

ORBIFOLD BREAKING OF LEFT-RIGHT
GAUGE SYMMETRY

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S. NANDI

OKLAHOMA STATE UNIVERSITY

WORK DONE IN COLLABORATION WITH
Y. MIMURA, hep-ph/0203126 (TO APPEAR IN PLB)

! INTRODUCTION TO ORBIFOLD SYMMETRY BREAKING
(INTERESTING ALTERNATIVE TO HIGGS MECHANISM)

! EXAMPLES OF GAUGE SYMMETRY BREAKING

! APPLY TO BREAKING OF LEFT-RIGHT
SYMMETRY ON $S^1/Z_2 \times Z_2'$ ORBIFOLD

! PHENOMENOLOGICAL IMPLICATIONS

→ OUR WORK

! CONSIDER ONLY ONE EXTRA DIMENSION
AT THE TEV⁻¹ SCALE

①

INTRODUCTION

STRING MOTIVATED

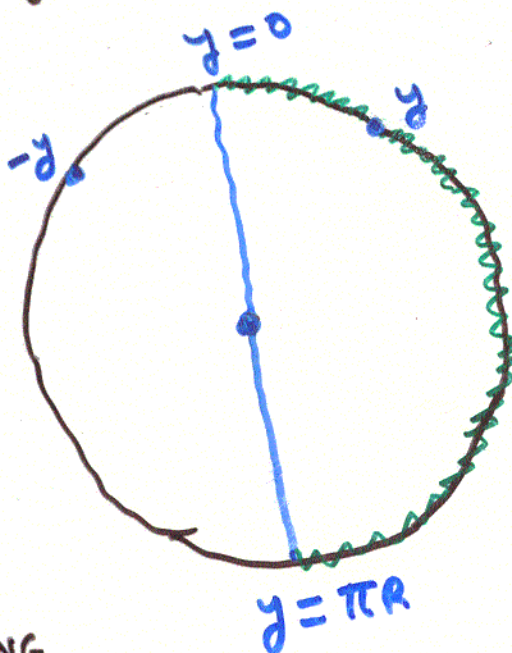
5-D FIELD THEORY : (X_μ, y) , $\mu = 0, 1, 2, 3$

$\Phi(X_\mu, y)$: Compactify y on a circle, S^1

$$5D \rightarrow M_4 \times S^1$$

: Z_2 symmetry : $y \rightarrow -y \Rightarrow \frac{S^1}{Z_2}$ orbifold

$y=0$ } Two
 $y=\pi R$ } fixed points



Physical space
in $M^4 \times \frac{S^1}{Z_2}$

SYMMETRY BREAKING

under Z_2 :

$$\Phi = \begin{pmatrix} \phi_1 \\ \phi_2 \\ \vdots \\ \phi_N \end{pmatrix}$$

$$\Phi(x, -y) = P \Phi(x, y)$$

$P =$ an $(N \times N)$ matrix,

$$P^2 = I, \text{ also } P^T P = I$$

: Z_2 -inv. does not require $P = I$ or $-I$

\Rightarrow Symmetry can be broken by giving different components of Φ different Z_2 -parity.

ORBIFOLD BREAKING OF

EXAMPLE: LOCAL (gauge) SYMMETRY

Idea: give different components of the gauge fields diff. transformation property under \mathbb{Z}_2 .

Those even under $\mathbb{Z}_2 \Rightarrow$ have zero mode

Those odd under $\mathbb{Z}_2 \Rightarrow$ no zero mode

$$\Rightarrow G \xrightarrow[\text{or}]{\text{breaks}} G'$$

Example: consider $SU(2)$ gauge symmetry

$$T_1 = \frac{1}{2} \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad T_2 = \frac{1}{2} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad T_3 = \frac{1}{2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$A_a, \quad a = 1, 2, 3$$

$$A_M(x, y) = T_a A_{Ma}(x, y), \quad M = \mu, 5$$

$$\mathcal{L}_5 = -\frac{1}{2} \text{Tr}(F_{MN} F^{MN})$$

Under Z_2 :

$$A_{\mu}(x, y) \rightarrow A_{\mu}(x, -y) = P A_{\mu}(x, y) P^{-1}$$

$$A_5(x, y) \rightarrow A_5(x, -y) = -P A_5(x, y) P^{-1}$$

Choose $P = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$, then

$$P T_1 P^{-1} = -T_1, \quad P T_2 P^{-1} = -T_2, \quad P T_3 P^{-1} = T_3$$

Now, $A_{\mu}(x, y) = T_a A_{\mu a}(x, y)$

Thus, Lagrangian to remain inv., we must have

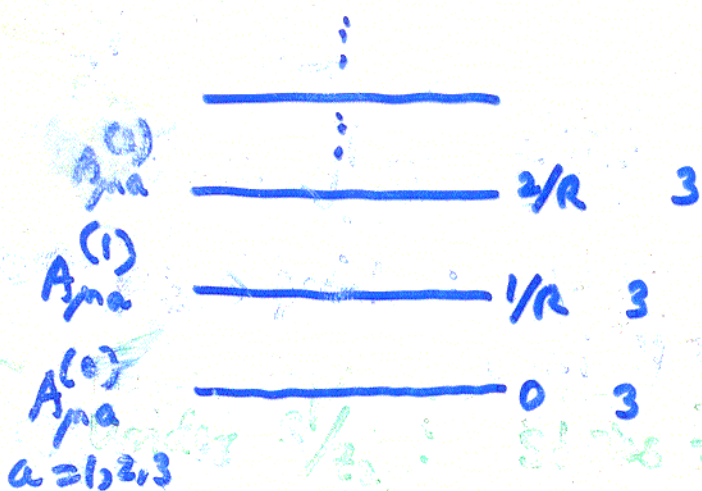
$$A_{\mu 1,2}(x, -y) = -A_{\mu 1,2}(x, y) \Rightarrow \underline{\text{no massless mode}}$$

$$A_{\mu 3}(x, -y) = A_{\mu 3}(x, y) \Rightarrow \underline{\text{have massless mode}}$$

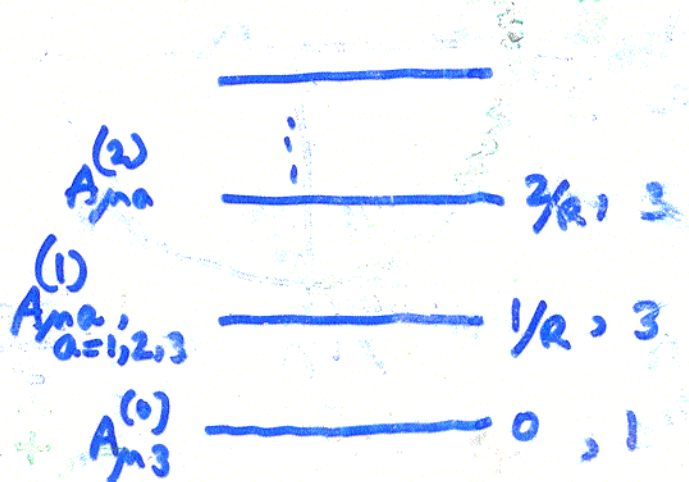
SPECTRUM UPON COMPACTIFICATION ON S^1

SPECTRUM UPON COMPACTIFICATION ON S^1/Z_2

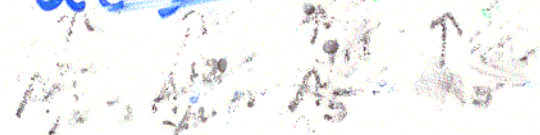
States mass #



States mass #



Under $SU(2) \xrightarrow[S^1/Z_2]{\text{broken to}} U(1)$



What happens to $A_5(x,y)$?

$$A_5(x,y) = T_a A_{5a}$$

$$A_5(x,-y) = -P A_5(x,y) P^{-1} \Rightarrow \begin{aligned} A_5^{1,2}(x,-y) &= A_5^{1,2}(x,y) \\ A_5^3(x,-y) &= -A_5^3(x,y) \end{aligned}$$

$\Rightarrow A_5^{1,2}(x,y)$: has massless mode
 $A_5^3(x,y)$: no massless mode
 \rightarrow scalar massless particle

We can make massless modes disappear from the Spectra by choose $\frac{S^1}{Z_2 \times Z_2'}$ orbifold.

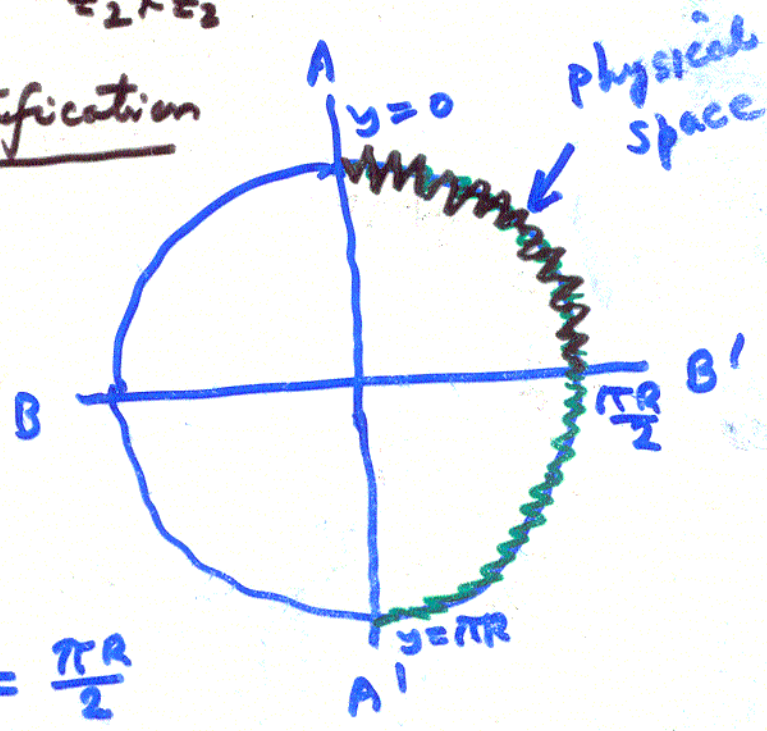
$\frac{S^1}{Z_2 \times Z_2'}$ orbifold compactification

$$Z_2: y \rightarrow -y$$

$$Z_2': y' \rightarrow -y'$$

$$\text{where } y' \equiv y + \frac{\pi R}{2}$$

Fixed points: $y=0$
and $y = \frac{\pi R}{2}$



Under S^1/Z_2 : States $\Rightarrow +, -$

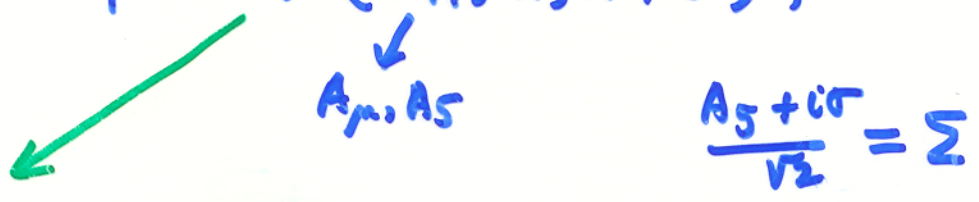
Under $S^1/Z_2 \times Z_2'$: States $\Rightarrow (++, -+, +-, --)$
 $\uparrow \quad \uparrow \quad \uparrow \quad \uparrow$
 $A_\mu^3, A_\mu^{1,2}, A_5, A_5$

SUPERSYMMETRY BREAKING

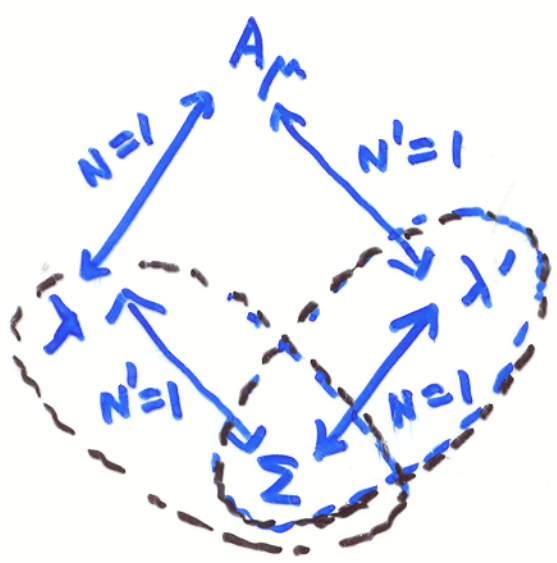
Basic idea: assign different $\mathbb{Z}_2 \times \mathbb{Z}'_2$ parity to the superpartners leading to splitting of mass degeneracy

5D SUSY:

Vector multiplet: $V(A_M, \lambda, \lambda', \Sigma)$,



Equivalent to 4D vector multiplet $V(A_M, \lambda)$ and a chiral multiplet in the adjoint representation, (λ', Σ) .

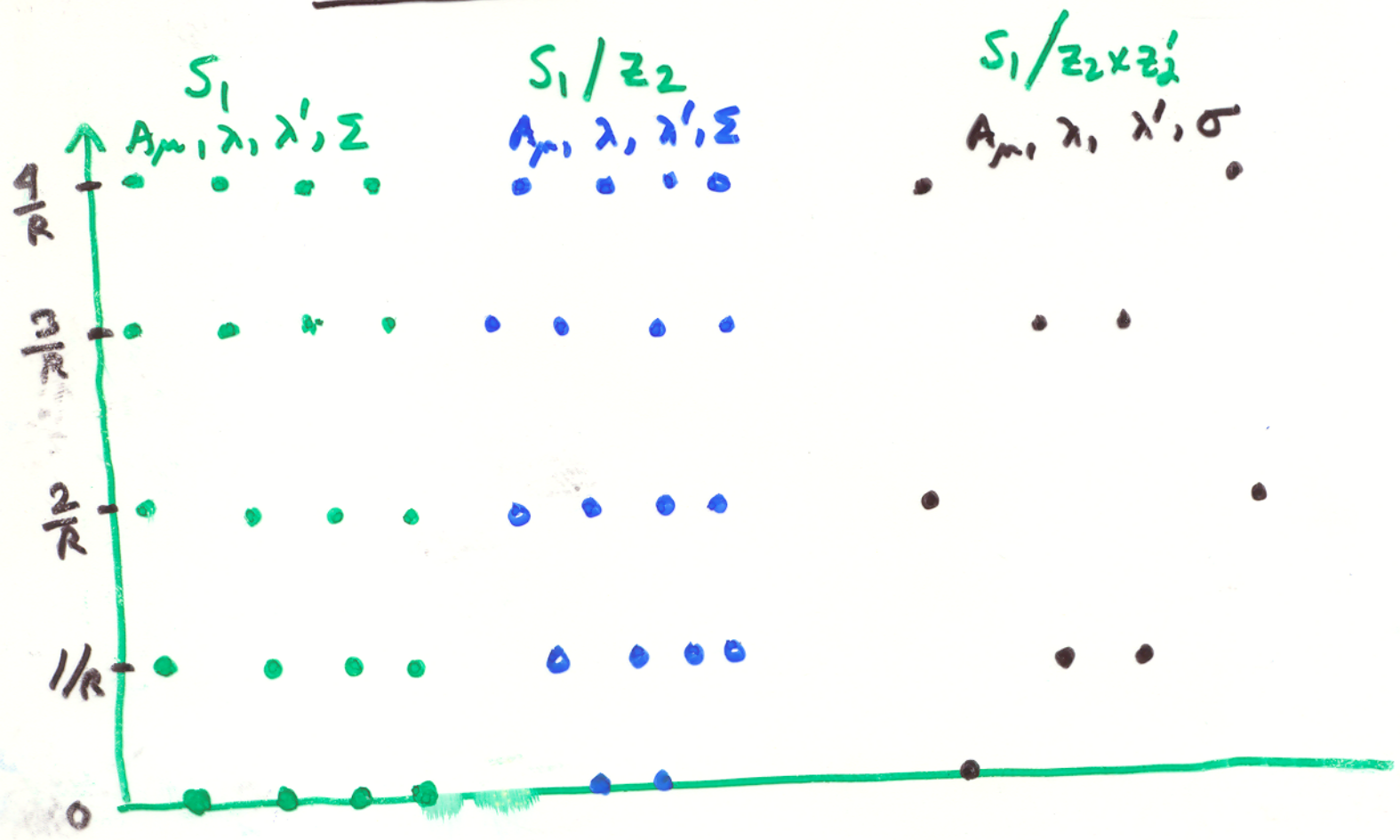


- S^1 : all have massless mode
- $\frac{S^1}{\mathbb{Z}_2}$: $V(A_M, \lambda)$: + \Rightarrow massless mode
- (λ', Σ) : - \Rightarrow no massless mode

$\frac{S^1}{\mathbb{Z}_2 \times \mathbb{Z}'_2}$: $A_M = (+, +) \rightarrow$ massless mode \Rightarrow Breaks $N'=1$ SUSY

$\left. \begin{matrix} \lambda = (+, -) \\ \lambda' = (-, +) \\ \Sigma = (-, -) \end{matrix} \right\} \rightarrow$ no massless mode \Rightarrow Breaks both supersymmetry

SPECTRUM



ORBIFOLD BREAKING OF LEFT-RIGHT GAUGE SYMMETRY

(Y. Mimura + S. Nandi)
hep-ph/0203126

$$SU(2)_L \times SU(2)_R \times U(1)_{B-L}$$

$\nearrow g_L \quad \nearrow g_R \quad \nearrow g'$

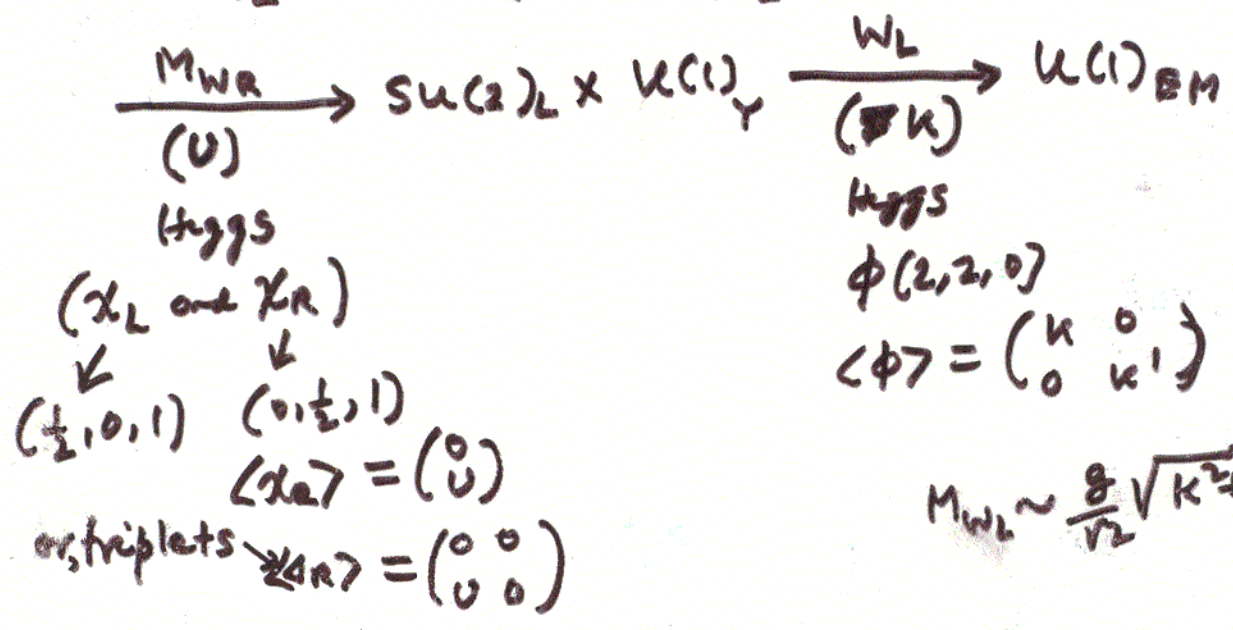
$L \leftrightarrow R : g_L = g_R \equiv g$

Two couplings : g, g'

$$Q = T_{3L} + T_{3R} + \frac{B-L}{2} \equiv \frac{Y}{2}$$

$(W_L^+, W_{3L}), (W_R^\pm, W_{3R}), B$

$$SU(2)_L \times SU(2)_R \times U(1)_{B-L}$$



$$M_{WL} \sim \frac{g}{\sqrt{2}} \sqrt{k^2 + k'^2}$$

$$M_{WR} \sim \frac{g v}{\sqrt{2}}$$

if $v \gg (k, k')$
 $M_{WR} \gg M_{WL}$

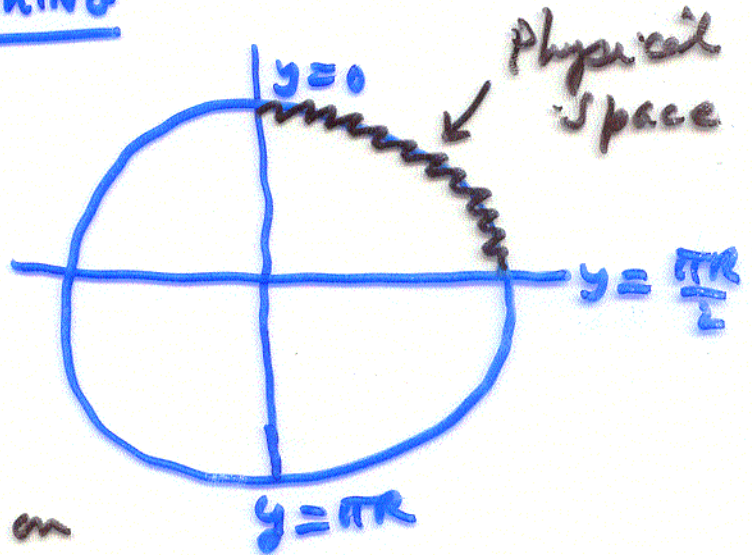
Left-right gauge symmetry

⇒ : Natural explanation of parity violation at low energy ($M_{NR} \gg M_{NL}$)

: non-vanishing of ν -masses

LEFT-RIGHT GAUGE SYMMETRY IN 5D AND ORBIFOLD BREAKING

S^1
 $Z_2 \times Z_2$ orbifold



Assume

- : Gauge & Higgs fields propagate into bulk
- : Fermions are confined on the 4D wall at $y=0$

Symmetry Breaking :

$SU(2)_L \times SU(2)_R \times U(1)_{B-L}$

↓ orbifold compactification, $\frac{1}{R}$

$SU(2)_L \times U(1)_R \times U(1)_{B-L}$

↓ Higgs $\chi_R (1, 2, \frac{1}{2}) \rightarrow U$

$SU(2)_L \times U(1)_Y$

↓ Higgs $\Phi (2, 2, 0) \rightarrow K$

$U(1)_{EM}$

(7)

We impose the following transformation for the 5D gauge fields:

$$Z_2: V_\mu(x, -y) = P V_\mu(x, y) P^{-1}$$

$$V_5(x, -y) = -P V_5(x, y) P^{-1}$$

$$Z_2': V_\mu(x, -y') = P' V_\mu(x, y') P'^{-1}$$

$$V_5(x, -y') = -P' V_5(x, y') P'^{-1}$$

5D Lagrangian is inv. under these tr.

$$V_\mu = T_a V_{\mu a}, \quad V_5 = T_a V_{5a}$$

For $SU(2)_L$ and $U(1)_{B-L}$ gauge fields,
we choose, $P = P' = I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

For $SU(2)_R$ gauge fields,

we choose $P = I, P' = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

$$V_R = \frac{1}{\sqrt{2}} \begin{pmatrix} W_{3R} & \sqrt{2} W_R^+ \\ \sqrt{2} W_R^- & -W_{3R} \end{pmatrix}$$

Then, under $Z_2 \times Z_2'$:

$$W_{R3} : (+, +), \quad W_R^\pm : (+, -)$$

$$W_5^3 : (-, -), \quad W_5^\pm : (-, +)$$

(8)

So, the fields can be Fourier expanded as

$$W_{R3}(x,y) = \sqrt{\frac{2}{\pi R}} \left[W_{R3}^{(0)}(x) + \sqrt{2} \sum_{n=1}^{\infty} W_{R3}^{(n)} \cos \frac{2ny}{R} \right], \text{ mass} = \frac{2n}{R}$$

$$W_R^{\pm}(x,y) = \frac{2}{\sqrt{\pi R}} \left[\sum_{n=0}^{\infty} W_R^{\pm(n)}(x) \cos \frac{(2n+1)y}{R} \right]; \text{ mass} = \frac{(2n+1)}{R}$$

$$W_S^{\pm}(x,y) = \frac{2}{\sqrt{\pi R}} \left[\sum_{n=0}^{\infty} W_S^{\pm(n)}(x) \sin \frac{(2n+1)y}{R} \right]; \text{ mass} = \frac{(2n+1)}{R}$$

$$W_S^3(x,y) = \frac{2}{\sqrt{\pi R}} \left[\sum_{n=0}^{\infty} W_S^3(n)(x) \sin \frac{(2n+2)y}{R} \right]; \text{ mass} = \frac{(2n+2)}{R}$$

⇒ only W_{R3} has massless mode

⇒ $SU(2)_R$ is broken to $U(1)_R$.

Now, we use Higgs fields to break

$$SU(2)_L \times U(1)_R \times U(1)_{B-L} \rightarrow SU(2)_L \times U(1)_Y \rightarrow U(1)_{EM}$$

need: $\chi_R(1, 2, \frac{1}{2}), \Phi_1(2, 2, 0)$

under $\mathbb{Z}_2 \times \mathbb{Z}'_2$:

$$\chi_R(x, -y) = P \chi_R(x, y)$$

$$\chi_R(x, -y') = -P' \chi_R(x, y)$$

$$\Phi_1(x, -y) = \Phi_1(x, y) P^{-1}$$

$$\Phi_1(x, -y') = \Phi_1(x, y) P'^{-1}$$

In order to break sym., χ_R and $\phi(2,2)$ must have VEVs \Rightarrow must have zero modes.

$$\chi_R = \begin{pmatrix} \chi_R^+ \\ \chi_R^0 \end{pmatrix}, \quad \phi = \begin{pmatrix} \phi_{11}^+ & \phi_{12}^+ \\ \phi_{21}^- & \phi_{22}^0 \end{pmatrix}$$

Choose $P = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, $P' = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

Then, under $Z_2 \times Z_2'$:

$$\chi_R^+ : (+, -), \quad \chi_R^0 : (+, +) \leftarrow \text{massless mode}$$

\Rightarrow So χ_R^0 can have VEV

$$\phi : \begin{pmatrix} \phi_{11}^+ \\ \phi_{21}^- \end{pmatrix} : (+, +), \quad \begin{pmatrix} \phi_{12}^+ \\ \phi_{22}^0 \end{pmatrix} : (+, -)$$

massless mode, can have VEV

no massless mode, can't have VEV.

$$\text{So: } \langle \chi_R \rangle = \begin{pmatrix} 0 \\ v \end{pmatrix}, \quad \langle \phi \rangle = \begin{pmatrix} k & 0 \\ 0 & 0 \end{pmatrix}$$

In the usual L-R model:

$$\langle \Phi(2,2,0) \rangle = \begin{pmatrix} k & 0 \\ 0 & k' \end{pmatrix}$$

L-R mixing $\sim k k'$

So, in orbifold compactified model

\Rightarrow NO L-R mixing

GAUGE BOSON-FERMION COUPLINGS

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Fermion multiplets are same as in the usual L-R model:

$$\begin{pmatrix} u \\ d \end{pmatrix}_L = \left(\frac{1}{2}, 0, \frac{1}{3}\right), \quad \begin{pmatrix} u \\ d \end{pmatrix}_R = \left(0, \frac{1}{2}, \frac{1}{3}\right)$$

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L = \left(\frac{1}{2}, 0, -1\right), \quad \begin{pmatrix} \nu_e \\ e \end{pmatrix}_R = \left(0, \frac{1}{2}, -1\right)$$

We assume fermions are confined to the 4D wall at one of the fixed points, $y=0$.

: Write down the gauge interaction in 5D, supplemented by $\delta(y)$, and integrate over y

$$\Rightarrow \mathcal{L}_4 = \frac{g}{\cos\theta_W} \sum_{i\mu}^{(\text{neutral})} \left[A_i^{(\mu)} J_{V-A}^\mu + B_i^{(\mu)} J_{V+A}^\mu \right]$$

where $J_{V\mp A}^\mu = \bar{\psi} \gamma^\mu \frac{1}{2} (1 \mp \gamma_5) \psi$

$Z_1^{(\mu)}, Z_2^{(\mu)} \Rightarrow$ KK excitations of $Z_1^{(0)}, Z_2^{(0)}$

$Z_3^{(\mu)} \Rightarrow$ KK excitation of the photon.

PHENOMENOLOGICAL IMPLICATIONS

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Qualitative differences with the usual L-R Model

① : KK excitations of both Left and right handed gauge bosons

: Lowest W_R state is a KK excitation with mass = $\frac{1}{R}$

: Lowest KK excitation of W_L has mass = $\frac{2}{R}$

We shall see that current data restrict $\frac{1}{R}$ can be as low as $\sim 2\text{TeV}$.

\Rightarrow At LHC, we may observe not only lightest W_R , but one or two KK excitations.

\Rightarrow Show evidence of higher dimensions.

② Unlike usual L-R model,

no left-right mixing in the charged sector.

\Rightarrow : $W_R \not\rightarrow W_L \gamma$

: no asymmetry in polarized muon decay.

③ $W_R^{(0)}$ is a KK state
 \Rightarrow its coupling to fermion is larger by a factor $\sqrt{2}$ than in usual L-R model.

$$\Gamma_{W_2^{(0)}} = 2 \text{ (usual L-R model)}$$

$$\Gamma_{W_2^{(0)}} = 2 \text{ (" ")}$$

But not for $Z_2^{(0)}$.

\Rightarrow can easily distinguish from usual L-R model.

④ For our model,

$$R \equiv \frac{M_{W_2^{(0)}}^2 \cos^2 \theta_W}{M_{Z_2^{(0)}}^2 \cos 2\theta_W} = 1 + \frac{1/R^2}{\frac{1}{2} g^2 V^2} > 1$$

whereas for usual L-R model, $R = 1$
 (for the same Higgs multiplets)

Also, since $1/R > gV/\sqrt{2}$

$\therefore W_2^{(0)}$ is expected to be heavier than $Z_2^{(0)}$
 whereas in usual L-R model, W_R is lighter than Z_R .

⑤ Less contribution to neutrinoless double beta decay (no L-R mixing).

LIMITS ON $Z_2^{(0)}$, $W_2^{(0)}$ MASSES AND ON $1/R$
FROM CURRENT DATA

$Z_2^{(0)}$ Mass: Use precision LEP data:

We used F-B asymmetry in $e^+e^- \rightarrow Z_{pole} \rightarrow e^+e^-$

$$\rho \equiv \frac{M_{W_1}^2}{M_{Z_1}^2 \cos^2 \theta_W}$$

For our model, $\rho = 1 + \Delta\rho_{LR} + \Delta\rho_{SM}$

$$\Delta\rho_{LR} \approx \cos 2\theta_W \frac{M_{Z_1}^2}{M_{Z_2}^2}, \quad \Delta\rho_{SM} \approx 3G_F M_t^2 / 8\sqrt{2}\pi^2$$

$$\text{Also, } \rho \equiv \frac{M_{W_1}^2}{\cos^2 \theta_W M_{Z_1}^2} = \frac{M_{W_1}^2}{M_{Z_1}^2} \frac{4}{3 + \frac{g_V}{g_A} (1 + \Delta\phi)}$$

$$\text{where } \Delta\phi \approx -(1 + \cos 2\theta_W) \frac{M_{Z_1}^2}{M_{Z_2}^2}$$

$$\text{Use } A_e = \frac{2g_V g_A}{g_V^2 + g_A^2} = 0.15138 \pm 0.00216$$

from F-B asymmetry

$$\Rightarrow M_{Z_2^{(0)}} > 935 \text{ GeV at } 95\% \text{ CL}$$

$$\text{and } \frac{g_V}{\sqrt{2}} > 780 \text{ GeV at } 95\% \text{ CL}$$

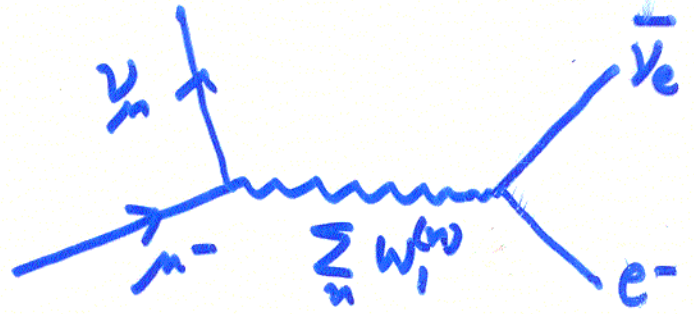
CONSTRAINTS ON $W_2^{(0)}$ MASS AND $1/R$:

- : TWO WAYS
- ① muon decay
 - ② $K^0 - \bar{K}^0$ mixing

① muon decay :

For our model, we get:

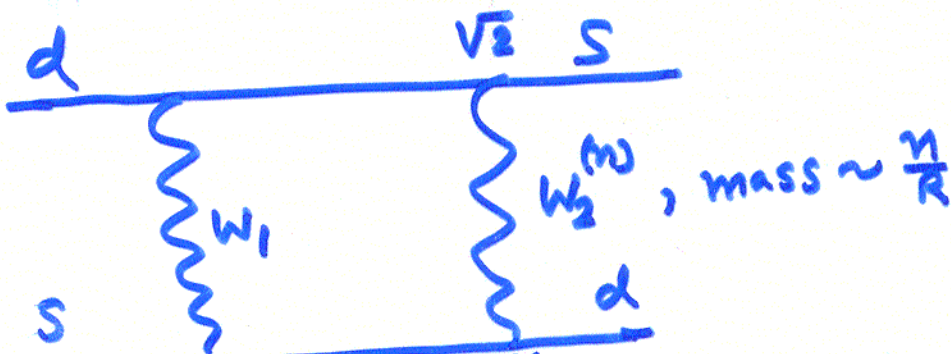
$$1 + 0.082 \Delta\phi + \frac{\pi^2}{12} M_{W_1}^2 R^2 = 0.99928 \pm 0.0015$$



$$\Rightarrow \frac{1}{R} > 1.2 \text{ TeV at } 95\% \text{ CL}$$

② $K^0 - \bar{K}^0$ mixing :

: receives additional contributions



$$\frac{1}{\langle M_{W_2}^2 \rangle} = 2 \sum \frac{1}{M_{W_2}^2} = \frac{\pi}{2} \frac{R^2}{d} \tan\left(\frac{\pi}{2} d\right)$$

where $d = R \sqrt{\frac{g^2}{2} (V^2 + K^2)}$

$$\Rightarrow M_{W_2^{(0)}} > 2.5 \text{ TeV} \Rightarrow \frac{1}{R} > 1.9 \text{ TeV}$$

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Also: in order to give mass to both up and down type quarks, need another $\phi(2, 2, 0)$ with $\langle \phi_1 \rangle = \begin{pmatrix} k_1 & 0 \\ 0 & 0 \end{pmatrix}$, $\langle \phi_2 \rangle = \begin{pmatrix} 0 & 0 \\ 0 & k_2 \end{pmatrix}$

GAUGE BOSON MASS MATRICES

Charged sector:

$$\begin{pmatrix} W_L^{+(n)}, W_R^{+(n)} \end{pmatrix} \begin{pmatrix} \frac{g^2}{2} k^2 + \left(\frac{2n}{\kappa}\right)^2 & 0 \\ 0 & \frac{g^2}{2} (k^2 + v^2) + \left(\frac{2n+1}{\kappa}\right)^2 \end{pmatrix} \begin{pmatrix} W_L^{-(n)} \\ W_R^{-(n)} \end{pmatrix}$$

where $k^2 = k_1^2 + k_2^2$.

\Rightarrow no left-right mixing

$\Rightarrow M_{W_L}^2, M_{W_R}^2$

Mass matrix for the neutral sector:

$$\frac{1}{2} \sum_{n=0}^{\infty} (W_{3L}^{(n)}, W_{3R}^{(n)}, B^{(n)}) M_{(n)}^2 \begin{pmatrix} W_{3L}^{(n)} \\ W_{3R}^{(n)} \\ B^{(n)} \end{pmatrix}$$

$$M_{(n)}^2 = \begin{pmatrix} \frac{g^2}{2} k^2 + \left(\frac{2n}{R}\right)^2 & -\frac{g^2}{2} k^2 & 0 \\ -\frac{g^2}{2} k^2 & \frac{g^2}{2} (k+v^2) + \left(\frac{2n}{R}\right)^2 & -\frac{gg'}{2} v^2 \\ 0 & -\frac{1}{2} gg' v^2 & \frac{g'^2}{2} v^2 + \left(\frac{2n}{R}\right)^2 \end{pmatrix}$$

\Rightarrow no KK level mixing

$$\Rightarrow M_{A^{(0)}}^2 = 0$$

$$M_{Z_1^{(0)}}^2 = \frac{M_{W_1^{(0)}}^2}{\cos^2 \theta_W} \left\{ 1 - (1 - \tan^2 \theta_W) \frac{k^2}{v^2} \right\}$$

$$M_{A^{(n)}}^2 \simeq M_{Z_1^{(n)}}^2 \simeq \left(\frac{2n}{R}\right)^2, \quad n \geq 1$$

$$M_{Z_2^{(n)}}^2 \simeq \left(\frac{2n}{R}\right)^2 + \frac{\cos^2 \theta_W}{2 \cos 2\theta_W} g^2 v^2, \quad n \geq 0$$

$\theta_W =$ weak mixing angle,

$$\tan \theta_W = \frac{g_Y}{g}, \quad g_Y = \frac{gg'}{\sqrt{g^2 + g'^2}}$$

CONCLUSIONS

: GAUGE SYM. BREAKING USING ORBIFOLD COMPACTIFICATION IS AN INTERESTING ALTERNATIVE TO HIGGS MECHANISM

: APPLIED TO L-R GAUGE SYM IN 5D

: $SU(2)_R$ (AS WELL AS PARITY) IS BROKEN TO $U(1)_R$ UPON COMPACTIFICATION

: SEVERAL DISTINGUISHING FEATURES FROM THE USUAL L-R MODEL IN 4D

⇒ EXISTENCE OF KK EXCITATIONS

⇒ NO L-R MIXING

⇒ LIGHTEST W_R IS A KK STATE, $Z_R^{(0)}$ IS NOT

⇒ BOUNDS ON $W_2^{(0)}$, $Z_2^{(0)}$ LOW ENOUGH SO THAT LHC CAN PRODUCE THOSE, AS WELL AS 1 OR 2 KK EXCITATIONS ⇒ EXISTENCE OF EXTRA DIM

⇒ PRODUCTION CROSS SECTIONS AS WELL AS DECAY WIDTH LARGE BY 2 FOR $W_2^{(0)}$, BUT NOT FOR $Z_2^{(0)}$.