Consequences of t-channel unitarity for pp, γp and $\gamma\gamma$ cross sections at $Q^2 = 0$ and in DIS

hep–ph/0207196

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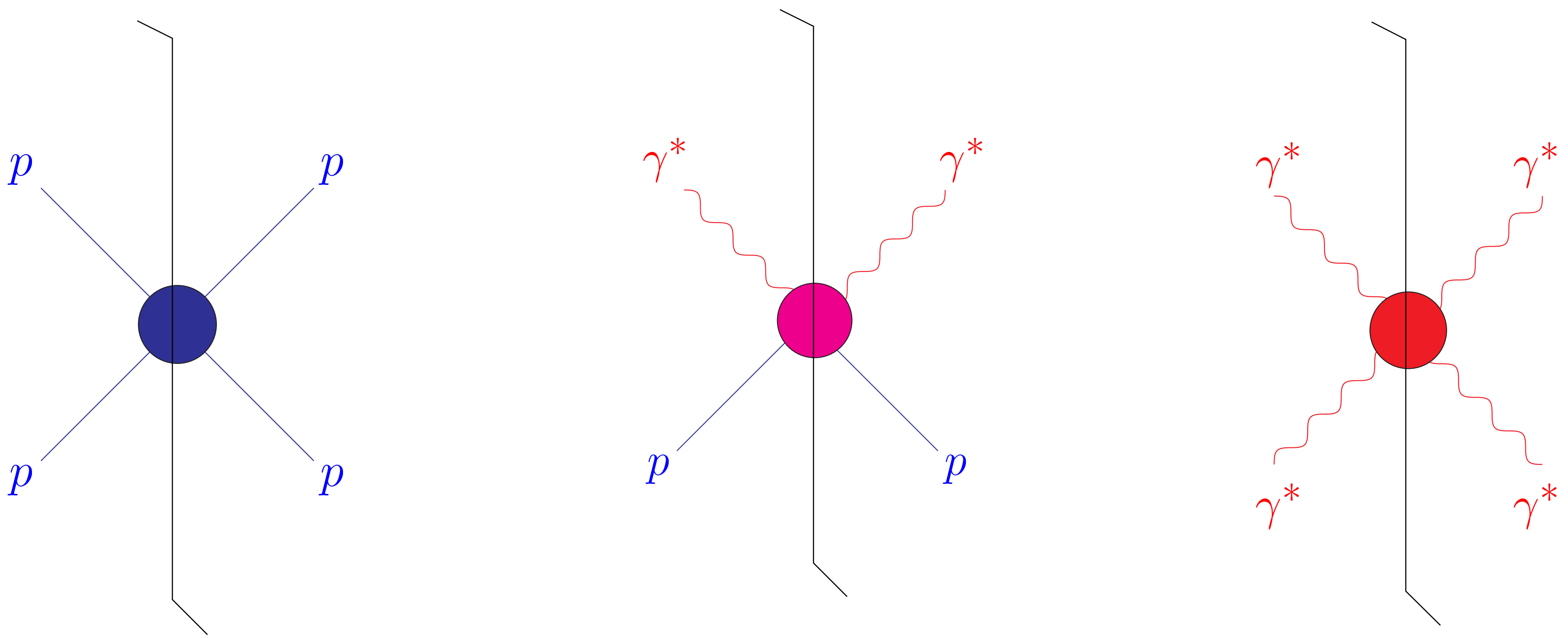
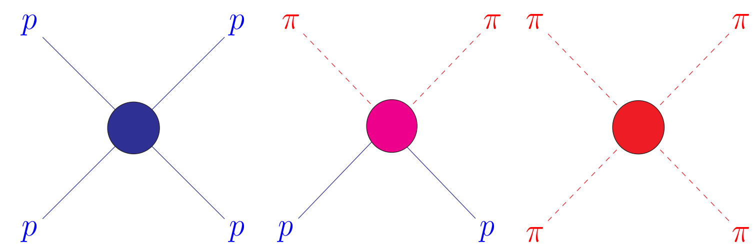


σ_{tot}^{pp}
RHIC: pp2pp

$\sigma_{tot}^{\gamma p}$
F_2^p
HERA: H1, ZEUS

$\sigma_{tot}^{\gamma\gamma}$
F_2^γ
LEP: L3, OPAL

Standard argument: p and π



Go above threshold in the crossed channel: $t > 4m_p^2$

$$S^\dagger S = SS^\dagger = 1\!\!\mathrm{I}$$

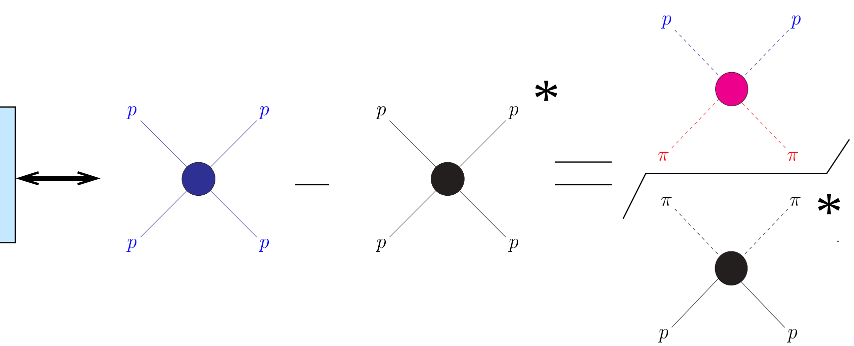
$$\underset{(S=1\!\!\mathrm{I}+iT)}{\Longrightarrow} T - T^\dagger = iT^\dagger T = iTT^\dagger$$

Watson-Sommerfeld transform to complex j plane:

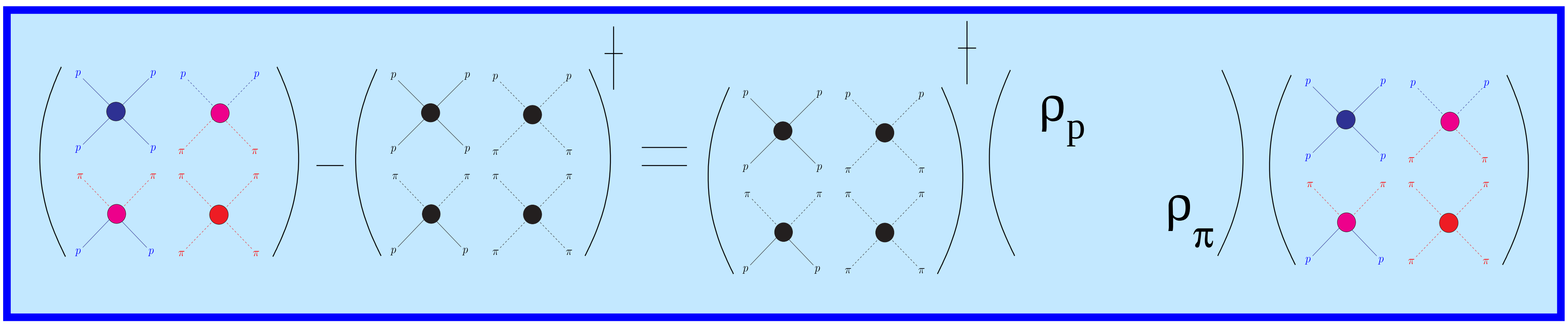
One π threshold:

$$T_{pp}(j,t) - T_{pp}(j,t)^\dagger = T_{p\pi}(j,t)\rho_\pi T^\dagger_{\pi p}(j,t)$$

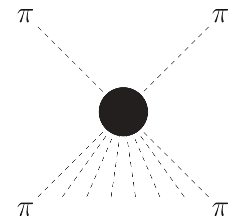
with $\rho_\pi = 2i\sqrt{\frac{t-4m_\pi^2}{t}}$.

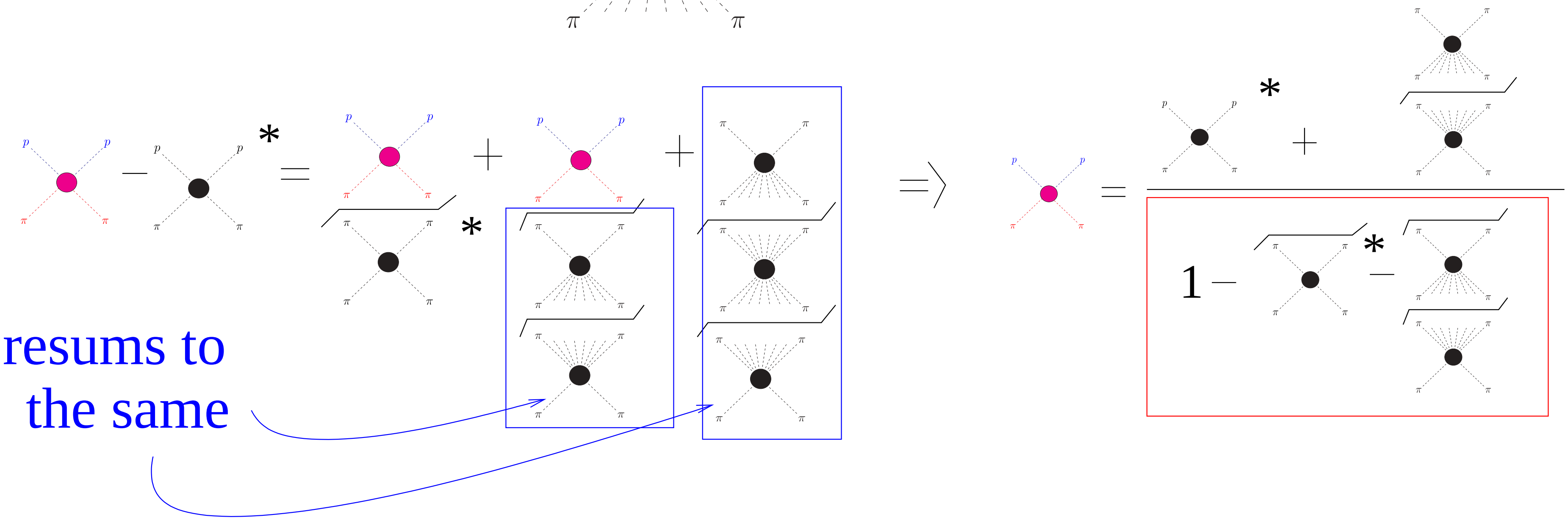


p and π thresholds:

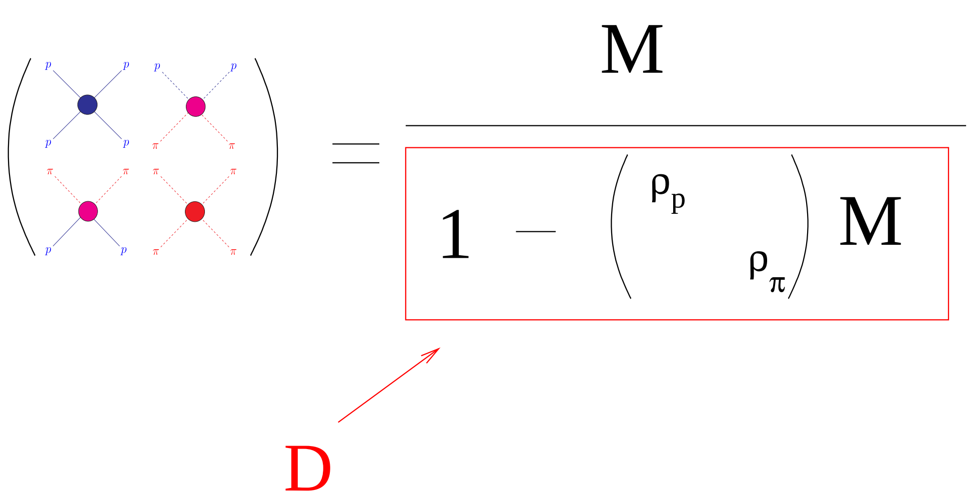


Multiple thresholds:

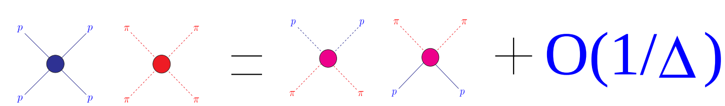




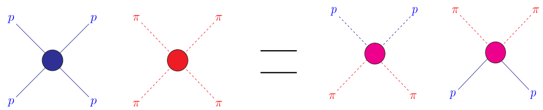
Multiple thresholds and several processes:



- All amplitudes have the same singularities
- They come from the zeroes of det(D)= Δ
- Factorisation:



Factorisation:



$+\,O(1/\Delta)$

$$\lim_{j\to z_m}\left[T_{\pi\pi}(j)-\frac{T_{p\pi}(j)T_{\pi p}(j)}{T_{pp}(j)}\right]=\mathit{finite\ terms.}$$

complex j−plane for t>0

relations for residues of each singularity

→ continue to t<0

Simple poles

$$T_{pq}=\frac{S_{pq}}{j-z}$$
$$\Downarrow$$
$$S_{22}S_{11}=(S_{12})^2$$

Simple and double poles

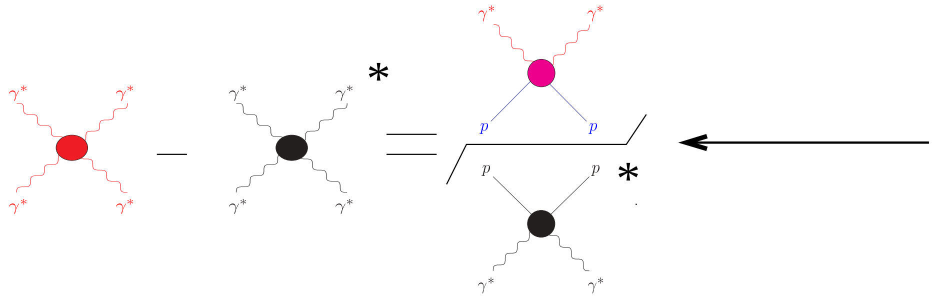
$$T_{pq}=\frac{S_{pq}}{j-z}+\frac{D_{pq}}{(j-z)^2}$$
$$\Downarrow$$
$$D_{11}D_{22}=(D_{12})^2$$
$$D_{11}^2S_{22}=D_{12}(2S_{12}D_{11}-S_{11}D_{12})$$

Simple, double and triple poles

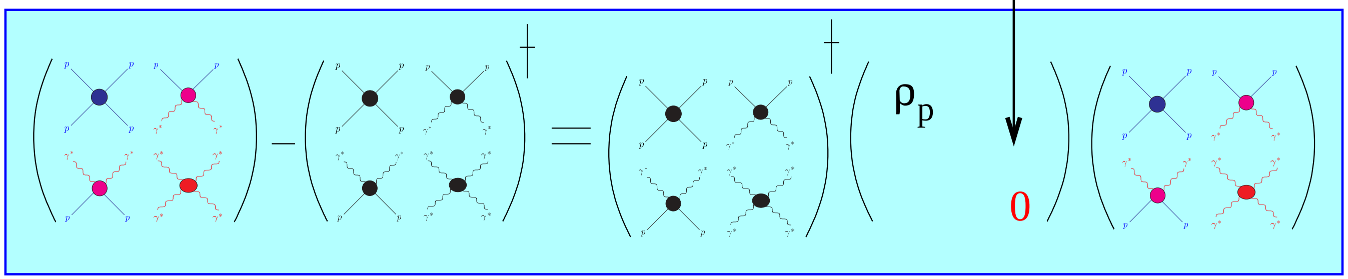
$$T_{pq}=\frac{S_{pq}}{j-z}+\frac{D_{pq}}{(j-z)^2}+\frac{F_{pq}}{(j-z)^3}$$
$$\Downarrow$$
$$F_{11}F_{22}=(F_{12})^2$$
$$\begin{aligned}F_{11}^2D_{22}&=F_{12}(2D_{12}F_{11}-D_{11}F_{12})\\F_{11}^3S_{22}&=F_{11}F_{12}\,(2S_{12}F_{11}-S_{11}F_{12})\\&+D_{12}F_{11}\,(D_{12}F_{11}-2D_{11}F_{12})\\&+D_{11}^2F_{12}^2\end{aligned}$$

New tCU relations

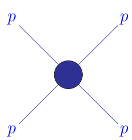
The case of off−shell photons



No γ* in the intermediate state

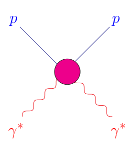


Several channels, multiple thresholds...



$= \frac{M}{D}$

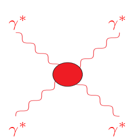
Same as before



$= \frac{N}{D}$

Possible new singularities

additionally to the old ones



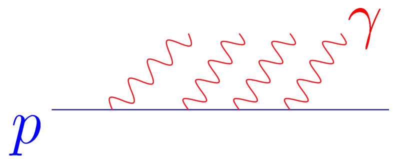
$= \frac{N^2}{D} + N'$

*γ*p singularities squared*

New (purely perturbative?) singularities

The case of on−shell photons

Problem:



Soft emissions may destroy the possibility
to define exclusive processeses
no S matrix for QED

$\Longrightarrow$ photons do not enter the tCU relations

$\Longrightarrow$ on−shell photons can have extra singularities, as off−shell photons

Remarks:

Existence of a pole mass for the electron?

Possibility of collective states?

cfr Lavelle, Mc Mullan, Bagan et al.

The existence of extra singularities in photon total cross sections would prove that one cannot recover an S−matrix formalism in QED

Models for σ

lower trajectories: a/f

$$Y(Q^2)(2\nu)^{-\eta}$$

rising term

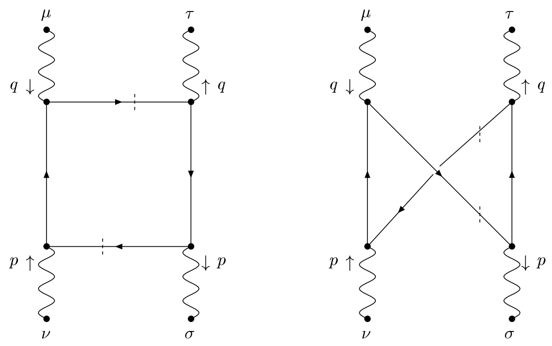
double pole

$$D(Q^2)\left[\log\left(1+\Lambda(Q^2)(2\nu)^{\delta}\right) + (s \to u)\right] + C(Q^2)$$

triple pole

$$t(Q^2)\left[\log(2\nu)+d(Q^2)\right]^2 + c(Q^2)$$

box diagram

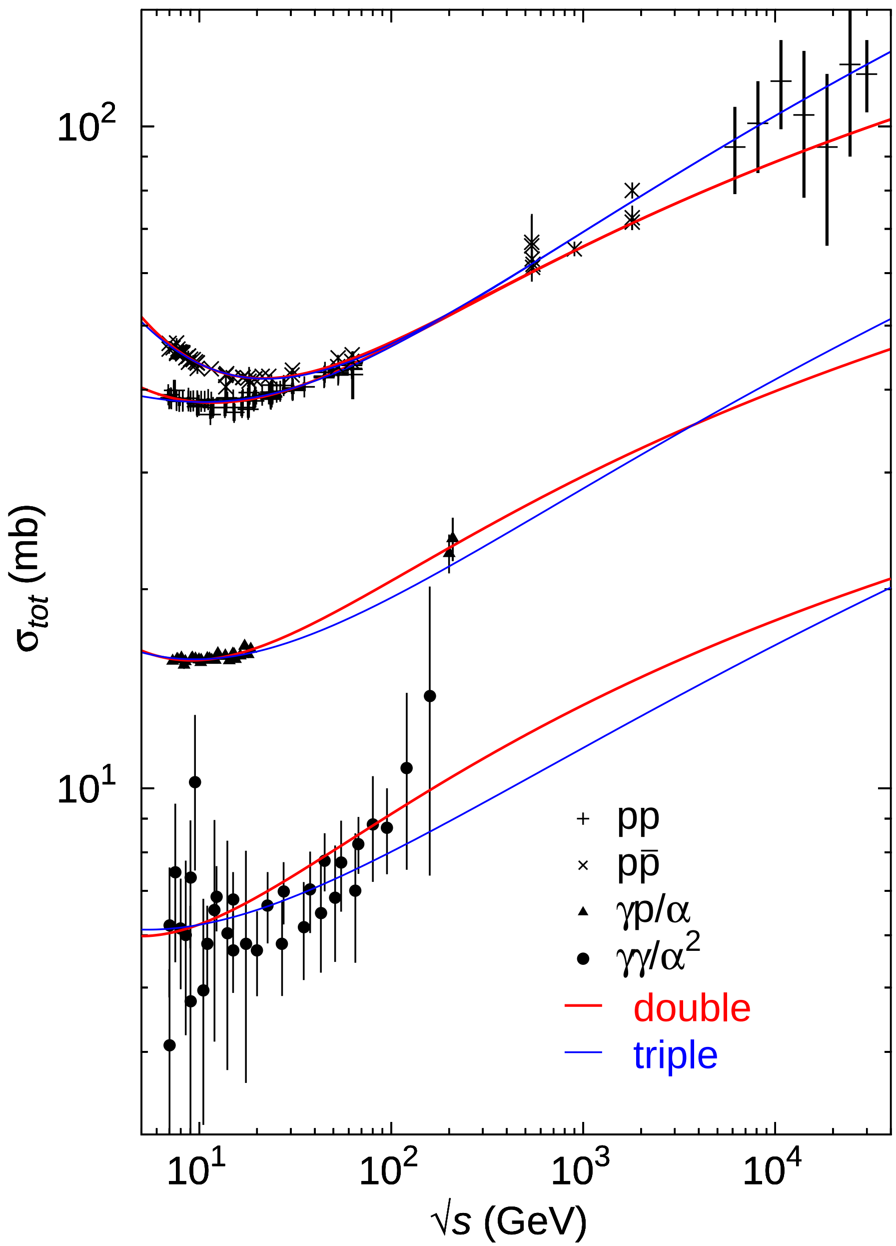


only for off−shell photons

Fit to total cross sections:

from 7 GeV to 30 TeV

γ down to 2 GeV!

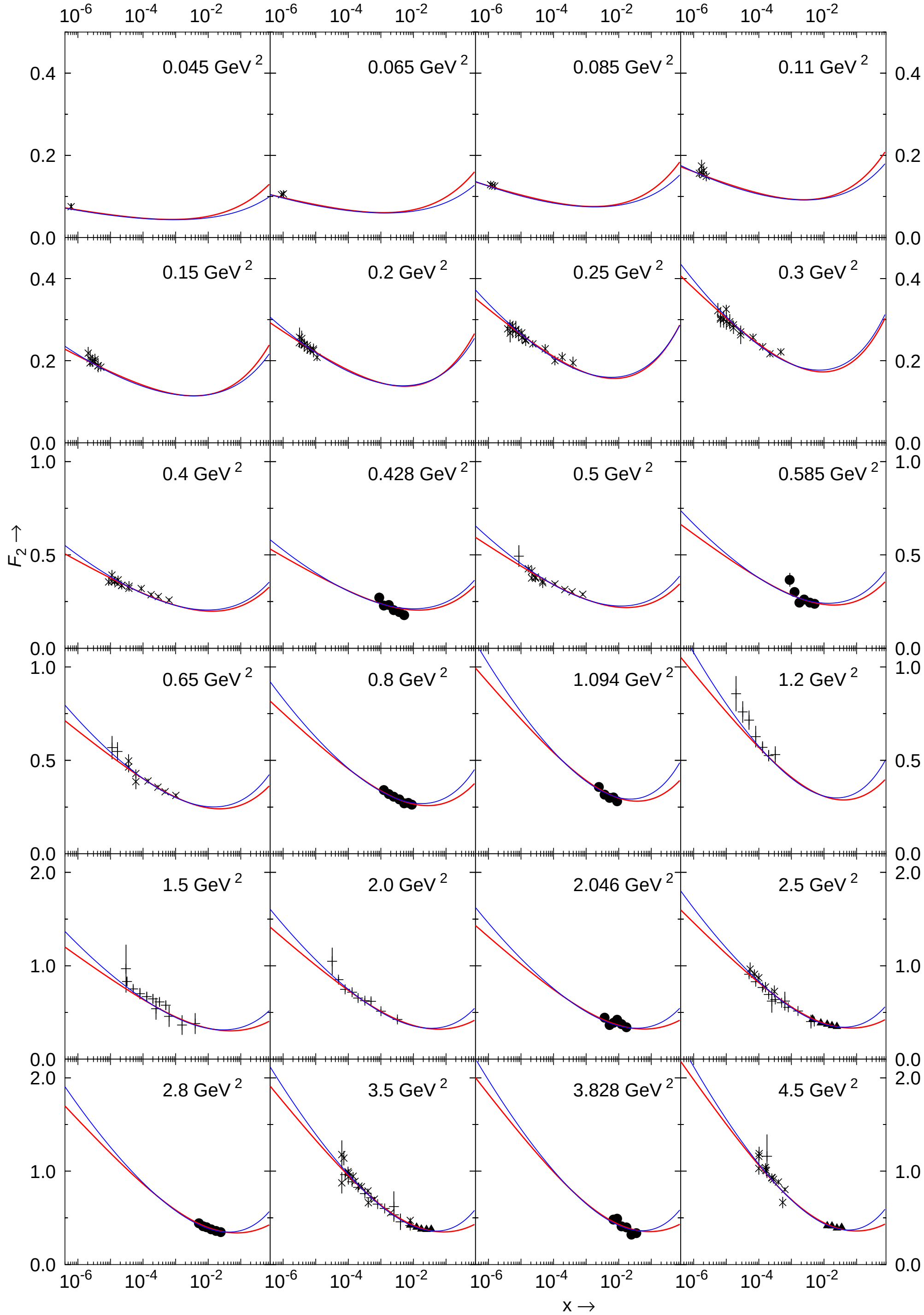


low side of error bars

no box diagram contribution

no extra singularities

F_2^p at low Q^2



$$x < 0.3$$
$$Q^2 < 400\ \mathrm{GeV}^2$$
$$2\nu > 7\ \mathrm{GeV}^2$$
$$|\cos(\vartheta_t)| = \frac{\nu}{m_p\sqrt{Q^2}} > \frac{49}{2m_p^2}$$

good fit to all data

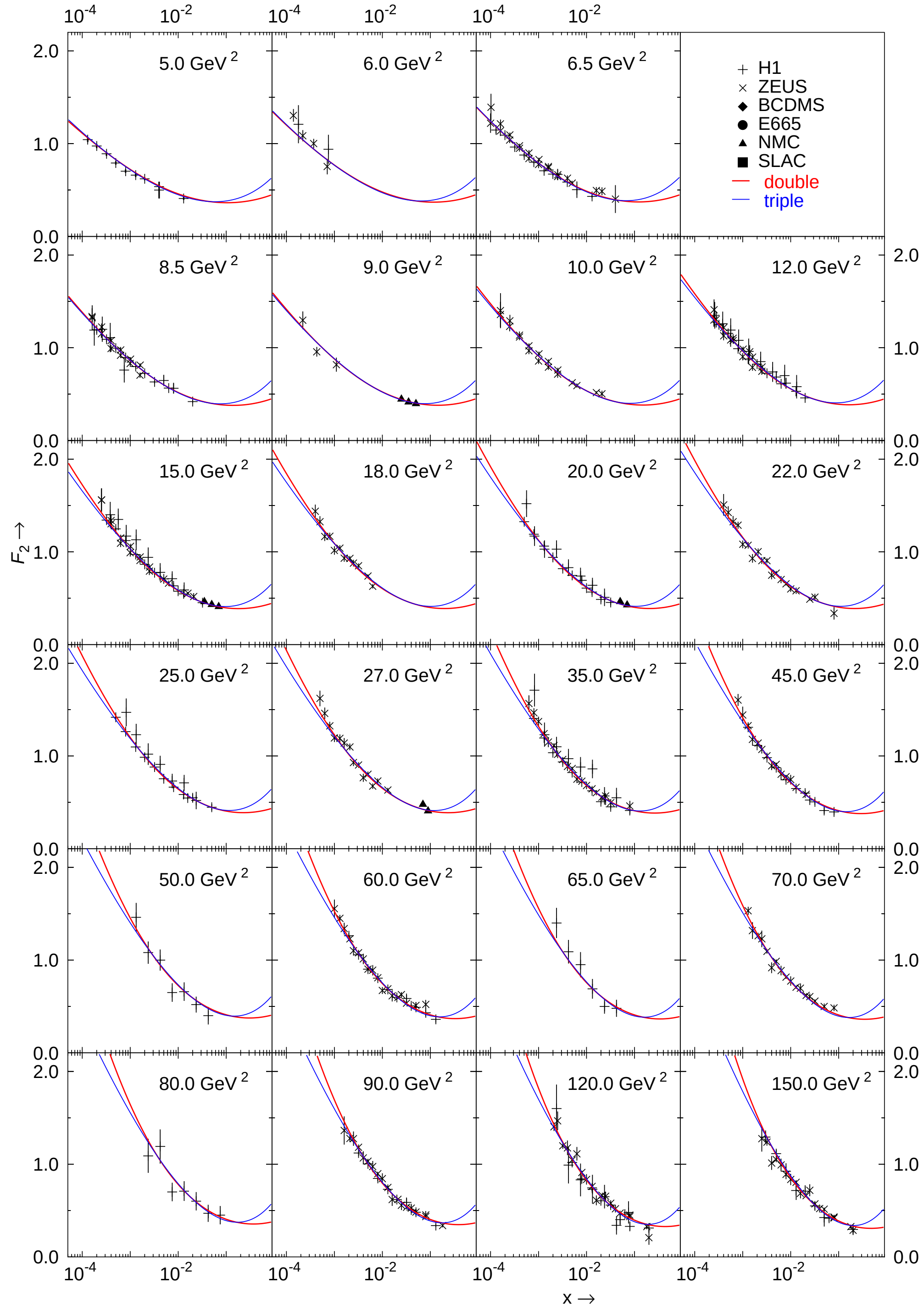
extrapolates to "low" total cross sections

F_2^p at higher Q^2

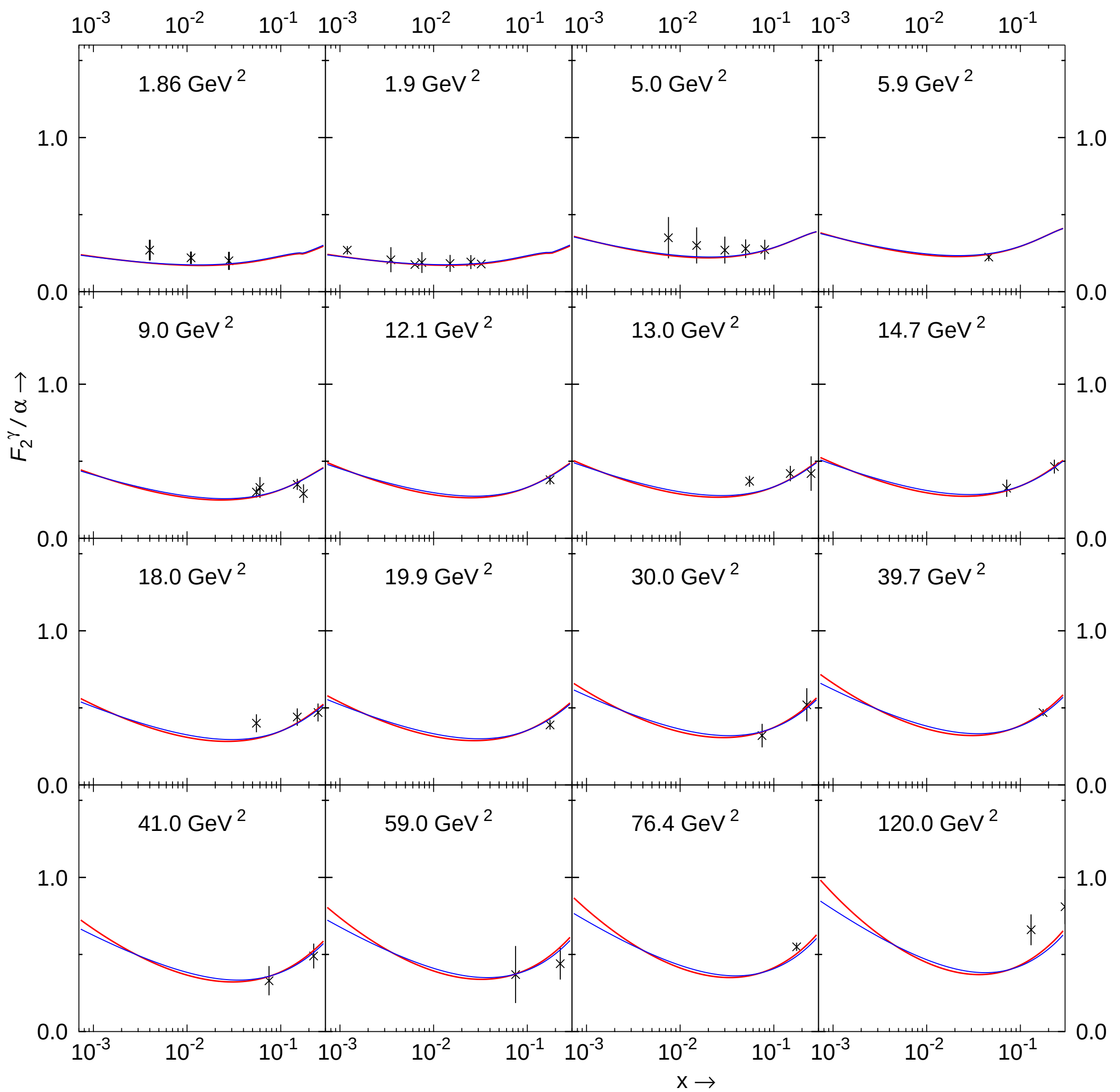
$$x < 0.3$$
$$Q^2 < 400\ \mathrm{GeV}^2$$
$$2\nu > 7\ \mathrm{GeV}^2$$
$$|\cos(\vartheta_t)| = \frac{\nu}{m_p\sqrt{Q^2}} > \frac{49}{2m_p^2}$$

no hard pomeron

no DGLAP evolution



F_2^γ



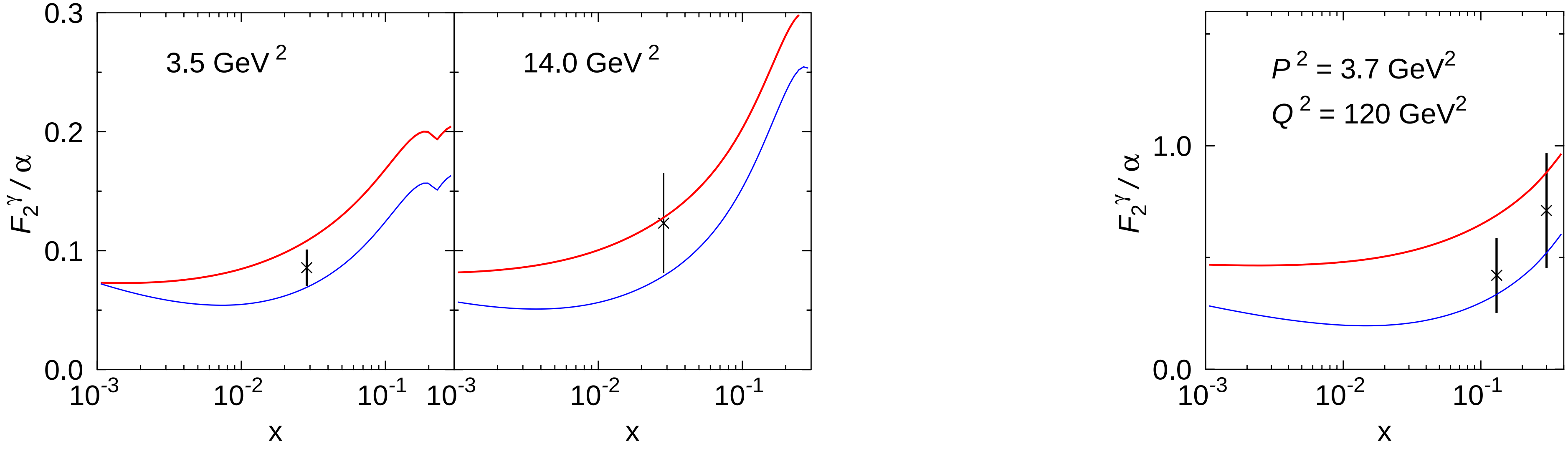
$$x < 0.3$$
$$Q^2 < 800 \text{ GeV}^2$$
$$2\nu > 7 \text{ GeV}^2$$
$$|\cos(\vartheta_t)| = \frac{\nu}{Q^2} > \frac{49}{2m_p^2}$$

test of tCU relations

good fit to all data

need box diagram

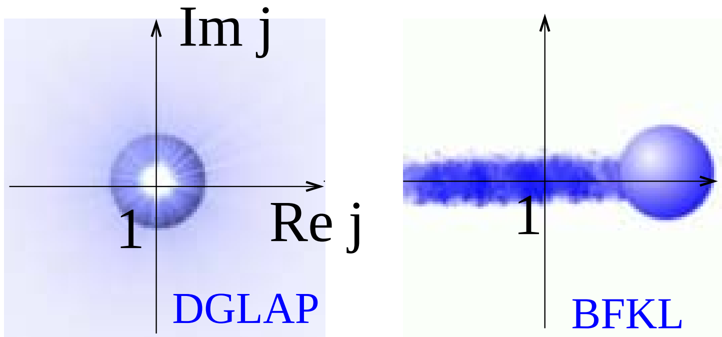
F_2^γ : perturbative (BFKL) region



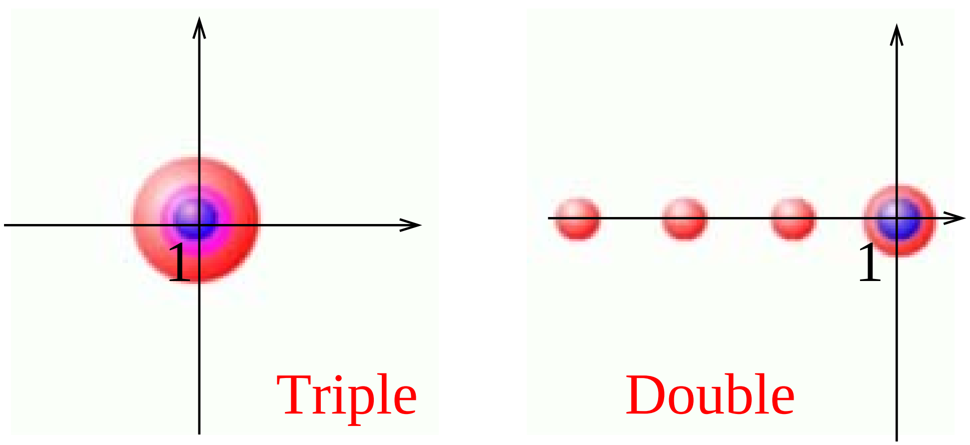
The soft fit + the box diagram seem to be sufficient to reproduce the data

Note: this can be done directly using the tCU relations and HERA data

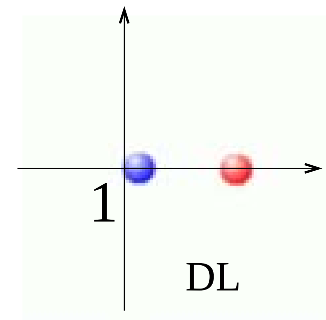
Conclusion: singularities at t=0



pQCD



New parametrisations of DIS
Respect t−Channel Unitarity
Predict $\gamma\gamma$ from γ p and pp



Global description, soft+hard