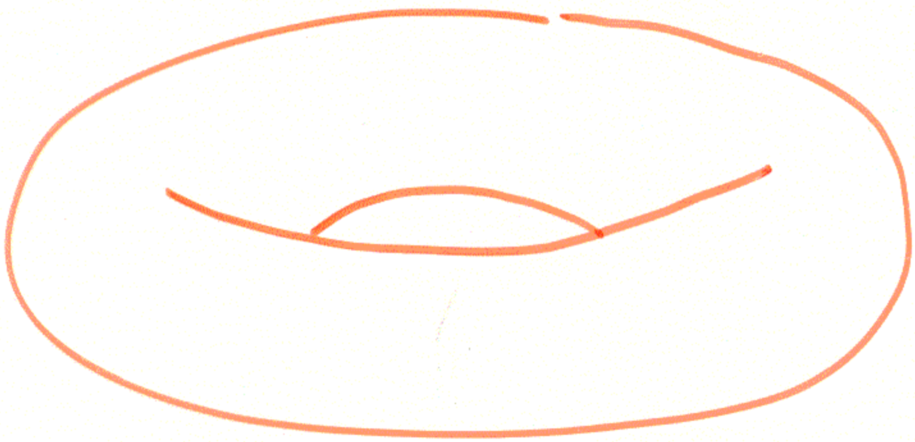
Q What are Instantons made of ?

Amsterdam July 02



Chris Ford (Leiden)

A Depends on who you ask!

Instantons

classical solutions of pure Yang-Mills in 4D Euclidean space

- Self dual $F = \tilde{F}$
- Finite action
- Smooth

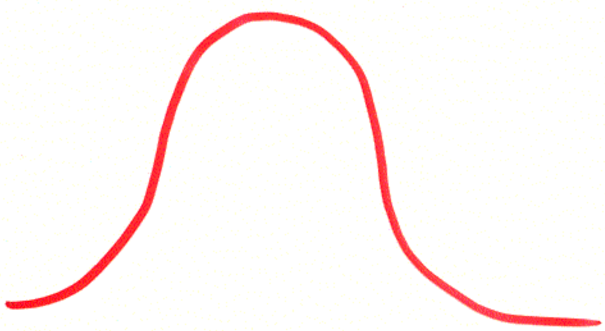
characterised by integer topological charge

$$k = \int d^4x \; F\tilde{F}$$

Number

$F\tilde{F}$ top. charge density

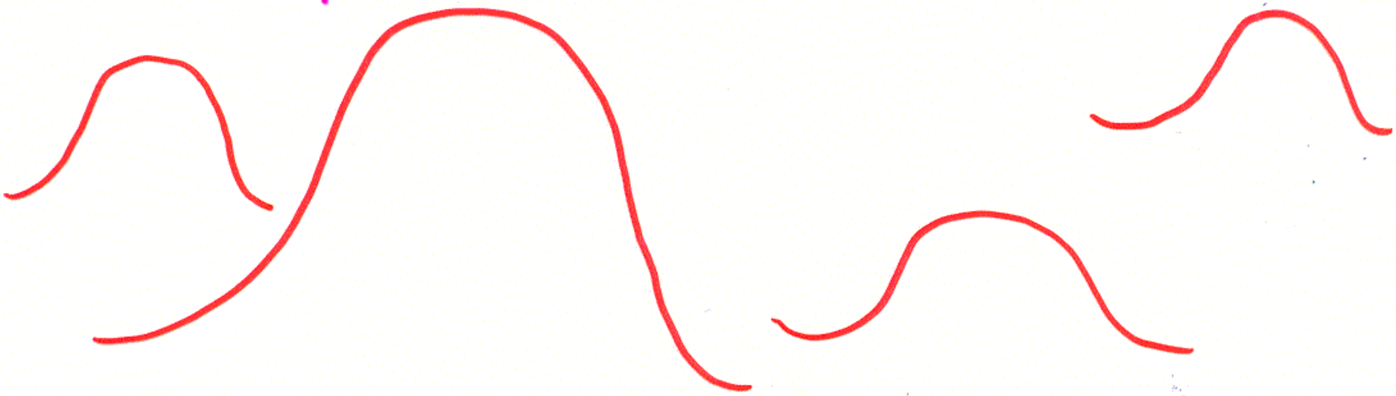
$\underline{k=1}$ single 'lump' in charge density



Multi-instantons

charge $k > 1$

k lumps



Very naive

(YM equations non-linear,
boundary conditions)

A lot of effort in

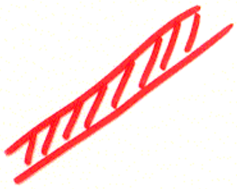
mid-late 70's

(infinite Euclidean space
R^4)

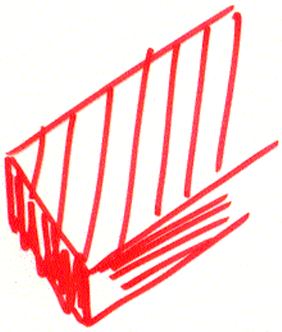
More exotic substructure ?

(Constituents)

c.f. baryons / quarks



monopole



vortex
sheet

instanton

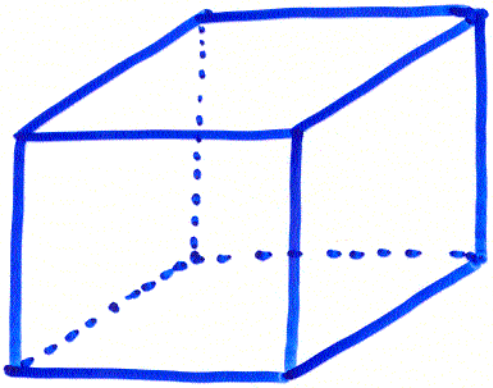
confinement scenarios

Pure YM : monopole
content ?

Monopoles / vortices versus finite action ?

Compactify on a torus

T^4



4D Euclidean box, opposite faces identified

4 periods L_0, L_1, L_2, L_3

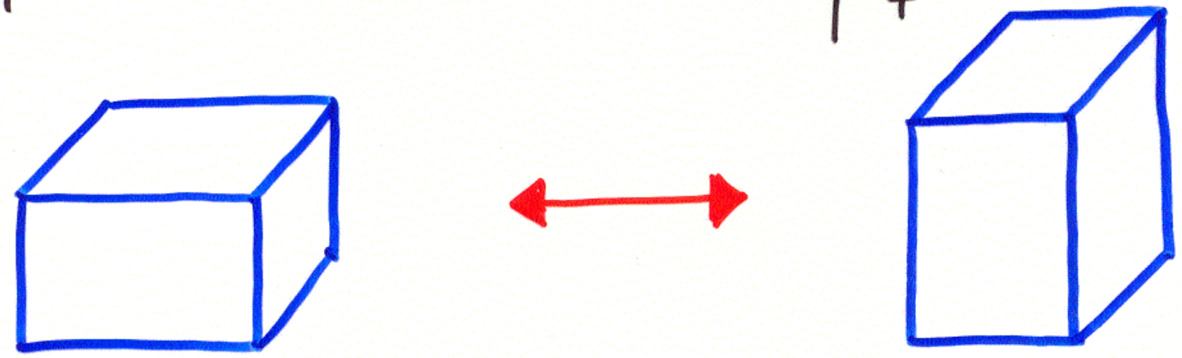
Instantons on T^4:

literature search

Nahm Duality

Dual torus

T^4 $\qquad\qquad\qquad\qquad$ $\tilde{T}^4$



periods

$L_0 \; L_1 \; L_2 \; L_3$ $\qquad\qquad$ $\frac{1}{L_0} \; \frac{1}{L_1} \; \frac{1}{L_2} \; \frac{1}{L_3}$

$SU(N)$ instanton $\qquad\qquad$ $U(k)$ instanton

top. charge k $\qquad\qquad$ charge N

Transformation involves zero modes of Dirac operator with instanton background

And Now ?

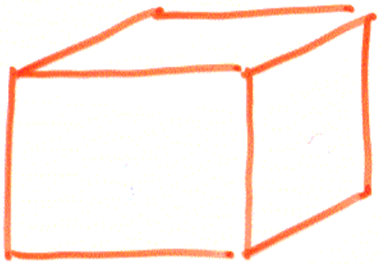
No explicit solutions on T^4:

NT doesn't really help;

maps one hard problem

to another

Way out Retreat a bit:



Take one or more

periods to be infinite

$L_0 \to \infty$	$T^3 \times R$	$\tilde{T}^3$
	$T^2 \times R^2$	$\tilde{T}^2$
Also	$S^1 \times R^3$ (calorons)	$\tilde{S}^1$
extreme case $L_0\ L_1\ L_2\ L_3 \to \infty$	R^4	$\{\cdot\}$

Dual torus lower dim

space !

A cheap lunch ?

An $SU(N)$ instanton $A_\mu(x)$ on $T^3 \times R$, $T^2 \times R^2$, $S^1 \times R^3$, R can always be Nahm-transformed to yield a self dual pot $\hat{A}_\mu^{ij}$ on $\tilde{T}^3$, $\tilde{T}^2$, $\tilde{S}^1$, $\{\cdot\}$

Such potentials are not really instantons; notion of top charge ('expected' to be N) makes no sense. But Nahm potential $\hat{A}$ has singularities. Number of sings. replaces top-charge

Plan determine 'simpler' $\hat{A}$ and then do inverse Nahm trans. to 'recover' $A_\mu(x)$

$T^2 \times R^2$ Doubly Periodic Instantons

Vortices and monopoles consistent with finite action

C. F. Jan Pawlowski

earlier work

Montero, Gonzalez Arroyo 98

Jardim 99

C. F, J. P, T. Tok, A. Wipf 00

One instanton sector, gauge group $SU(2)$. Nahm transform abelian self dual potential 2 singularities

Technically Nahm transform pot. on $\tilde{T}^2$ easy

Hard perform NT to 'recover' $T^2 \times R^2$ instanton

Restrict radially symmetric (R^2) case.

2d subspace origin of R^2

Soft zero modes $\tilde{T}^2$ zero modes singular (but square-integrable)

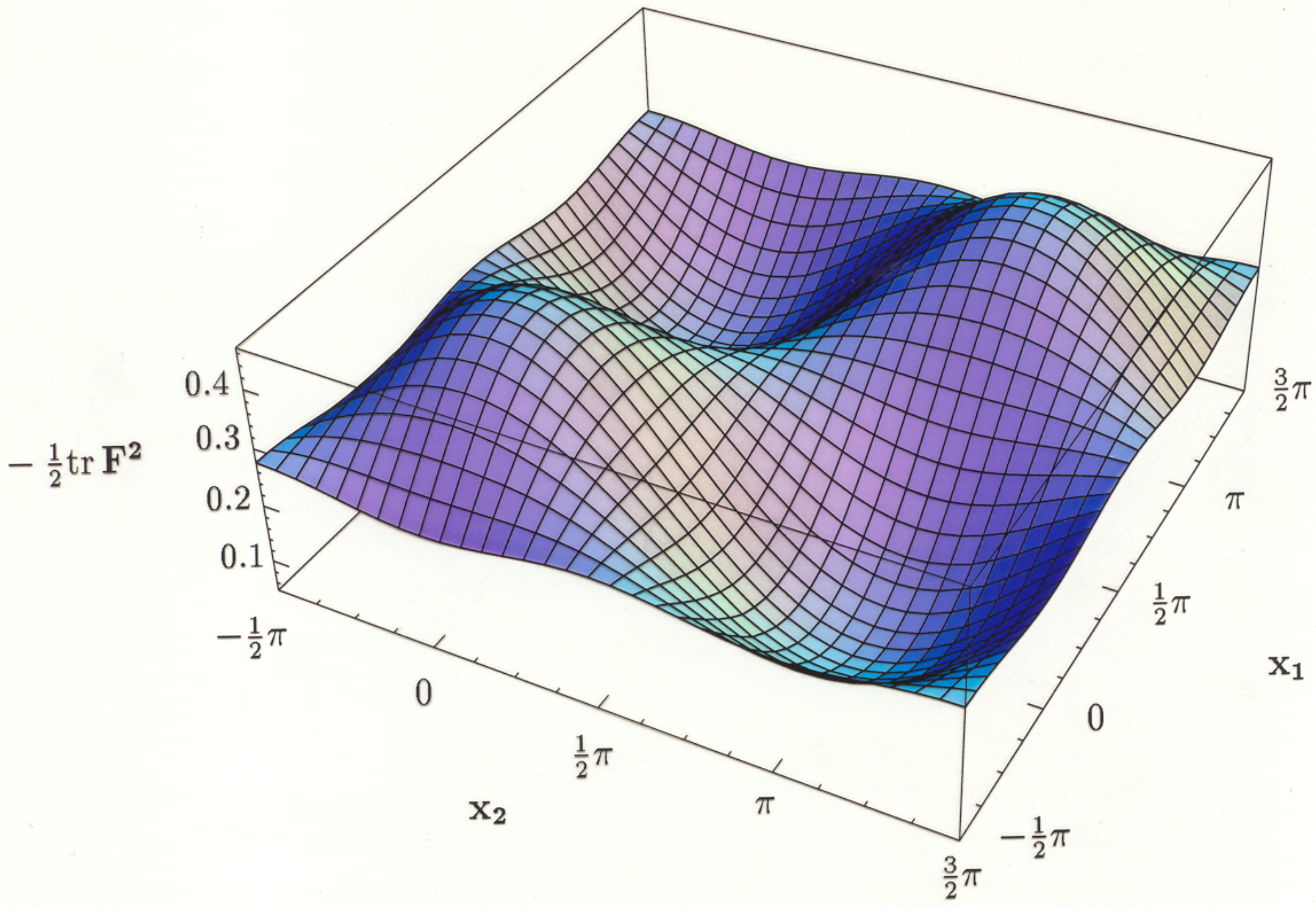
Two POINTS in $T^2 \times R^2$ zero modes SOFTER

Constituent locations ?

Plot and see !

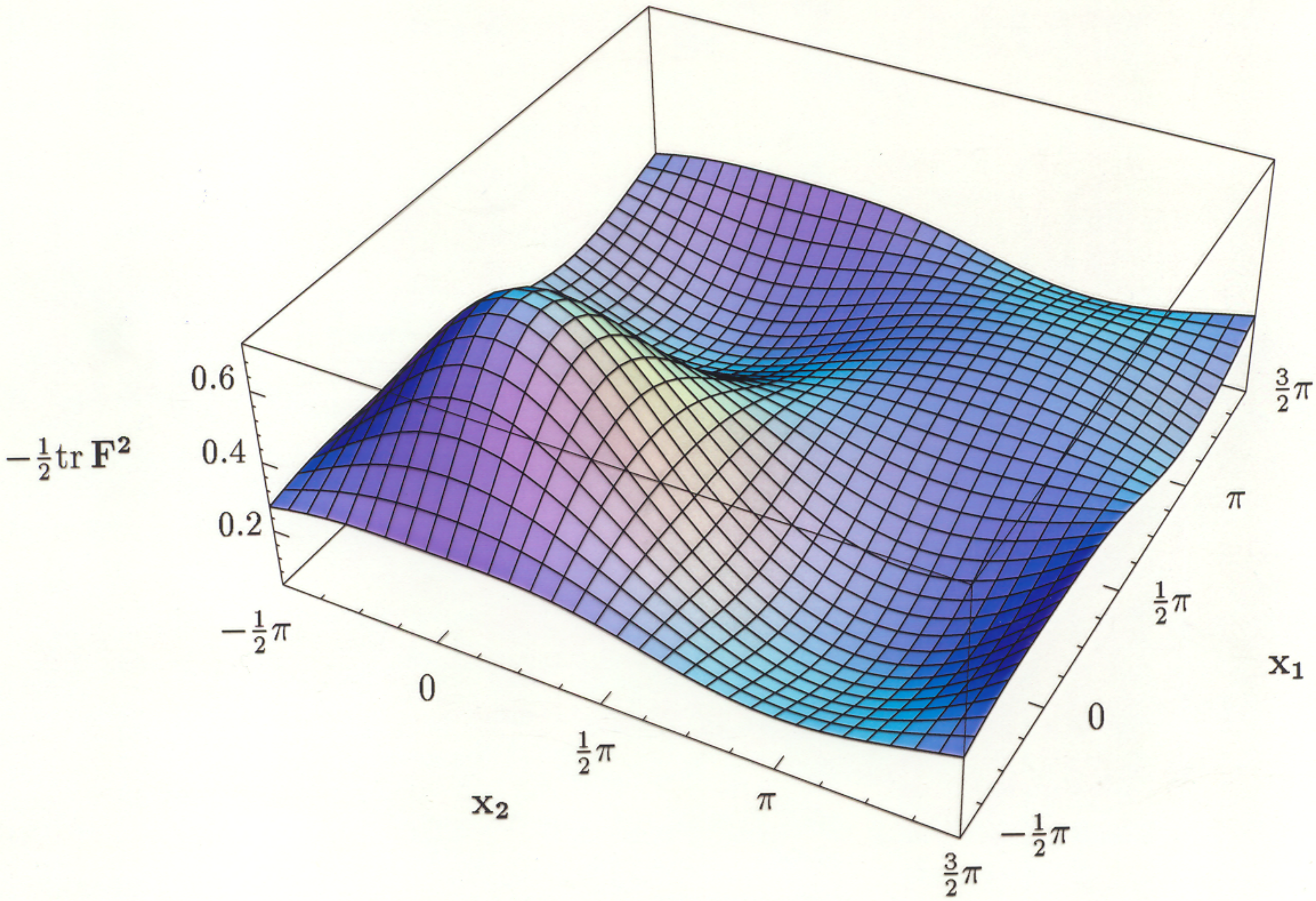
Plot of action density

$x_\perp = 0$ and $\kappa = \frac{1}{2}$ and $\omega_1 = \omega_2 = \frac{1}{4}$



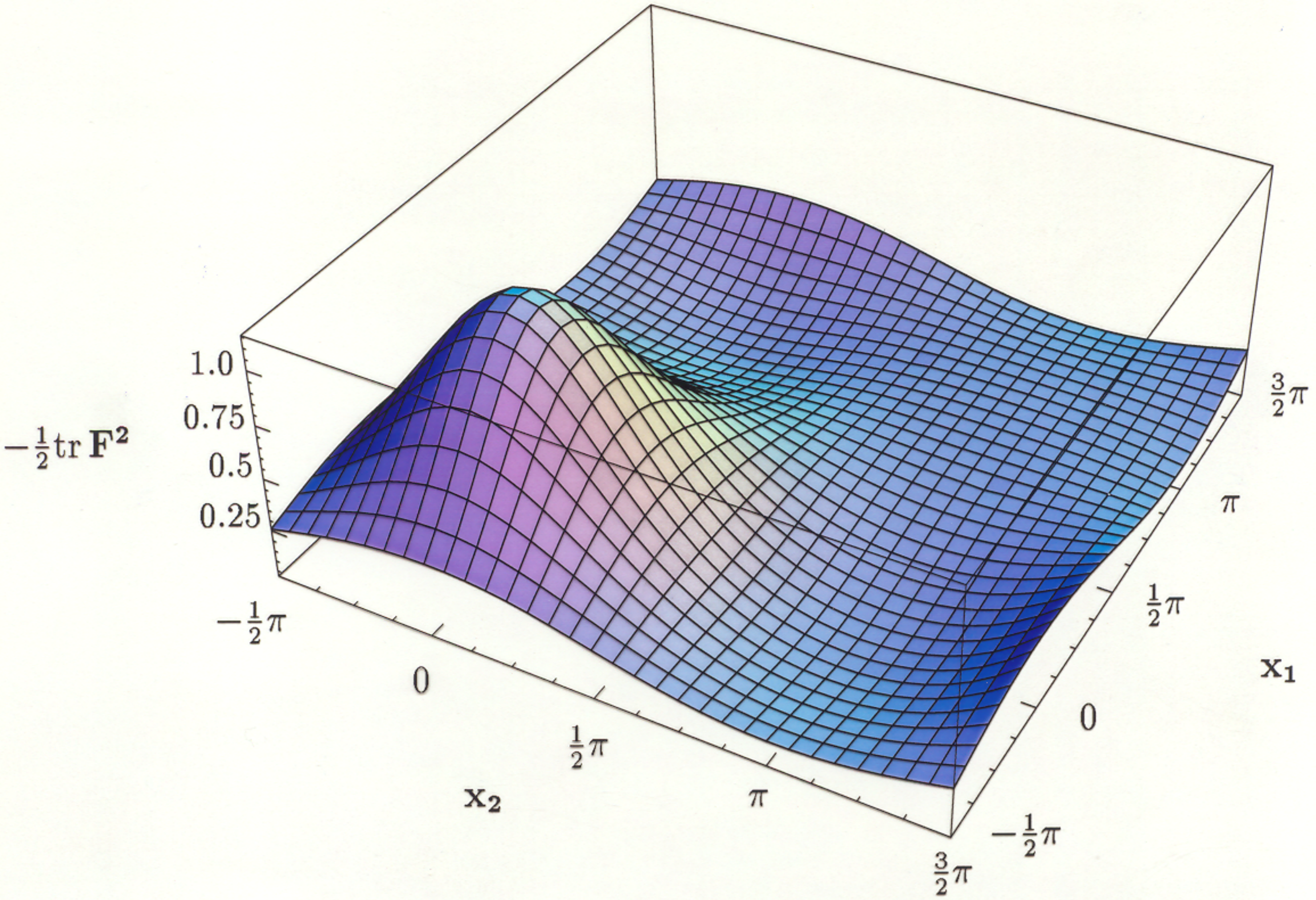
Plot of action density

$x_\perp = 0$ and $\kappa = \frac{7}{16}$ and $\omega_1 = \omega_2 = \frac{1}{4}$



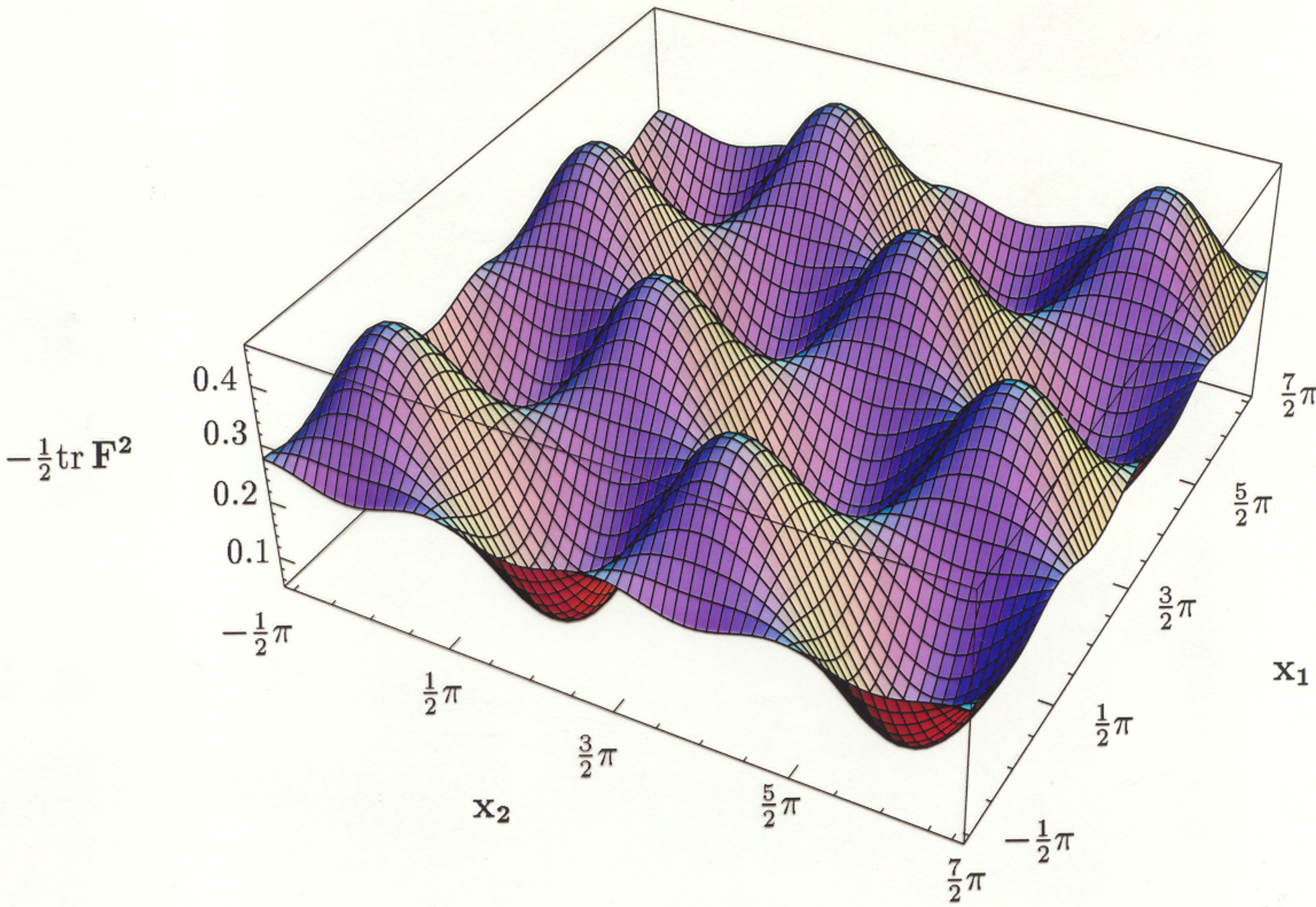
Plot of action density

$x_\perp = 0$ and $\kappa = \frac{3}{8}$ and $\omega_1 = \omega_2 = \frac{1}{4}$



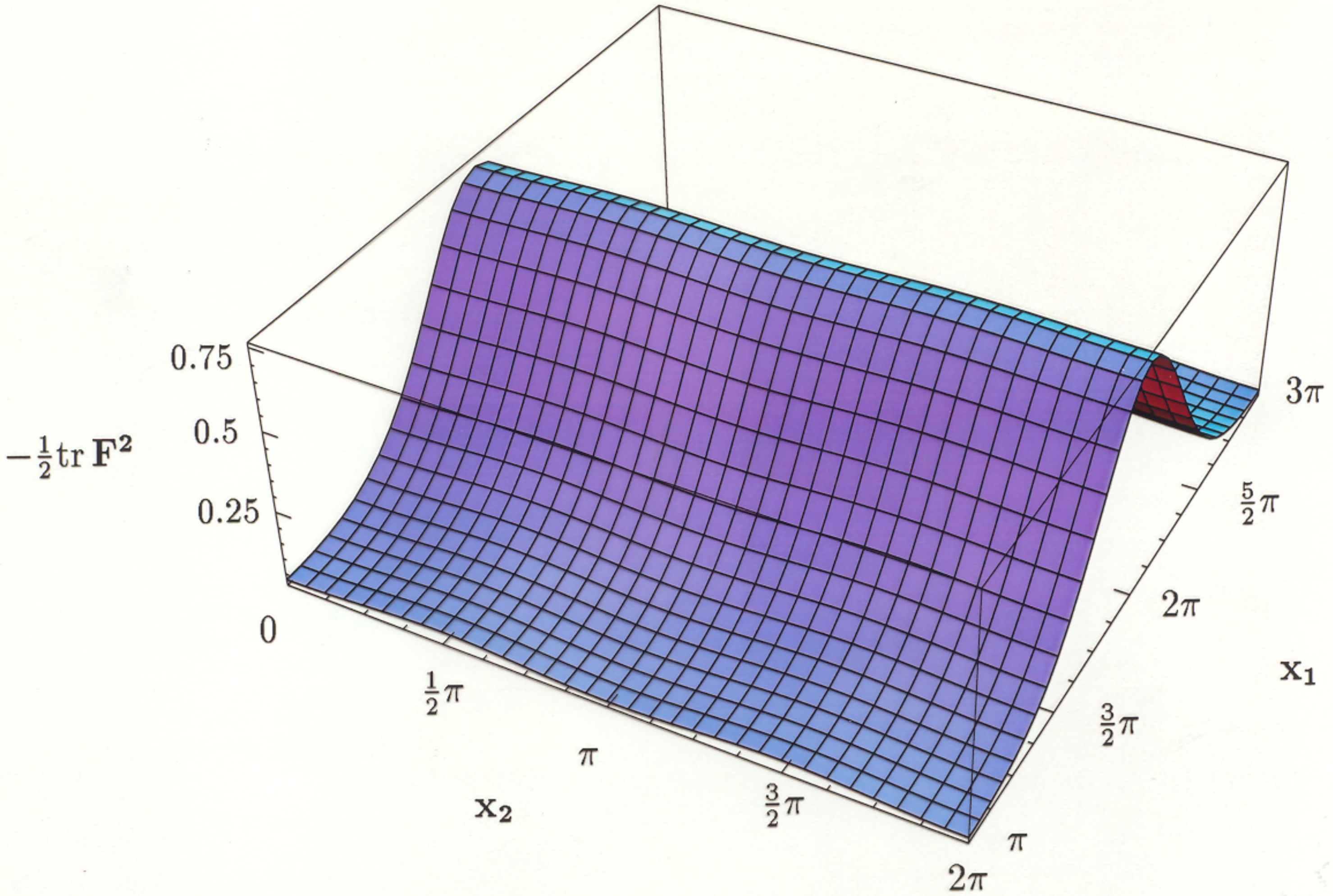
Plot of action density for two periods

$x_\perp = 0$ and $\kappa = \frac{1}{2}$ and $\omega_1 = \omega_2 = \frac{1}{4}$



Plot of action density

$x_\perp = 0$ and $\kappa = \frac{1}{2}$ and $\omega_1 = \frac{1}{4}$, $\omega_2 = 0$



What are Instantons made of ?

SU(N) charge k instantons made of <u>Nk</u> 'strongly' overlapping <u>instantons</u>

Overlap often leads to monopole / vortex structures