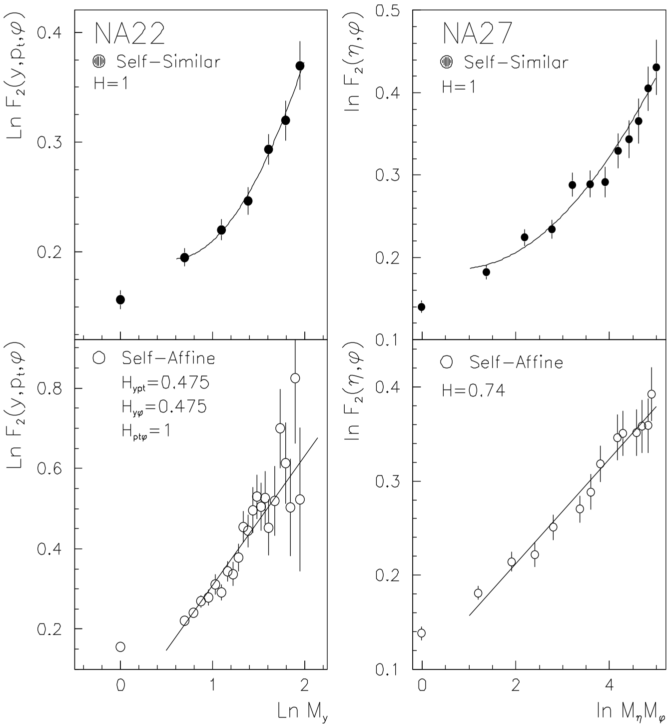
2D phase-space partition

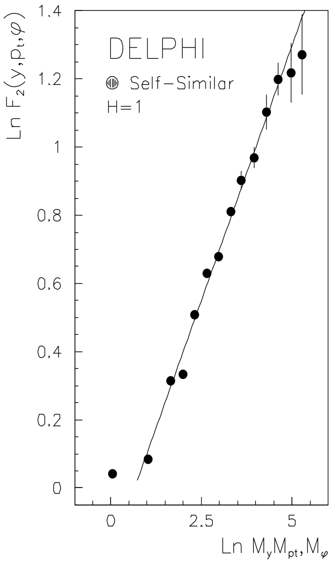
- **The Hurst exponent**
  $$H_{xy} = \ln M_x / \ln M_y$$
- (a) self–similarity
  $$M_x = M_y \;\Rightarrow\; H = 1$$
- (b), (c) self–affinity
  $$M_x \neq M_y$$

- **Experimental indications:** factorial moments in

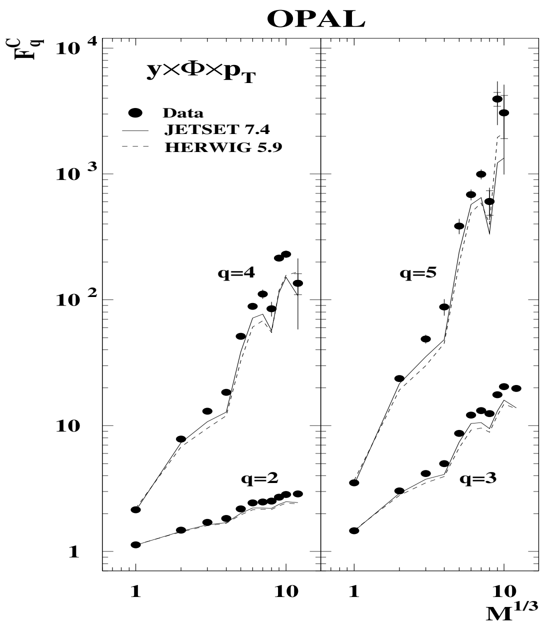
hadronic collisions — $e^+e^- \rightarrow Z^0 \rightarrow$ hadrons

self–*affinity* $\rightarrow$ anisotropy — self-*similarity* $\rightarrow$ isotropic fluctuations





3D



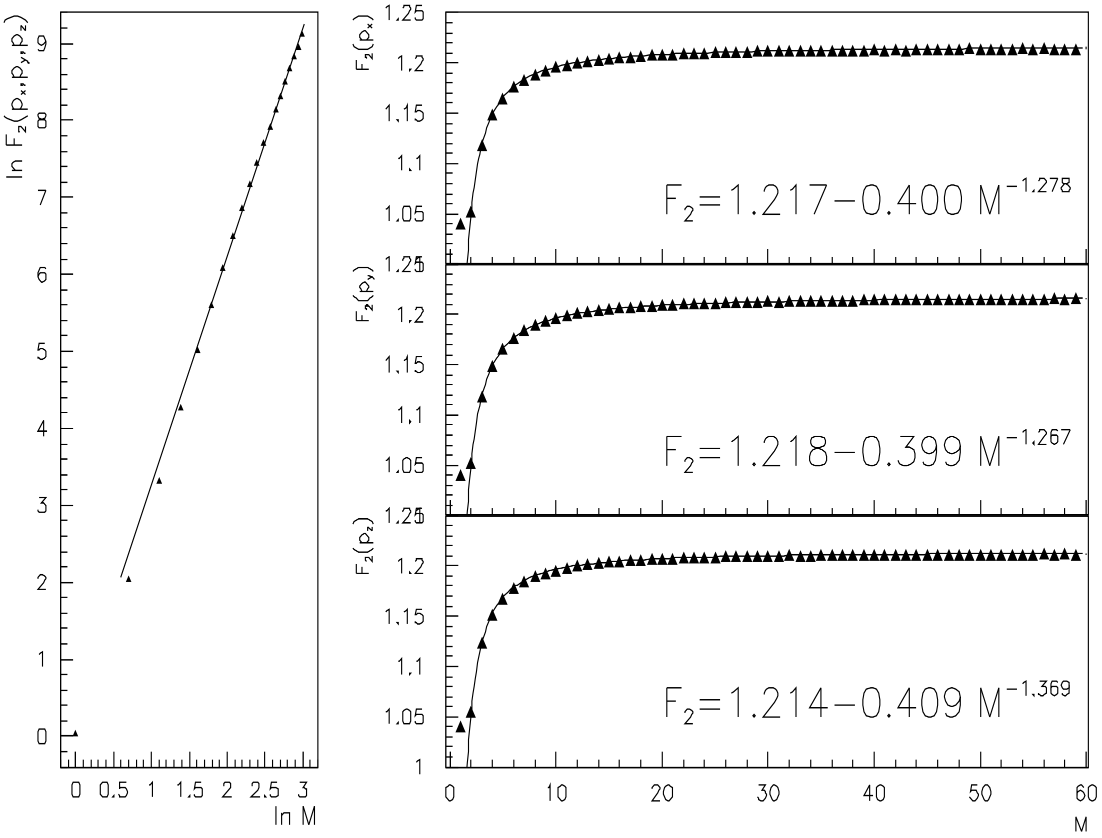
F_2: 3D **2D**

$y - p_T - \Phi$ phase space
(w.r.t. event axis)

Liu Lianshou et al.

Self-affinity vs. self-similarity. II
A quantitative estimate

- To measure **1D 2nd-order** factorial moments



3D **1D**

- **To fit** 1D 2nd-order factorial moments with

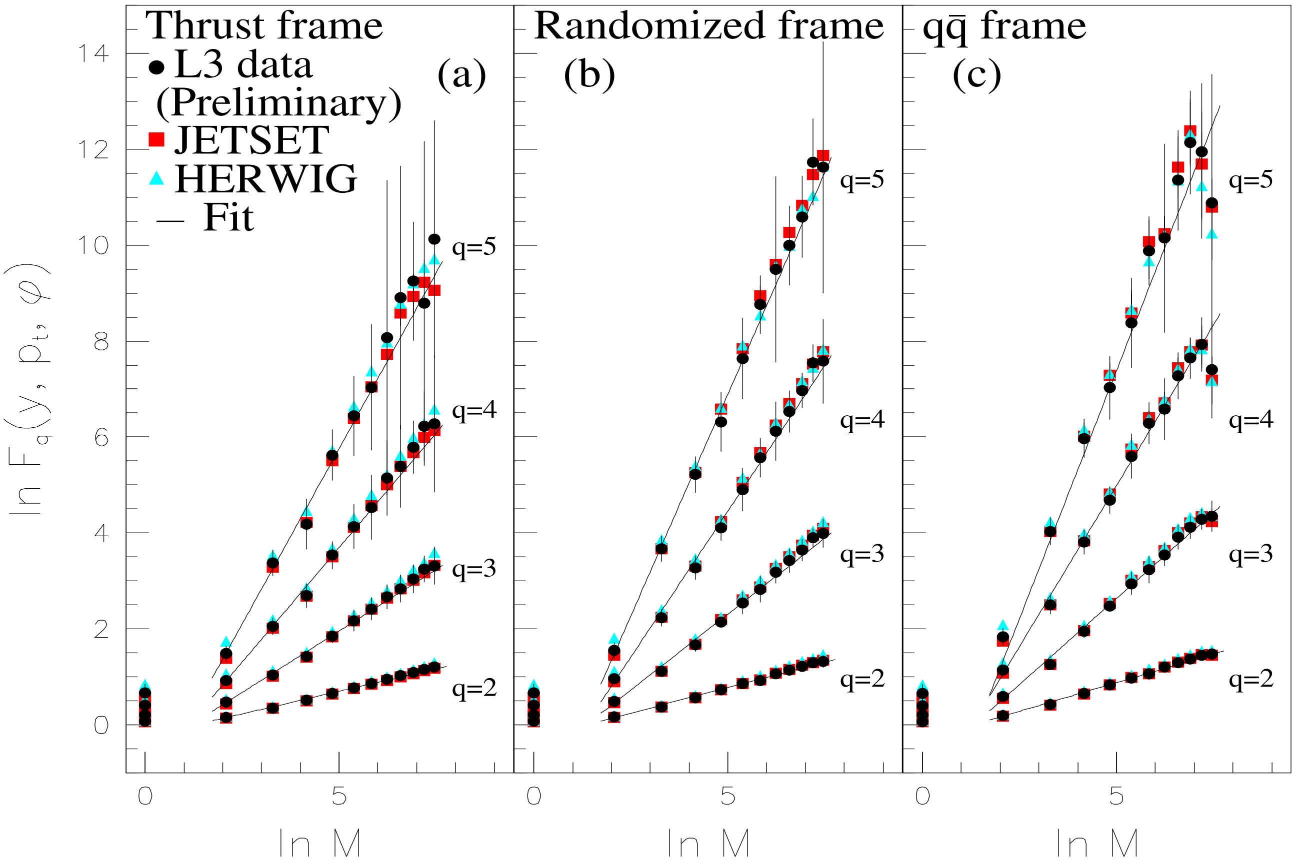
$$F_2(M_x) = A_x + B_x\, M_x^{-\gamma_x}$$

$\Rightarrow$ the Hurst exponent $H_{xy} = \frac{\ln M_x}{\ln M_y} = \frac{1+\gamma_y}{1+\gamma_x}$

Liu Lianshou et al.

Scaling of 3D factorial moments

L3 Collaboration, Abs. 494

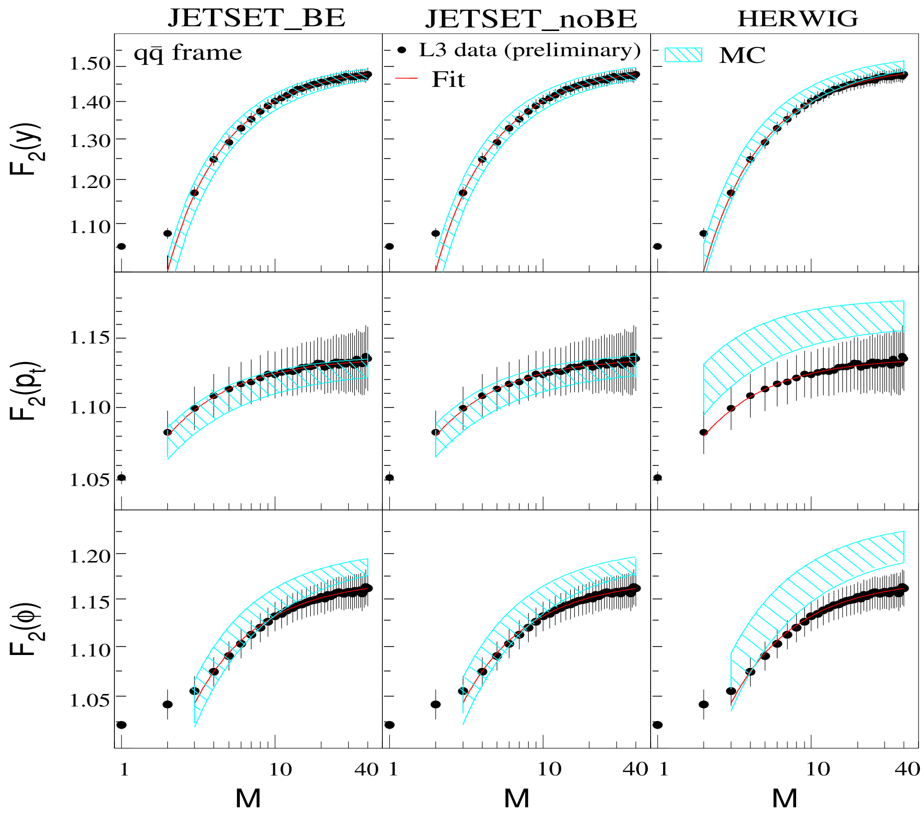


L3 Note 2758 (2002): $e^+e^- \to Z^0 \to$hadrons

- Factorial moments in 3D, $M = M_y M_\varphi M_{p_t}$
  rapidity $-5 \leqslant y \leqslant 5$, azimuthal angle $-\pi \leqslant \varphi < \pi$, transverse momentum $0.1 \leqslant p_t \leqslant 3$ GeV w.r.t. thrust
- Three frames: thrust, φ-random., q$\bar{\text{q}}$ MC corr.
- Self–similarity, i.e. isotropical fluctuations
  directly indicated
  (earlier seen by DELPHI, OPAL in the event frame)

Isotropic fluctuations from 1D F_2

L3 Collaboration, Abs. 494



L3 Note 2758 (2002): $e^+e^- \rightarrow Z^0 \rightarrow$hadrons

- 1D 2nd-order factorial moments in $q\bar{q}$ frame
- Fit: $F_2(M_a) = A_a + B_a\, M_a^{-\gamma_a}$ with
  the Hurst exponent $H_{ab} = \frac{\ln M_a}{\ln M_b} = \frac{1+\gamma_b}{1+\gamma_a}$
  $a, b = \{y, \varphi, p_t\}$
- Isotropic fluctuations obtained: $H_{ab} \approx 1$
- The same **conclusion** from other frames
- MC reproduces the isotropy
- 2-jet analysis: anisotropic fluctuations
  $\Rightarrow$ **hard gluon gives isotropy**

Cumulants and correlations

- Divide a phase space into **M cells of equal size**
  - Calculate **cell-averaged normalized cumulant moments**, cumulants, of order q:

$$\mathrm{K_q} = \frac{1}{\mathrm{M}} \sum_{\mathrm{m=1}}^{\mathrm{M}} \int_{\delta \mathrm{y}} \prod_{\mathrm{i}} \mathrm{dy_i} \frac{\mathrm{C_q(y_1, ..., y_2)}}{[\int_{\delta \mathrm{y}} \mathrm{dy} \rho_1(\mathrm{y})]^{\mathrm{q}}}$$

- $C_q(y_1, ..., y_2)$ – qth order correlation function
- For example,

$$C_2(y_1, y_2) = \rho_2(y_1, y_2) - \rho_1(y_1)\rho_1(y_2)$$

$$C_3(y_1, y_2, y_3) = \rho_3(y_1, y_2, y_3) - \sum_{(3)} \rho_1(y_1)\rho_2(y_2, y_3) + 2\rho_1(y_1)\rho_1(y_2)\rho_1(y_3)$$

- **Cumulants calculated** according to:

$$\mathrm{K_q} = \frac{1}{\mathrm{M}} \sum_{\mathrm{m=1}}^{\mathrm{M}} \frac{\mathrm{k_q^{(m)}}}{\langle \mathrm{n_m} \rangle^{\mathrm{q}}}$$

- $k_q^{(m)}$ – **unnormalized** cumulants (Mueller moments), e.g.:

$$k_2^{(m)} = \langle n_m^{[2]} \rangle - \langle n_m \rangle^2,$$

$$k_3^{(m)} = \langle n_m^{[3]} \rangle - 3\, \langle n_m^{[2]} \rangle \langle n_m \rangle + 2\, \langle n_m \rangle^3$$

via unnormalized factorial moments,

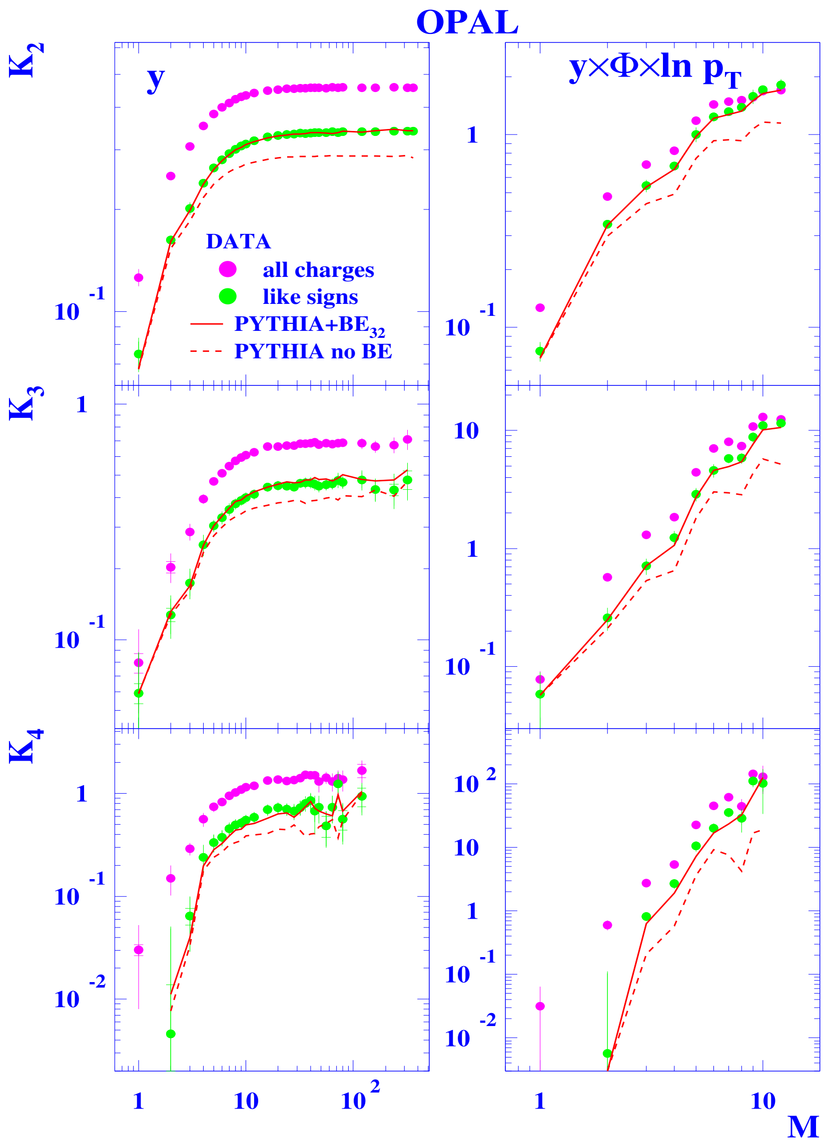
$$n_m^{[q]} = \langle n_m\, (n_m - 1) \cdots (n_m - q + 1) \rangle$$

$\langle \cdot \rangle \equiv$ averaging over events

E.A. De Wolf, I.M. Dremin, W. Kittel, Phys. Reports 270 (1996) 1

High-order genuine correlations 1D and 3D cumulants

OPAL Collaboration, Abs. 319

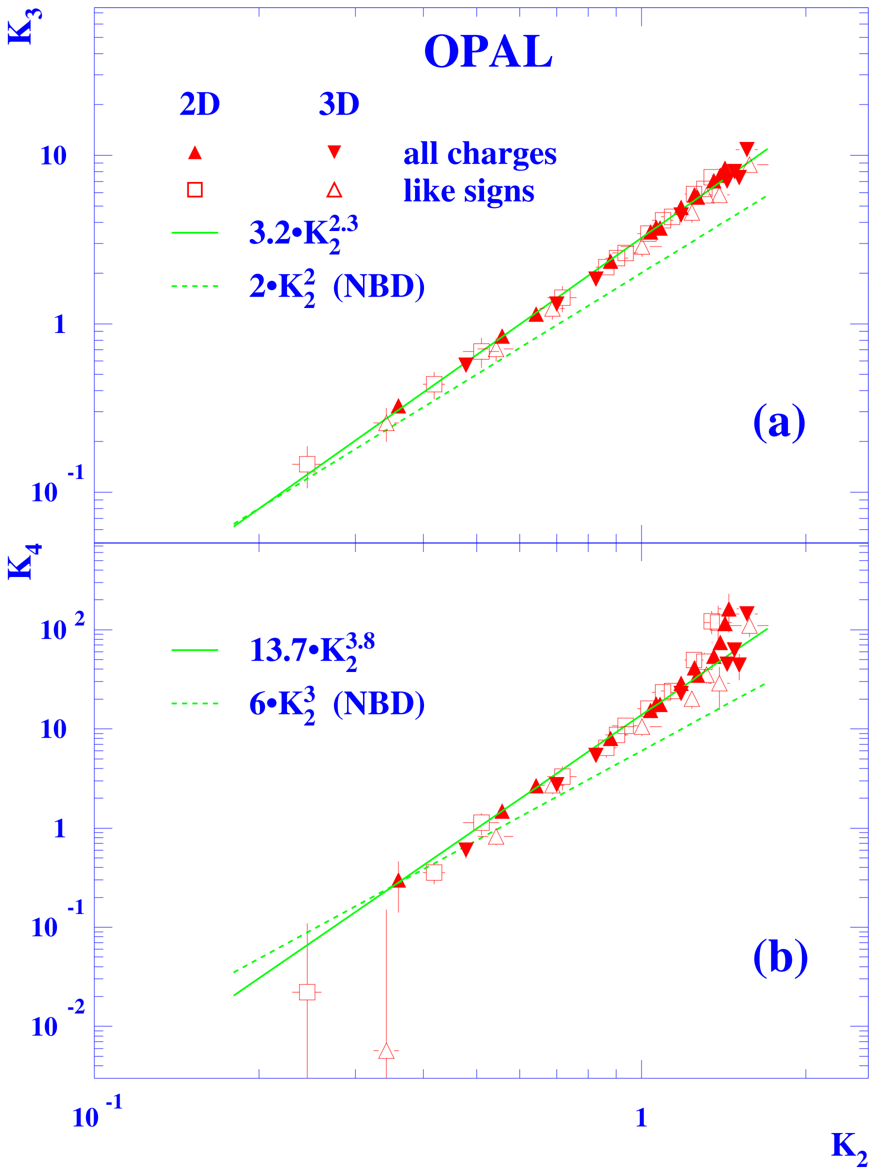


Phys. Lett. B523 (2001) 35: $e^+e^- \to Z^0 \to$hadrons

- Genuine 2-, 3-, 4-particle correlations observed
- Model **without BEC fails**, BEC clearly needed
- Like-sign cumulants increase faster, drive 'all charged' Ks at small bins
- BEC model **simultaneously** reproduces 1D to 3D Ks
- Pair-wise adjusted BEC account for higher-order Ks (?)
- Model adjusted to like-sign Ks reproduces all-charge Ks

Interdependence of correlations Ochs-Wosiek relation

OPAL Collaboration, Abs. 319



Phys. Lett. B523 (2001) 35: $e^+e^- \to Z^0 \to$hadrons

- Hierarchy between cumulants, $K_2 \leftrightarrow K_3$, $K_2 \leftrightarrow K_4$

  $\Rightarrow$ **Same mechanism** for $K_2 \to K_3 \to K_4$?
- Same for like-sign and all-charged multiplets
- MD deviates **from NBD** in small bins

Correlations in $\bar{p}p$ annihilation at 22.4 GeV/c

E.G. Boos et al., Abs. 37

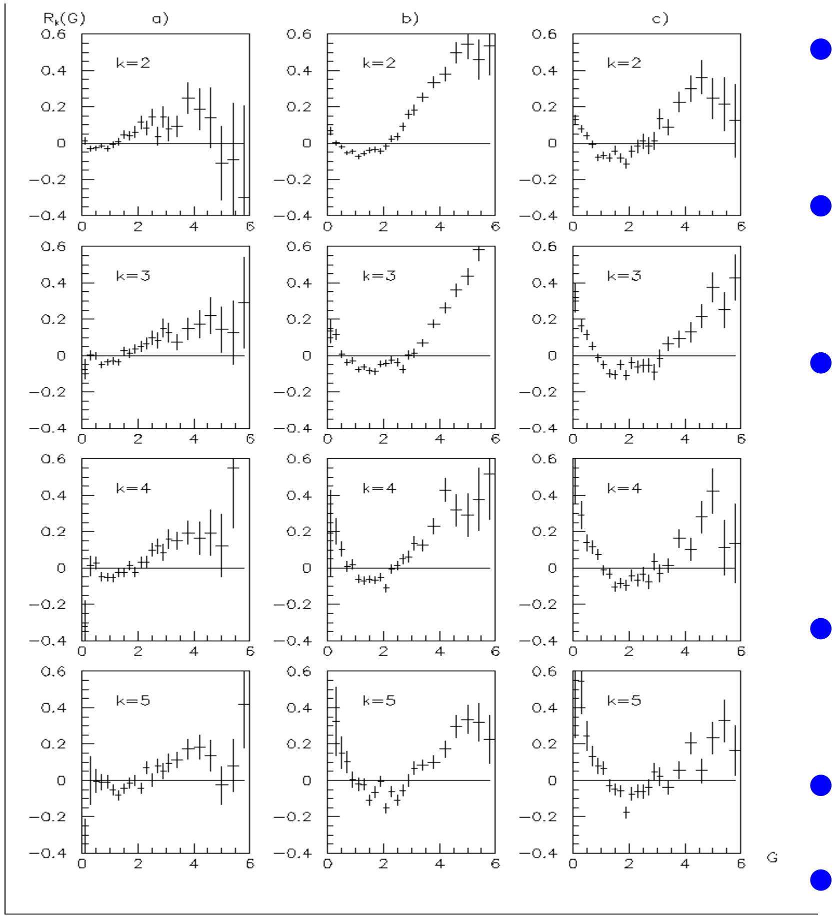


Fig. 2

(a) – $\bar{p}p$ annihilation at 22.4 GeV/c
(b) – non-annihilation channel
(c) – inelastic pp at 69 GeV/c

- Data from hydrogen bubble chambers "Ludmila" ($\bar{p}p$), "Mirabelle" (pp) at Serpukhov accelerator
- Annihilation and non-annihilation events separ. with statistical identification and missing mass
- Correlation function $R_k(G) = F_k(G)/F_k^{\phi}(G)$
  F_k, F_k^{ϕ} – measured and bkg differential spectra
  $G = \eta_{i+k} - \eta_i$ differense in pseudarapidities between i-th and $(i+k)$-th particles
- Non-annihilation $\bar{p}p$ is similar to inelastic pp: mesons from intermediate nucleon isobars (?)
- G-dip bin width $\langle \Delta G \rangle = \mathrm{const}(k) \sim 1.8$
- Number of clusters estimated $\langle n_{cl} \rangle \simeq 2-3$ with two particles per cluster in average
- Irregularities for $n > 3$-prong events observed

SUMMARY

- Multiplicity, fluctuation and correlation studies in high-energy physics gives further important information about hadron production processes
- Multiplicity **flavour dependence** at LEP1–2
  Abstracts 232, 320, 373
  - No flavour dependence for **light** quarks
  - Energy independence of $\delta_{bl} = \langle n_{b\bar{b}} \rangle - \langle n_{l\bar{l}} \rangle$ **consistent with QCD**
- Scaling of **local fluctuations** at LEP1
  Abstract 494
  - Self-similarity confirmed for **multi-jet** events
  - Self-affinity observed for **two jets**
- **Multiparticle correlations** at LEP1
  Abstract 319
  - High-order correlations **observed**
  - BEC is **a necessary ingredient**
  - Order hierarchy between correlations
- Multiparticle correlations in $\bar{\mathrm{p}}\mathrm{p}$ at 22.4 GeV/c
  Abstract 37
  - Weak correlations in $\bar{\mathrm{p}}\mathrm{p}$ **annihilation**
  - Similarity in **non-annihilation** $\bar{\mathrm{p}}\mathrm{p}$ and pp **inelastic** interactions
  - Some mechanisms proposed