SCATTERING AND RESONANCES IN QCD_2

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SUMMARY:

Extending previous works on the spectrum of QCD_2, we now investigate the 2D analogue of meson-baryon scattering.

We use semi-classical methods, perturbing around classical soliton solutions.

In the case of one flavor, we find that the effective potential is reflectionless.

In the case of several flavors, the method yields a potential which depends on the momentum of the incoming particle.

In this case there is both transmission and reflection.

In both cases no resonances appear.

GENERAL:

In 1+1 dimensions it is often possible to obtain analytic solutions. Thus instead of studying an approximate effective action in 3+1 dimensions, one can analyze the exact effective action in 1+1 dimensions.

For many purposes it is useful to have a strong analytic grip on the 1+1 dimensional analogues of the problems in 3+1 dimensions.

For References see hep-ph/0206001.

The Baryon spectrum of QCD_2 for general N_f and N_c was computed in

G. D. Date, Y. Frishman and J. Sonnenschein, Nucl. Phys. B **283**, 365 (1987)

Y. Frishman and J. Sonnenschein, Nucl. Phys. B **294**, 801 (1987)

The $\bar{q}q$ content of baryons was calculated in

Y. Frishman and M. Karliner, Nucl. Phys. B **344**, 393 (1990).

The physical picture of baryons composed of constituent quark solitons was obtained in

J. R. Ellis, Y. Frishman, A. Hanany and M. Karliner, Nucl. Phys. B **382**, 189 (1992)

Here we will compute the meson-baryon scattering in QCD_2 at strong coupling, following the techniques of

M. P. Mattis and M. Karliner, Phys. Rev. D **31**, 2833 (1985)

applied to the bosonized action

Y. Frishman and J. Sonnenschein, Phys. Rept. **223**, 309 (1993)

FORMULATION:

QCD in 1+1 dimensions, bosonized in the scheme

$$[\underbrace{SU(N_c)}_{h}]_{N_F} \times [\underbrace{U(N_F)}_{g}]_{N_c}$$

is

$$S[g,h,A_+A_-] = N_c S[g] + N_f S[h]$$
$$-\frac{1}{2e_c^2}\int d^2x \,\mathrm{Tr}_c\, F_{\mu\nu}F^{\mu\nu}$$
$$+\frac{N_F}{2\pi}\int d^2x \,\mathrm{Tr}_c \left[i\left(A_+ h\partial_- h^\dagger + A_- h^\dagger \partial_+ h\right)\right]$$
$$-\frac{N_F}{2\pi}\int d^2x \,\mathrm{Tr}_c \left(A_+ h A_- h^\dagger - A_- A_+\right)$$
$$+m'^2 N_{\widetilde{m}} \int d^2x \,\mathrm{Tr}\left(g\, h + h^\dagger\, g^\dagger\right)$$

where

$$m'^2 = m_q\, C\, \widetilde{m}$$

$\widetilde{m}$ is to be fixed, $C = e^\gamma \approx 0.891$,

$S[u] \equiv S_{WZW}[u] = \frac{1}{8\pi} \int d^2x \, \mathrm{Tr} \left(\partial_\mu u \partial^\mu u^\dagger \right)$

$+\frac{1}{12\pi} \int_B d^3y \, \epsilon_{ijk} \, \mathrm{Tr}$

$\left[\left(u^{-1} \partial_i u \right) \, \left(u^{-1} \partial_j u \right) \, \left(u^{-1} \partial_k u \right) \right]$

In the strong coupling limit

$e_c / m_q \to \infty,$

$S_{\text{eff}} = N_c S[g] + m^2 N_m \int d^2x \left(\mathrm{Tr}\, g + \mathrm{Tr}\, g^\dagger \right)$

$$m = \left[N_c C m_q \left(\frac{e_c \sqrt{N_f}}{\sqrt{2\pi}} \right)^{\Delta_c} \right]^{\frac{1}{1+\Delta_c}}$$

$\Delta_c = \frac{N_c^2 - 1}{N_c (N_c + N_F)}$

Equation of motion, as coefficient of $(\delta g) g^\dagger$,

$\frac{N_c}{4\pi} \partial_+ \left[(\partial_- g) \, g^\dagger \right] + m^2 \left(g - g^\dagger \right) = 0$

Expanding in small fluctuations around a given static classical solution

$$g = e^{-i\Phi_c(x)}\, e^{-i\tilde{\delta}\phi(x,t)}$$

$$\approx e^{-i\Phi_c(x)} - ie^{-i\Phi_c(x)}\tilde{\delta}\phi(x,t)$$

Equations of motion,

$$\frac{N_c}{4\pi}\partial_+ \left[e^{-i\Phi_c(x)}\left(\partial_-\tilde{\delta}\phi(x,t)\right)e^{i\Phi_c(x)}\right]$$

$$+m^2\left[e^{-i\Phi_c(x)}\tilde{\delta}\phi(x,t) + \tilde{\delta}\phi(x,t)e^{i\Phi_c(x)}\right]$$

$$= 0$$

Choose $\Phi_c(x)$ to have only the 11 entry non-zero, which we denote as $\phi_c(x)$. Then

$$\phi_c(x)'' - \frac{8\pi}{N_c}m^2 \sin\phi_c = 0$$

$$\phi_c(x) = 4\text{arctg}\left(e^{\mu x}\right), \qquad \mu = m\sqrt{\frac{8\pi}{N_c}}$$

ONE FLAVOR:

Denote this case by $\delta\phi_A$, where the subscript "A" stands for "Abelian".

Then

$$\delta g = -i\delta\phi_A(x)e^{-i\phi_c(x)}$$

$$\Box\delta\phi_A + \mu^2(\cos\phi_c)\delta\phi_A = 0$$

$$\cos\phi_c = \left[1 - \frac{2}{\cosh^2\mu x}\right]$$

Get

$$\mathcal{L}_{\text{eff}} = \frac{1}{2}\left(\partial_\mu\delta\phi_A\right)^2 - \frac{1}{2}V(x)\left(\delta\phi_A\right)^2$$

$$V(x) = \mu^2\cos\phi_c(x) = \mu^2\left[1 - \frac{2}{\cosh^2\mu x}\right]$$

Take

$$\delta\phi_A(x,t) = e^{-i\omega t}\chi_A(x)$$

Then

$$-\omega^2\chi_A - \chi_A'' + V(x)\chi_A = 0$$

When $x \to \pm\infty$, the potential $\to \mu^2$, and so

$$\chi_A''(\pm\infty)+\omega^2\chi_A(\pm\infty) = \mu^2\chi_A(\pm\infty)$$

Take

$$\chi_A(x) \longrightarrow_{|x|\to\infty} e^{\pm ikx}$$

which results in

$$\omega^2 = k^2 + \mu^2$$

It turns out that for the particular potential above, there is no reflection at all. The transmission T is

$$T = e^{i\delta}$$

$$\mathrm{ctg}\,\tfrac{1}{2}\delta = \frac{k}{\mu}$$

δ varies smoothly and decreases monotonically from $\delta = \pi$ at $k = 0$ to $\delta = 0$ at $k = \infty$. Thus there is no resonance.

For ACTIONS that lead to solitons with potentials of NO-REFLECTION, see paper.

MULTI-FLAVOR CASE:

$$\Box\tilde{\delta}\phi - i\left(\partial_{+}\Phi_c\right)\left(\partial_{-}\tilde{\delta}\phi\right) + i\left(\partial_{-}\tilde{\delta}\phi\right)\left(\partial_{+}\Phi_c\right)$$

$$+\frac{1}{2}\mu^2\left[\tilde{\delta}\phi e^{-i\Phi_c(x)} + e^{i\Phi_c(x)}\tilde{\delta}\phi\right] = 0$$

The equation for $\tilde{\delta}\phi_{ij}$ with $i, j \neq 1$ is like for the free case

$$\Box\tilde{\delta}\phi_{ij} + \mu^2\tilde{\delta}\phi_{ij} = 0\,, \qquad i \text{ and } j \neq 1$$

whereas the $i = 1, j = 1$ matrix element is like in the abelian case

$$\Box\tilde{\delta}\phi_{11} + \mu^2\left(\cos\phi_c(x)\right)\tilde{\delta}\phi_{11} = 0$$

with no reflection and no resonance.

So in order to proceed beyond these results, we need to consider $\tilde{\delta}\phi_{1j}$, $j \neq 1$, or $\tilde{\delta}\phi_{i1}$, $i \neq 1$. As $\tilde{\delta}\phi$ is hermitean, it is sufficient to discuss one of the above.

Thus we take

$$\tilde{\delta}\phi_{1j} = e^{-i\omega t}u_j(x) \qquad j \neq 1$$

resulting in

$$u_j''(x) - i\phi_c'(x)u_j'(x) +$$
$$\left[\omega^2 + \omega\phi_c'(x) - \tfrac{1}{2}\mu^2\left(1 + e^{i\phi_c(x)}\right)\right]u_j(x)$$
$$= 0$$

Define

$$u_j \equiv e^{\frac{i}{2}\phi_c}v_j$$

Then

$$v_j'' +$$
$$\left[\omega^2 + \omega\phi_c' - \tfrac{1}{2}\mu^2(1+\cos\phi_c) + \tfrac{1}{4}\left(\phi_c'\right)^2\right]v_j$$
$$= 0$$

Using

$$\tfrac{1}{2}\left(\phi_c'\right)^2 = \mu^2(1-\cos\phi_c)$$

we get

$$v_j'' + \left[\omega^2 + \omega\phi_c' - \mu^2 \cos\phi_c\right] v_j = 0$$

or

$$-v_j'' - \omega^2 v_j + V(x) v_j = 0$$

where

$$V(x) = -\omega\phi_c' + \mu^2 \cos\phi_c$$

$$= \mu^2 - 2\mu^2 \left[\frac{(\omega/\mu)}{\cosh \mu x} + \frac{1}{\cosh^2 \mu x}\right]$$

with $\omega = \sqrt{k^2 + \mu^2}$ as before. Note that the potential depends on the momentum of the incoming particle.

NUMERICAL RESULTS:

T is the transition amplitude and R is the reflection amplitude, with

$$|T|^2 + |R|^2 = 1$$

We take

$$v_j(x) = Te^{ikx}, \; x \to +\infty$$

$$v_j(x) = e^{ikx} + Re^{-ikx}, \; x \to -\infty$$

Since the potential is symmetric, the symmetric and anti-symmetric scattering amplitudes don't mix, yielding two independent phase shifts δ_S and δ_A, respectively. This leads to

$$T = \frac{1}{2}\left(e^{i\delta_S} + e^{i\delta_A}\right)$$

$$R = \frac{1}{2}\left(e^{i\delta_S} - e^{i\delta_A}\right)$$

Define

$\delta_{\pm} = \frac{1}{2}(\delta_S \pm \delta_A)$

Then

$T = e^{i\delta_+} \cos \delta_-$

$R = ie^{i\delta_+} \sin \delta_-$

Note that R/T is purely imaginary.

The transmission and reflections probabilities are

$|T|^2 = \cos^2 \delta_-$

$|R|^2 = \sin^2 \delta_-$

The numerical results for the transmission probability $|T|^2$ and for the phase of T, δ_+ are presented in Figs 1 and 2.

For comparison and as an extra check we also plot the WKB result for δ_+.

Note that no resonance appears.

Note also that the asymptotic value of the phase shift is π. This can also be obtained from a WKB calculation, which becomes exact at infinite energies.

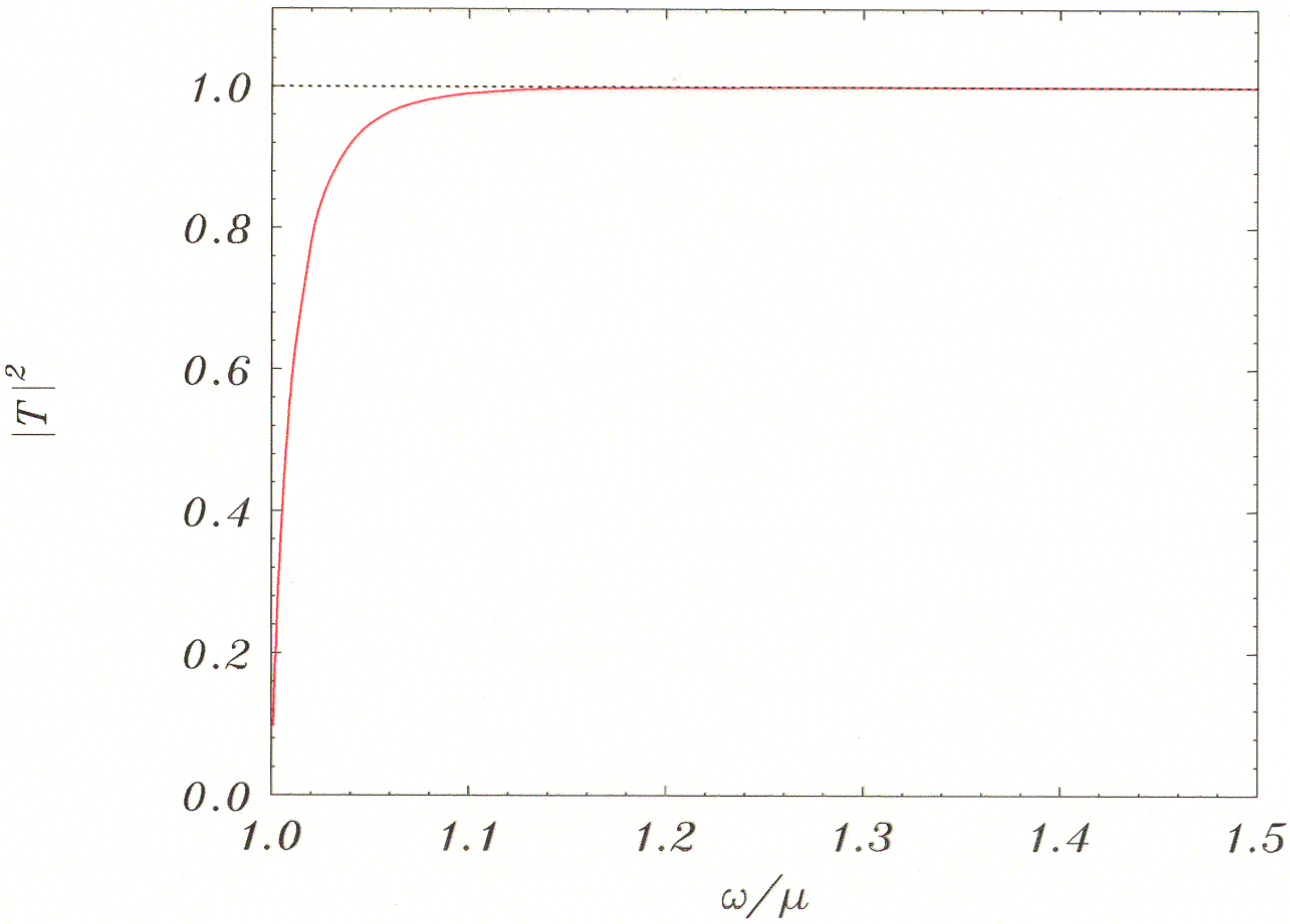

Fig 1

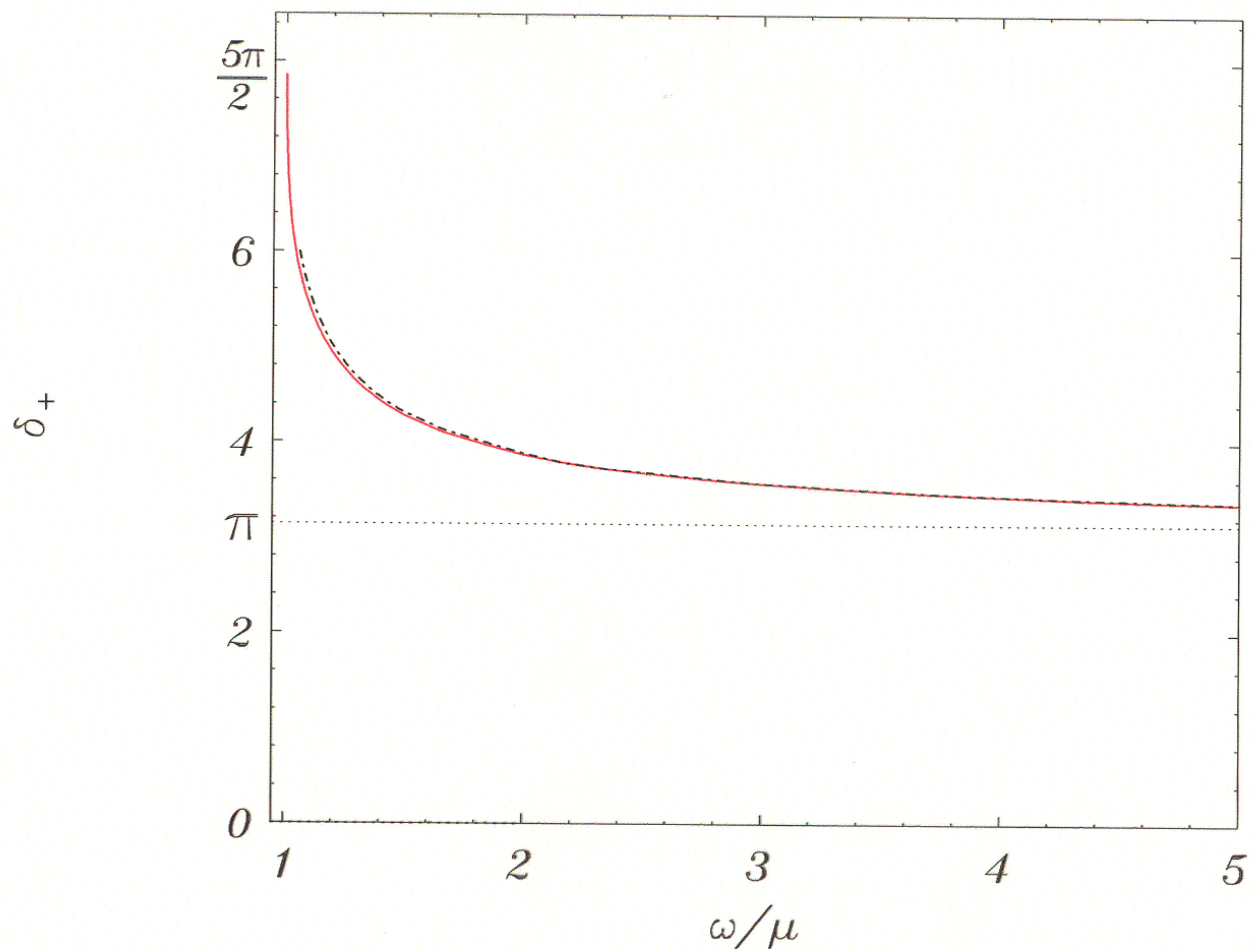

Fig 2