Reggeon exchange from AdS/CFT

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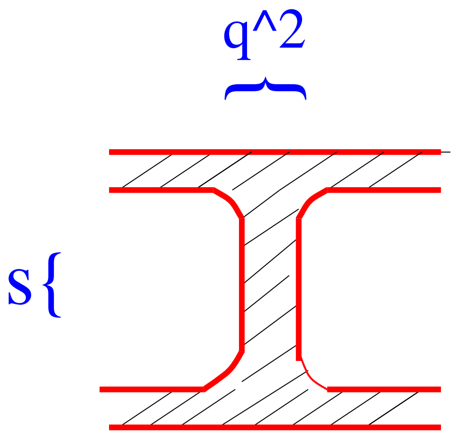
Contents

A 35-years-old Puzzle

A String Theory for **Strong** Interactions ?

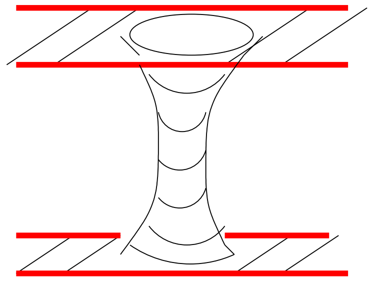
Strings $\Rightarrow$ Gauge Fields $\Rightarrow$ Strings

1968 $\Rightarrow$ 1974 $\Rightarrow$ 1998



Veneziano Amplitude

$A_R(s,q^2)$



Shapiro-Virasoro Amplitude

$A_P(s,q^2)$

A 35-years-old Puzzle

Scattering Amplitudes: are they 4-D String Amplitudes?

Problems before 1974 ...

- **Conformal anomaly: D=26,10**
- **Gauge, Gravitation in the spectrum:**
$$\alpha_{\mathcal{R}} = 1 \ , \ \alpha_{\mathcal{P}} = 2$$
- **Zero mass asymptotic states**
- **Strong Interaction Phenomenology: not quantitatively described**

...And after 1974

- **Where is QCD_4 in String Theory?**
- **Can we elaborate a String Theory out of Gauge Fields: Cf. Polyakov**

AdS/CFT Correspondence

- $1^{rst}\,Step:$ **Duality Property for Conformal theories:**

  (Super)String / $SU(N)$ Gauge Theory

  AdS Backg. / CFT $(\Rightarrow \mathcal{N}_{SUSY} = 4)$

  Weak coupling / Strong Coupling

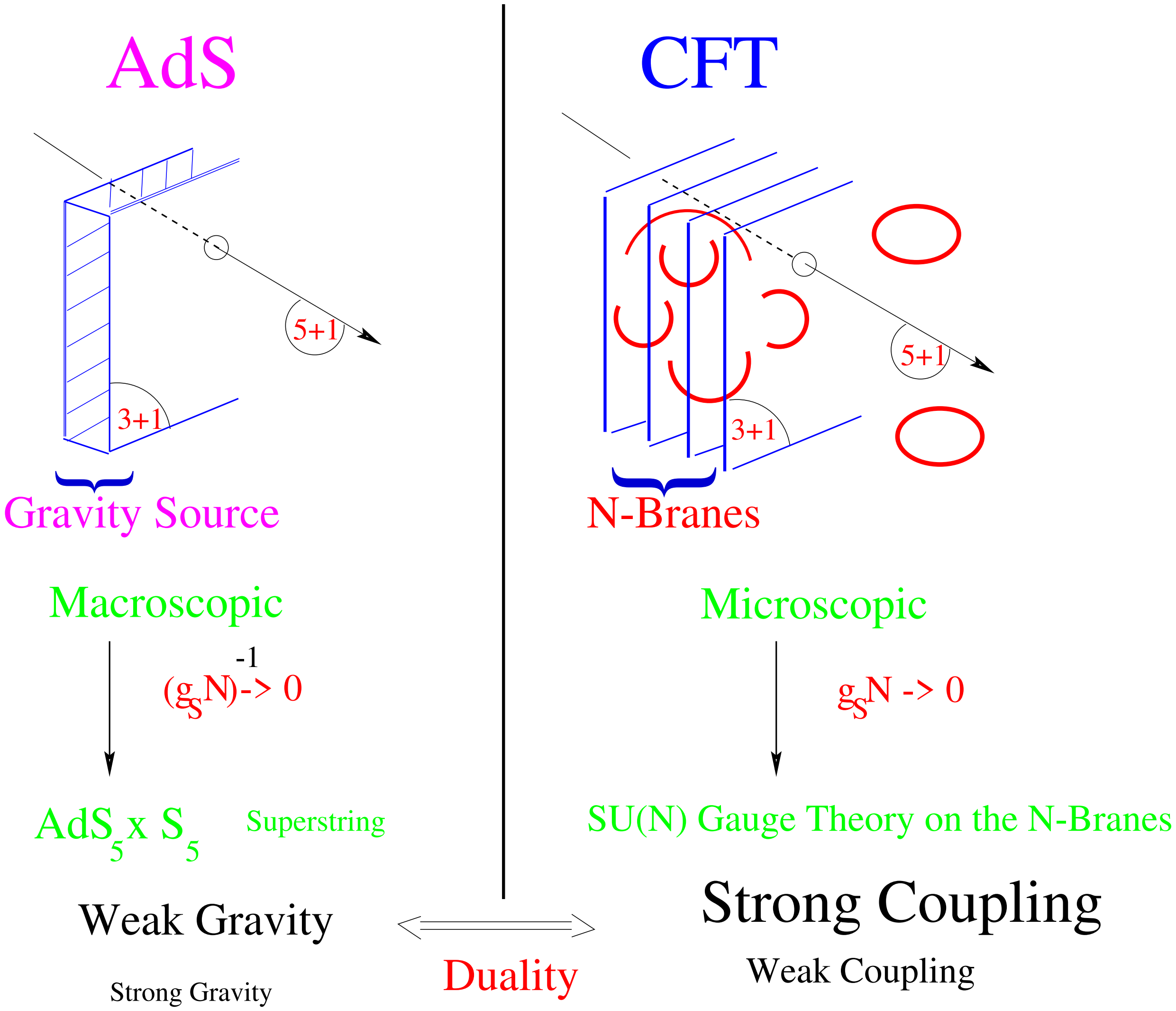
- 2^{nd} **Step: Duality extended to Confining Theories**

  (Super)String / Confinement scale

  AdS Black Hole / $\mathcal{N}_{SUSY} = 2, 1, 0$

  Weak coupling / Strong Coupling

$\mathbf{AdS_5}$ / $\mathbf{CFT_4}$



More on AdS$_5$

- **D_3-brane Solution of Super Gravity:**
  $\equiv$ low energy IIB SuperString

$$ds^2 = f^{-1/2}(-dt^2 + \sum_{1-3} dx_i^2) + f^{1/2}(dr^2 + r^2 d\Omega_5)$$

"On-Branes$\times$Out-Branes"

$$f = 1 + \frac{R^4}{r^4} \; ; \; R = 4\pi g_{YM}^2 \alpha'^2 N$$

- **"Maldacena limit":**

$$\frac{\alpha'(\to 0)}{r(\to 0)} \to z \; , \; R \; fixed \; \Rightarrow g_{YM}^2 N \to \infty$$

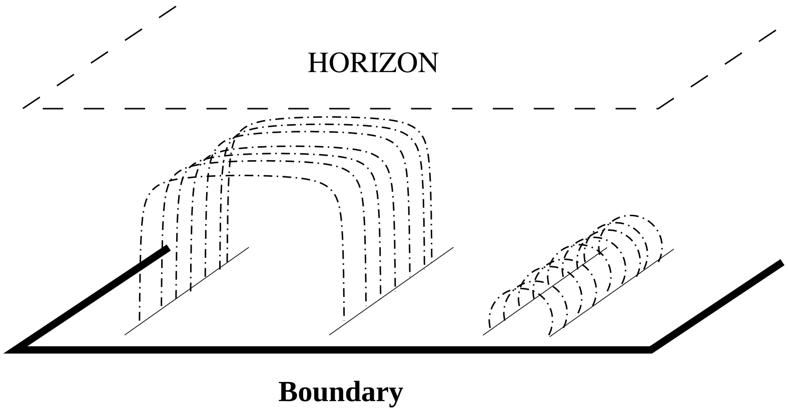
Strong coupling limit

$$ds^2 = \frac{1}{z^2}(-dt^2 + \sum_{1-3} dx_i^2 + dz^2) + R^2 d\Omega_5$$

Background Structure: AdS$_5$ $\times$ S_5

Writing the Correspondence:

Exemple of $\langle Wilson\ Lines \rangle$



$$\langle e^{iP \int_C \vec{A}\cdot\vec{dl}} \rangle = \int_\Sigma e^{-\frac{Area(\Sigma)}{\alpha'}} \approx e^{-\frac{Min.Area}{\alpha'}} \times Fluct.$$

- **Conformal case :** AdS_5

$$\langle Wilson\ Lines \rangle = e^{V(L)} \sim e^{1/L}$$

- **Confining case :** $AdS - BlackHole$

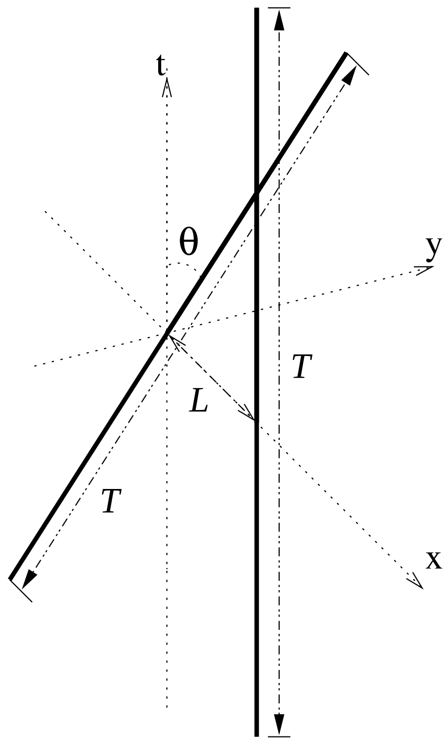
$$\langle Wilson\ Lines \rangle = e^{V(L)} \sim e^{L}$$

Conclusion: AdS/CFT **a tool for Strong Coupling Gauge Theories**

Supergravity Dual of Scattering Amplitudes

- $q - q(\ or\ \bar{q}) scattering$

$$A(s,q^2) = 2is \int d\vec{l}\ e^{i\vec{q}\cdot\vec{l}}\ \langle W_1 W_2 \rangle_{L=|\vec{l}|}^{\chi=\log s/m^2}$$



- **Analytic Continuation:**

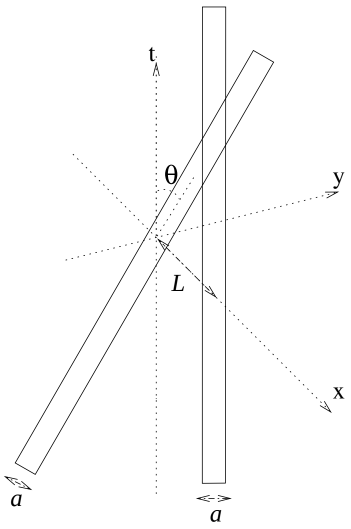
$$\chi \rightarrow i\theta \quad ; \quad t \rightarrow -it_{Eucl.}$$

Conclusion: **Helicoidal** **Geometry in Euclidean AdS Spaces**

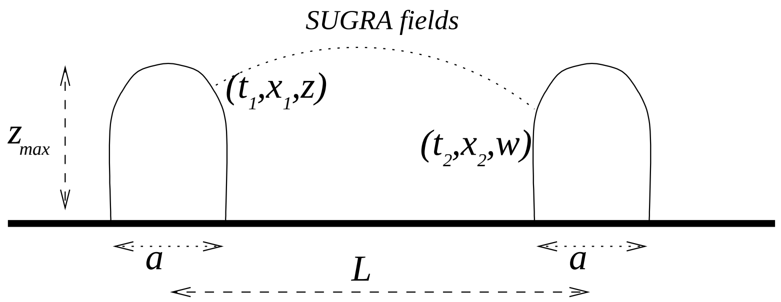
Elastic Amplitude

- **Dipole-Dipole scattering**

$$A_{\mathcal{P}}(s,q^2) = -2is \int d\vec{l}\; e^{i\vec{q}\cdot\vec{l}} \left\langle \frac{W_1 W_2}{\langle W_1\rangle \langle W_2\rangle} - 1 \right\rangle$$



- **Minimal Area and Supergravity limit**

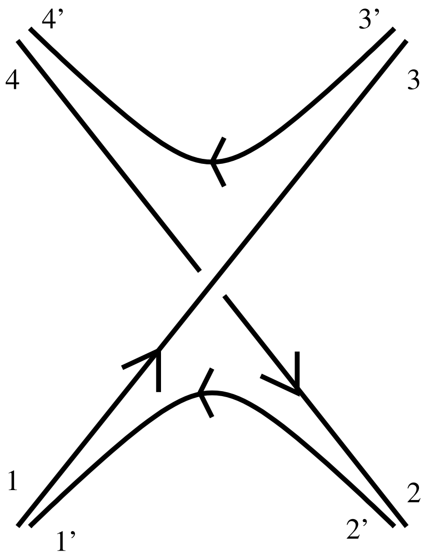


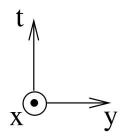
Conclusion: Geometrical Transition
Small *vs.* **Large Impact Parameter**

Inelastic Amplitude

- **Exchanged quarks:** **τ-trajectory on "helicoid"**

$$\int \mathcal{D}\tau \; \langle W(1 \to 3' \to 4' \to 2' \to 1') \rangle_{\mathcal{A}} [\tau] \; e^{-2m Length[\tau]}$$





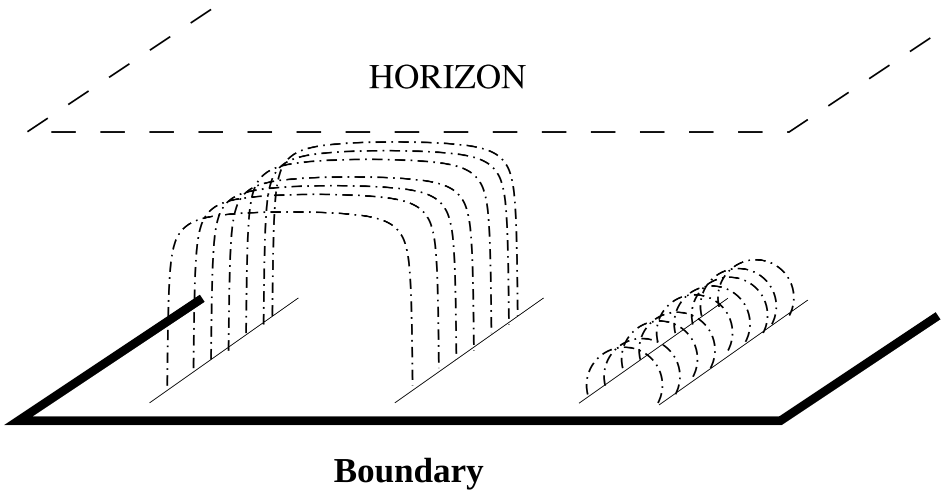
- **Inserting AdS/CFT:** **Summing over $\mathcal{A}$, Gauge d.o.f.**

$$\langle W(1 \to 3' \to 4' \to 2' \to 1') \rangle_{\mathcal{A}} = e^{-\frac{1}{2\pi\alpha'} Area[\tau]}$$

$\Rightarrow$Helicoidal Geometry with boundary

Dual Calculations:

- **Confining** *vs.* **non-Confining Geometry:**

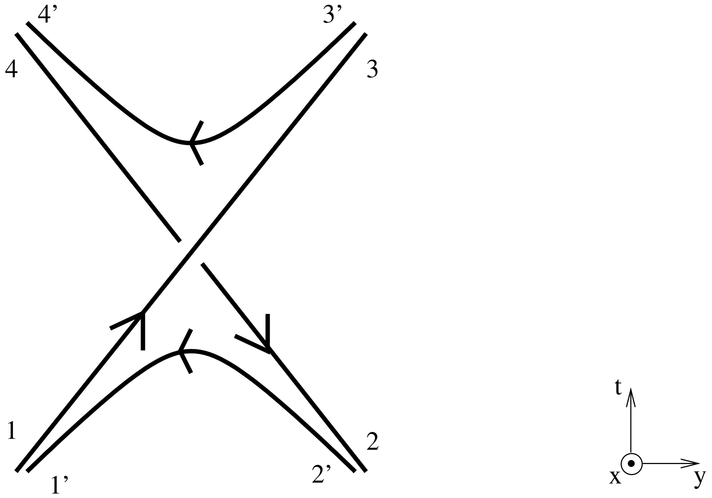


- **Confining Geometry:** $\approx$ **Classical Helicoid**

$$\begin{aligned} t &= \tau \cos \theta\sigma / L \\ y &= \tau \sin \theta\sigma / L \\ x &= \sigma \end{aligned}$$

More on Inelastic (1)

$$(11') + (22') \longrightarrow (33') + (44')$$



$$A_{\mathcal{R}}(s, L^2) \propto \langle <\!out| S_F(3', 4') S_F(1,3) S_F(4,2) S_F(2', 1') |in\!> \rangle_{\mathcal{A}}$$

- **1^{rst} order Path Integral**

$$S_F(X,Y) = \int_0^\infty dT e^{-mT} \int \mathcal{D}X^\mu(\tau) \delta(\dot{X}^2 - 1)\, I[X^\mu(\tau)]\, W_{\mathcal{A}} \dot{X}^\mu d\tau$$

- **Fermionic Spin Factor**

$$I[X^\mu(\tau)] = P \prod \frac{1 + \dot{X}^\mu \gamma_\mu}{2} = \lim_{N \to \infty} \frac{1 + \dot{X}^\mu(T)\gamma_\mu}{2} \ \ldots$$

$$\ldots \ \frac{1 + \dot{X}^\mu(\frac{2}{N}T)\gamma_\mu}{2} \frac{1 + \dot{X}^\mu(\frac{1}{N}T)\gamma_\mu}{2}$$

More on Inelastic (2)

Technics

- **Fermionic Spin Factor:** $\tau(\sigma) \subset$ **3D**

$$I[\dot{X}] \rightarrow \frac{1+\dot{X}^{\mu}(T)\gamma^{\mu}}{2}\frac{1+\dot{X}^{\mu}(0)\gamma^{\mu}}{2}\cdot\left(\frac{1+\dot{X}(T)\cdot\dot{X}(0)}{2}\right)^{-1} \Rightarrow \frac{1}{s}$$

- **Minimization: Euler-Lagrange**

$$A_{\mathcal{R}}(s,L^2) = \lim_{\alpha'\rightarrow 0}\int \mathcal{D}\tau\, e^{-\frac{1}{2\pi\alpha'}Area[\tau]}\, e^{-2mLength[\tau]}$$

$$\frac{\partial(Area-2mLength)}{\partial\tau} = 0 \Rightarrow \sqrt{1+\left(\frac{\theta\tau}{L}\right)^2} = \frac{\partial\tau(\sigma)}{\partial\sigma} = 0$$

$$\tau(\sigma)_{min} \equiv \mathcal{T} = \pm iL/\theta$$

- **Minimal Area: Analytically Continued**

$$e^{-\frac{1}{2\pi\alpha'_{eff}}Area[-iL/\theta]} = e^{-\frac{iL^2}{4\alpha'_{eff}\theta}} \longrightarrow e^{-\frac{L^2}{4\alpha'_{eff}\chi}}$$

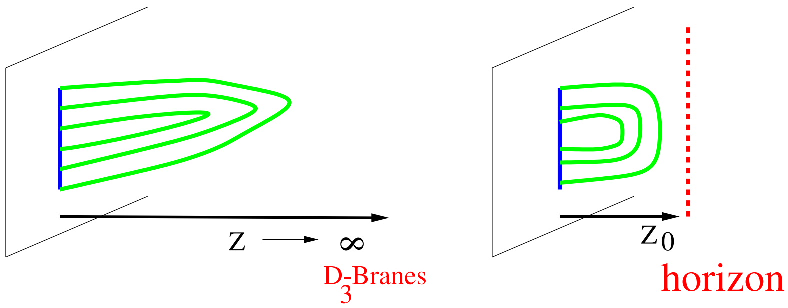
- **Reggeon Amplitude: Classical Approx.**

$$A_{\mathcal{R}}(s,q^2) \propto s^{-\alpha'_{eff}q^2}$$

Linear Regge trajectory: $\alpha'_{eff} \hookrightarrow$ **potential q-q; 'classical' 0 intercept**

More on Inelastic (3)

- **Origin of Reggeization**

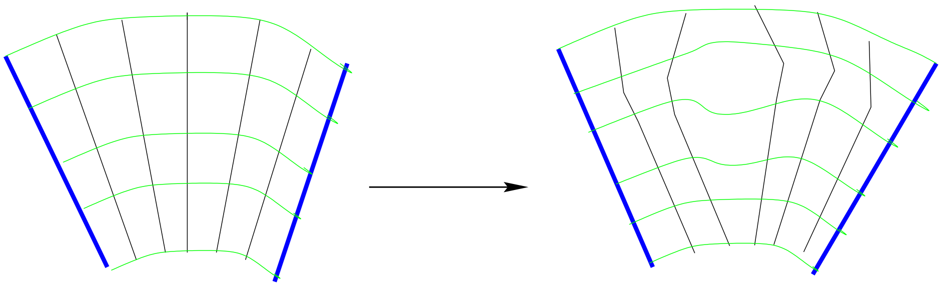


$$Area^{AdS}_{min} = \lim_{\chi\to\infty} L/(L/\chi) \qquad Area^{BH}_{min} = \lim L \times (L/\chi)$$

Maths : Minimal Surfaces in AdS, AdS/BH, ...?

Beyond Classical AdS/CFT

Fluctuations



- **Fluctuations: "universal" prefactor**

$$e^{-\frac{1}{2\pi\alpha'_{eff}}\left\{Area+[L]^0\alpha'_{eff}+[L]^{-2}\alpha'^2_{eff}\cdots\right\}} = A_{class.}(s,L^2)\times e^{-[L]^0}\times\ldots$$

- **Exemple: Lüscher term**

$$\langle W(T)\rangle_{\mathcal{A}} = e^{TV(L)} \sim e^{-\frac{TL}{2\pi\alpha'_{eff}}}\cdot e^{\frac{\pi n_\perp}{24}\frac{T}{L}}\ ;\ n_\perp \equiv \#(0{-}modes_\perp)$$

- **Critical String : flat metric on helicoid**

$$\{\sigma,\tau \Rightarrow \left\{\sigma,\rho = \frac{L}{\theta}\log\left[\frac{\theta\tau}{L}+\sqrt{1+\left(\frac{\theta\tau}{L}\right)^2}\right]\right\} \in (\pm L/2, \pm\rho(\mathcal{T}))$$

Results

- **Bosonic 0-modes : Polyakov action**

$$(\det \Delta)^{-\frac{D-2}{2}} = (\det \Delta)^{-\frac{n_\perp}{2}} \quad ; \Delta \equiv \mathit{Laplacian}$$

- **Fluctuation Factors: Inelastic case**

$$Fluct.(\rho(\mathcal{T}) = -i\pi/2) \; = \; e^{\frac{n_\perp}{24} i\theta} \rightarrow_{(i\theta \rightarrow \chi)} \; s^{\frac{n_\perp}{24}}$$

Elastic case

$$Fluct.(\rho(\mathcal{T}) = -2i\pi) \; = \; e^{\frac{n_\perp}{96} i\theta} \rightarrow_{(i\theta \rightarrow \chi)} \; s^{\frac{n_\perp}{96}}$$

- **Amplitudes** **Inelastic case**

$$\Rightarrow A_{\mathcal{R}}(s, q^2) \propto s^{\frac{n_\perp}{24} - \alpha'_{eff} q^2}$$

Elastic case

$$\Rightarrow A_{\mathcal{P}}(s, L^2) \sim s^{1 + \frac{n_\perp}{96} - \frac{\alpha'_{eff}}{4} q^2}$$

- **AdS-BH/CFT**

$n_\perp = 7, 8 \;\Rightarrow \alpha_{\mathcal{R}}(0) \sim .3 \;;\; \alpha_{\mathcal{P}}(0) \sim .08$

Conclusions

The 35-years-old Puzzle

- **String dimensionality:** $D \to 4 + 1 + 5$
- **Gravitation : Decoupled**
- **Phenomenology:** $\alpha_{\mathcal{R}} = 0 + \frac{n_\perp}{24}$, $\alpha_{\mathcal{P}} = 1 + \frac{n_\perp}{96}$

Some Problems ahead

- **Flavor/Spin dependence**
- **Prefactors**
- **Unitarity**

Deep Problems ahead

- **Dual of QCD?**
- **Horizon** $\longleftrightarrow \Lambda_{QCD}$**?**
- **Strings in AdS backgrounds**
- **Proof of Duality Conjecture:**

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